Examples of Partials for the $e^{f(x,y)}$ Function

From MyMathLab

Example 1
If

$$f = y^9 e^{xy^7}$$

Find f_x and f_y

For f_x there is only ONE x in the expression, so just a chain rule using this rule

$$\begin{aligned} z &= e^{f(x,y)} \\ \frac{\partial z}{\partial x} &= e^{f(x,y)} \cdot f_x \end{aligned}$$

Treat the other variables as constants

$$\begin{aligned} f_x &= y^9 e^{xy^7}(y^7) \\ &= y^{16} e^{xy^7} \end{aligned}$$

note, the derivative of xy^7 with respect to x is y^7 and the derivative of xy^7 with respect to y is $7xy^6$

For f_y, you use the product rule since y appears twice

$$\begin{aligned} f &= \Big(\underbrace{y^9}_{g(x,y)}\Big)\Big(\underbrace{e^{xy^7}}_{h(x,y)}\Big) \\ f_y &= g_y h + g h_y \\ f_y &= 9y^8 e^{xy^7} + y^9\left(e^{xy^7} 7xy^6\right) \\ f_y &= \left(9y^8 + 7xy^{15}\right) e^{xy^7} \end{aligned}$$

Example 2

Finding the CROSS Partials (f_{xy}, f_{yx}). Note that Cross Partials are a type of SECOND derivative

If

$$f(x,y) = e^{4xy^2}$$

Then the first partial derivatives are

$$\begin{aligned} f_x &= e^{4xy^2}(4y^2) = 4y^2 e^{4xy^2} \\ f_y &= e^{4xy^2}(8xy) \end{aligned}$$

(note that the derivative of $4xy^2$ with respect to x is $4y^2$ and the derivative of $4xy^2$ with respect to y is $8xy$

To find f_{xy} you take the derivative of the function f_x with respect to y

i.e.

$$f_{xy} = \frac{\partial(f_x)}{\partial y} = \frac{\partial\left(4y^2 e^{4xy^2}\right)}{\partial y}$$

Since the function $f_x = 4y^2 e^{4xy^2}$ has the letter Y in two places, this is a product rule

$$f_x = \overset{g}{\left(4y^2\right)} \overset{h}{\left(e^{4xy^2}\right)}$$

where

$$\begin{aligned} g_y &= 8y \\ h_y &= e^{4xy^2}(8xy) \end{aligned}$$

and the product rule is

$$\begin{aligned} f_{xy} &= g_y \times h + g \times h_y \\ f_{xy} &= (8y) \times \left(e^{4xy^2}\right) + \left(4y^2\right) \times \left(e^{4xy^2} 8xy\right) \\ f_{xy} &= 8ye^{4xy^2} + 32xy^3 e^{4xy^2} \end{aligned}$$

Similarly, we can find f_{yx}

$$\frac{\partial f_y}{\partial x} = \frac{\partial\left(e^{4xy^2}(8xy)\right)}{\partial x}$$

This also requires the product rule

$$f_y = \overset{j}{e^{4xy^2}} \overset{k}{(8xy)}$$

$$\begin{aligned} f_{yx} &= j_x \times k + j \times k_x \\ f_{yx} &= \left(e^{4xy^2}(4y^2)\right) \times (8xy) + \left(e^{4xy^2}\right) \times (8y) \\ f_{yx} &= 32xy^3 e^{4xy^2} + 8ye^{4xy^2} \end{aligned}$$

Note that MyMathlab has a glitch when it comes to this question. It may mark you wrong when you are right.

NOTE: you can check using the rule $f_{yx} = f_{xy}$. Do it both ways and see if you get the same answer