

Here is how to do this type of two variable cubic

Given

$$f(x, y) = x^3 - 18xy + y^3$$

The FOC's are

$$f_x = 3x^2 - 18y = 0$$

$$f_x = x^2 - 6y = 0$$

and

$$f_y = 3y^2 - 18x = 0$$

$$f_y = y^2 - 6x = 0$$

Re-write f_x as

$$x^2 = 6y$$

or

$$\frac{x^2}{y} = 6$$

and f_y as

$$y^2 = 6x$$

or

$$\frac{y^2}{x} = 6$$

Since they both equal 6, then it is true that

$$\frac{x^2}{y} = \frac{y^2}{x}$$

or

$$x^3 = y^3$$

$$x = y$$

Use the fact that $x = y$ and f_x to solve for x

$$x^2 - 6(\overset{=y}{x}) = 0$$

$$(x - 6)x = 0$$

This gives two solutions: $x = 0, x = 6$. Since $x = y$ this means two pairs of solutions

$$x = 0, \quad y = 0$$

and

$$x = 6, \quad y = 6$$