Chapter 0: Functions Review

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1 Equations Review

1.1 Equations

In mathematics, an equation is used to express equalities, in the form of

$$A = B \tag{1}$$

,where A and B can be expressions containing one single, or a combination of variables. numbers, and functions. We have lots of experiences with equations containing only numbers since pre-school. An example of this type of equation is

$$6 - 2 = 4 \tag{2}$$

An algebraic, or polynomial equation is no different. Simply, instead of having numbers like equation (2), A and B are allowed to contain one or more variables. We have some experience with these types of equations, too. For example

$$2 + x = 6 \tag{3}$$

One of the many useful properties of an equation is that you can do the same operation on both sides of the equation sign, and the equality will still hold. Let's test this out with a simple example

$$5 + 6 = 11 \tag{4}$$

$$(5 + 6) + 2 =?11 + 2 \tag{5}$$

Test this out by evaluating both sides of the equation.

Because the equal sign means that the expressions on the two sides are the same, doing the same operation on two identical objects will yeild the same result. With that in mind, we can extend this property to equations with terms containing other entries, such as variables

$$2 + x = 6 \tag{6}$$

$$(2 + x) - 2 = (6) - 2 \tag{7}$$

$$x = 4 \tag{8}$$

As demonstrated in the example above, we can use this equality under operation to find the values of variable in equations.

This process of finding the value of unknown variables is called "solving equations". Namely, in this example, we have *solved for x in the equation* $2x = 6$. Here are some more examples.

Example 0.1: Solve the equation for y.

$$3y = 12 \tag{9}$$

$$(3y)/3 = (12)/3 \tag{10}$$

$$y = 4 \tag{11}$$

Example 0.2: Solve the equation for a.

$$6a + 8 = 12 \tag{12}$$

$$6a = 12 - 8 \tag{13}$$

$$6a = 4 \tag{14}$$

$$a = \frac{2}{3} \tag{15}$$

There are many ways to solve an equation, which you will develop through this course by solving practice problems.

The ability to solve polynomial equations is fundamental to almost all content of this course. Let's try some practice questions before moving on to the next concept. (Exercise 1)

1.2 Inequalities

Once we have mastered the art of solving linear, single variable equations, we shall look into a slightly more difficult, yet still simple, relationship: inequalities. For now, let's just take the rules for granted. To solve an inequality, isolate the variable by manipulating the inequality the same way as manipulating equations, with one exception: flip the direction of the inequality if you multiply or divide both sides by a negative number. And, obviously, you can not multiply both sides by 0. Here is an example.

Example 0.3: Solve the inequality for y.

$$-3y < 12 \tag{16}$$

$$(-3y)/(-3) > 12/(-3) \tag{17}$$

$$y > 4 \tag{18}$$

Think about why the sign flips when you multiply the equation by a negative number. We'll talk more about inequalities in Chapter 5. For now, just do some practice problems. (Exercise 2)

2 First Order (Linear) Functions

A **function** expresses the relationship between an input, the **independent variable**, and the output, the **dependent variable**. Intuitively, we can picture a function to be a machine, which take a value of the independent variable, performs some operation, and returns the value of the dependent variable. Since the operation will be the same every time, there can only be one output for an input. By convention, a function is most commonly referred to y or f(x), where the x inside the bracket is the independent variable. In this chapter, we will only focus on first order equations with one independent variable.

Algebraically, functions look very similar to equations. Let's look at an example of a linear function.

$$f(x) = 3x + 1 \tag{19}$$

$$g(x) = 3x + 1 \tag{20}$$

$$y(x) = 3x + 1 \tag{21}$$

For the students who just started learning functions, it is very important not to be intimidated by the different names of a function. Regardless of whether the function is called $f(x)$, $g(x)$, or $y(x)$, they all describe the same relationship between the input x, and the output function. The independent variable doesn't have to be x either: it is allowed to be any variable. One can name his function $banana(timbit)$, and $banana(timbit)$ can be treated just the same way as the good old $f(x)$.

2.1 The standard form of a linear function

As we have seen in equations (19)-(21), there are 4 components in a typical linear function. There's the dependent variable (y), the coefficient of the independent variable (3), the independent variable (x), and a constant (1). We can generalize a function like this into:

$$y = mx + b \tag{22}$$

, where m and b are constants (numbers).

Equation (22) is called the standard form of a linear function. We can rearrange any first order functions into this standard form. In the standard form, m is called the slope, and b is called the y-intercept. We will see why they are named like this when we start graphing linear functions.

2.2 Graphing linear functions

Graphing, or sometimes referred to plotting, functions is a commonly used way to visualize how the functions behave in an interval. In this section we will only concern graphing linear functions. This is reasonably simple because it only takes 2 points to define a line. Let's start with an example.

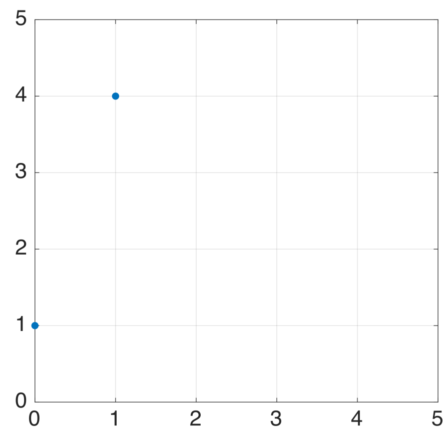
Example 0.4: Graph the function in equation (21).

All we have to do is to find 2 points that satisfies $y = 3x + 1$. In order to do this, we can substitute x with any 2 values. In the sake of reducing the amount of calculation, I will choose x to be 0 and 1.

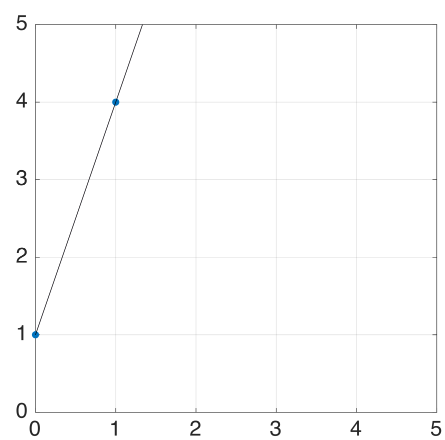
x	y
0	1
1	4

I will leave you to verify the computation for y. Please do so as an exercise for substitution.

Now, let's plot the 2 points.



We then connect the 2 points, and there we get our graph for $y = 3x + 1$



Examine the graph and think about why m and b are called the slope and the y-intercept respectively.

2.3 Application of linear functions

Linear functions naturally arise from everyday problems. Conversion between Celsius and Fahrenheit, calculating wages, and Newton's Second Law are all examples that require establishing a linear relationship between 2 variables. We will explore these scenarios in more detail in the following example.

Example 0.5: We know that Celsius and Fahrenheit are related linearly. When it's 0 Celsius, it's 32 Fahrenheit; when it's 20 Celsius, it's 68 Fahrenheit. Find the degree in Fahrenheit, f, as a function of Celsius, c.

Because Celsius (c), and Fahrenheit (f) are related linearly, we can put the 2 variables in the standard form just like equation (22):

$$f = mc + b \tag{23}$$

We then substitute the 2 known values into the standard form, and get the following equations:

$$32 = 0m + b \tag{24}$$

$$68 = 20m + b \tag{25}$$

By simplifying equation (24), we get $b = 32$.

By substituting $b = 32$ into equation (25), we get $m = 1.8$

Therefore

$$f = 1.8c + 32 \tag{26}$$

The function $f = 1.8c + 32$ gives the relationship between Fahrenheit and Celsius.

It's important to note that since this is a equation of application, we should complete our answer with a brief conclusion of the result.

Graph this function for your own practice.