Due: Friday, November 29th 11:59 p.m.

Problems for Math 408 and Math 708

1. You have now found several classes of inequalities for your team multi-constrained knapsack problem. These are:

   a. Non-negativity constraints, $x_i \geq 0$.

   b. Variable upper bounds, $x_i \leq 1$.

   c. The initial knapsack constraints from the 4 students in your group.

   d. Gomory cuts (2) from Team Homework 3.

   e. Cover inequalities (3 per person) from Individual and Team Homework 4.

   f. Lifted cover inequalities (1 per person) from Team Homework 4.

   g. Targeted cover inequalities (1 per person who submitted Individual Homework 5), which may have been extended and lifted, from Individual Homework 5.

Make a table that shows the optimal solution and objective values of:

- The initial team multi-constrained knapsack problem relaxation (from Team Homework 1).
- The relaxation given by adding each single cut (alone) from d., e., f., and g. above to this initial multi-constrained knapsack problem relaxation.
- The relaxation given by adding all cuts listed above to the initial multi-constrained knapsack problem relaxation.
- The optimal integer solution (from Team Homework 1).

2. Examine the optimal solution to the relaxation given by adding all cuts listed in question 1 to the initial multi-constrained knapsack problem relaxation. Which inequalities from your table hold with equality at the this solution?

3. Returning to your team routing problem, add upper bounds $x_{ij} + x_{ji} \leq 1$ for pairs of opposite edges to the relaxation that you solved in question 5 of the previous team assignment. This will eliminate any 2-cycles that appear in the previous solution, and perhaps also cause it to be non-integral as the constraint matrix is no longer T.U.M. Solve this relaxation in `AMPL`. Illustrate this relaxed solution by showing edges with weight 1 in black, and fractional edges in a different colour.

4. Can you find a subtour inequality that is not satisfied by your relaxed solution? If so, add it to your formulation and resolve. If you can, repeat this process up to 2 times.

5. You can also ask `AMPL` to try to solve the above relaxations using integer variables. This may give you a tour, in which case you it is optimal for your problem. More likely, there will be subtours, which you can eliminate with subtour elimination constraints. Do this until you have solved your problem, or added at least 3 subtour constraints.

6. Make a table with rows corresponding to:

   a. The degree constraints relaxation from the previous assignment.

   b. The degree constraints relaxation with 2-cycle elimination constraints from question 3.

   c. The sequence of improved relaxations that you get by adding subtour constraints in questions 4 and 5.

For each row show the value of the LP relaxation of your formulation, along with the value you get from adding integrality constraints to the formulation.

How close are these values to the shortest heuristic tour you discussed in question 4 of the third team homework assignment?

This assignment will be submitted directly to the instructor by e-mail. Please submit a single file named `team_hw5_name.pdf` containing all your written work, with your group identifier substituted in place of `name`. Please also send the `AMPL` files corresponding to the tightest relaxation that you found in question 5, as `team_hw5_name.mod` and `team_hw5_name.dat`.