Due: Friday, November 22nd (11:59 p.m.)

The final exam will take place Friday, December 6th at 3:30 p.m. in SUR 5100.

Reading

Chapter 8 through Section 8.2

Chapter 4 of Applegate, Bixby, Chvátal and Cook reviews some topics that we have seen in class, and a few that we haven't, in their historical context.

Problems for Math 408 and Math 708

Solutions to be submitted `Crowdmark`.

1. Returning once more to your personal knapsack problem, or your amended personal knapsack problem in case your personal knapsack relaxation was integer, you should see that your relaxed optimal solution has exactly one non-integer coordinate.

   a. Explain why in this situation, the cover inequality corresponding to the non-zero terms in your solution will cut your relaxed optimal solution.

   b. Add this new cover cut to your formulation and resolve.

   c. What is the dimension of the new face that you have found?

2. Consider the following 0-1 knapsack polyhedron:

$$X = \{\mathbf{x} \in \{0,1\}^6 \mid 5x_1 + 3x_2 + 8x_3 + 9x_4 + 11x_5 + 8x_6 \leq 14\}.$$

   a. What is the cover inequality corresponding to variables $\{1,2,3\}$?

   b. What is the dimension of the face of $P_I = \text{conv}(X)$ represented by this cover inequality?

   c. Lift the inequality you found in part (a) in variable 6, and then lift the resulting inequality in variable 5.

   d. Lift the inequality you found in part (a) in variable 5, and then lift the resulting inequality in variable 6.

3. If C is a cover for a 0-1 knapsack inequality, the *extended cover* $E(C)$ of C is $C \cup \{j \mid a_j \geq a_i \text{ for all } i \in C\}$.

   a. Show that if C is a cover for a given 0-1 knapsack inequality, then the *extended cover inequality* given by $\sum_{i \in E(C)} x_i \leq |C| - 1$ is valid for $\text{conv}(S)$, the set of integer solutions to the 0-1 knapsack problem.

   b. Can your cover inequality from question 1 be replaced by an extended cover inequality?

   c. For each variable that is not in your extended cover inequality (or simply your cover inequality if it did not extend), try to lift the inequality in that variable. If you succeed in lifting a variable, if you like, you can work with the new inequality for subsequent liftings. This will lead to stronger cuts, but requires solving more complicated knapsack subproblems.

   d. Compute the dimension of any new cuts you have found.

4. List the feasible (integer) points $(\vec{x}, \vec{u})$ to the Miller-Tucker-Zemlin formulation of the TSP (pages 62-63 of the text) on the complete directed graph with 4 vertices. You should take the variables u_i for $i = 2, 3, 4$ to be integers between 2 and 4.

5. Write an integer programming relaxation of the asymmetric travelling salesman on 6 vertices with degree constraints and a single subtour elimination constraint corresponding to vertices 1, 2 and 3. Then find an integer point satisfying these constraints that does not represent a tour.

Additional Problems for Math 708

6. Textbook Exercise 3.13.

7. Consider the problem of finding a maximum stable set of a graph (a maximum set of vertices with no two vertices sharing an edge). Formulate this problem as:

$$\max \sum_{v \in V} x_v \quad \text{subject to} \quad x_{v_1} + x_{v_2} \leq 1 \quad \forall (v_1, v_2) \in E \quad \text{and} \quad x \in \{0,1\}^{|V|}$$

Show that for any complete subgraph (*clique*) W of G, you can obtain the clique inequality $\sum_{v \in W} x_v \leq 1$ by repeatedly applying rounding cuts.

8. Compute the Chvátal closure of $P := \{(x, y) \in [0, 1]^2 \mid 2x + 2y \leq 3\}$.

Graduate projects and presentations

We will have graduate student presentations on Monday, December 2nd and possibly on Wednesday, November 27th.

For graduate students who are doing option B (writing introduction and review of a recent research paper). This should include a brief review of any necessary background beyond what is covered in this course. You should explain the main results of the paper, framing them in terms of recent results, and mention potential future research directions (which may already be past research directions if the paper is a few years old).

Try to do this within 10 pages, with reasonable spacing and margins. You should include a bibliography.