Due: Thursday, October 3rd (4:30 p.m.)

Reading

From the textbook, Sections 3.3 (should be review), 4.1, 4.2 and the introduction to Section 4.3.

Chapters 3 and 4 of the AMPL book, available at:

`http://ampl.com/resources/the-ampl-book/chapter-downloads/`

The remainder of Chapter 1 of Applegate, Bixby, Chvátal and Cook.

Problems for Math 408 and Math 708

Written solutions will submitted via `Crowdmark`. For question 4, where `AMPL` code is requested, submit files `hw2.dat` and `hw2.mod` to `Canvas`. Please make sure to write your name on the first page of the assignment and in the comments of `hw2.dat` and `hw2.mod`.

1. Consider the personal knapsack problem that you made in the first homework assignment. Look at the LP relaxation from part 1 b. If the LP relaxation is an integer vector, then before proceeding you will add 1 to the right hand side of your knapsack constraint and work with your *amended personal knapsack problem* instead of the original problem that you were working with.

At this point you should be working with a problem whose LP relaxation is not integral. Perform one step of branch-and-bound on this problem. That is, branch on a non-integer variable to get two subproblems. For each of the two subproblems:

a. Construct a primal (lower) bound for the optimum using the following greedy algorithm: begin with the feasible point $\vec{x} = 0$. For each i from 1 to 9 in turn, see if the point remains feasible if x_i is set to 1. If it is, then keep $x_i = 1$. Otherwise, reset to $x_i = 0$.

b. Construct a dual (upper) bound for the optimum by solving the LP relaxation of the problem, where $\vec{0} \leq \vec{x} \leq \vec{1}$.

c. Find the optimal (integer) solution to the subproblem using AMPL. Compare this to the two bounds you have found for that subproblem. You do not submit the AMPL files.

2. Solve your [amended] personal knapsack problem using branch-and-bound. You don't have to give full details, but draw the branch-and-bound tree as is done in Figure 1.4 in the textbook.

Note that you can use the dual bound from question 1a. to help prune the tree. If you have expanded at least 15 nodes without verifying optimality, that is sufficient – you can stop there. In this case note the best solution found and the gap to optimality.

3. Textbook Exercise 1.4.

4. Exercise 20-1 from the AMPL book. (It references Exercise 1-1, but doesn't require it.) Submit the only the AMPL files for part (b), you do not submit the modified files related to the relaxation.

5. Textbook Exercise 4.1.

Additional Problems for Math 708

6. Textbook Exercise 2.13.

7. Textbook Exercise 4.7.

8. Consider modelling a scheduling problem where machine can be switched on at most k times, with discrete time segments indexed by t. This problem can be modelled using variables y_t representing whether the machine is on during period t, and z_t which representing whether the switching on happened during period t. This can be formulated via the following inequalities:

$$y_t - y_{t-1} \leq z_t \leq y_t \text{ for all } t; \qquad \sum_t z_t \leq k; \qquad 0 \leq y_t, z_t \leq 1 \text{ for all } t.$$

Show that the matrix encoding these constraints is totally unimodular.

Graduate Student Projects

Math 708 students will do a project, choosing from the following three plans in consultation with the instructor:

A. Present and introduction to an additional topic in discrete optimization. The presentation will take place at the end of the semester, either in class or in the Operations Research Seminar. Presentations will last for 30 minutes, followed by a 5 minute question period. Well prepared slides will be submitted as part of the evaluation.

Possible sources for topics include sections of the textbook not covered in class, and the book *Algebraic and Geometric Ideas in the Theory of Discrete Optimization* by De Loera, Hemmecke and Köppe or the surveys from *50 Years of Integer Programming 1958-2008* by Jünger.

B. Write an introduction and review of a recent (in the past 10 years) research paper. This should include necessary background (what is needed to understand the paper beyond what is in this course), context in terms of recent results, and a discussion of importance and future research directions.

C. Use the methods of this course to model and (attempt to) solve a challenging discrete optimization problem.

In each case, students must consult with the instructor about topics to confirm suitability and avoid overlap with other students.

Students can choose to do the projects by themselves or in pairs. For projects of type A, two person presentations will last for 50 minutes with each student speaking for about half the time on a common topic.