Due: ~~Wednesday, October 25th~~ Thursday, October 26th (9:30 a.m. PT.)

Notes

There will be no class on Mondays October 9th. However, Tuesday, October 10th will be treated as Monday: class and office hour will meet, but there will not be a tutorial.

The first midterm is on Wednesday, October 11th in class, and will cover material covered through the end of class on Friday, October 6th.

Reading

For Friday, October 6th, through Section 3.6 of *Probability Models*.

For Tuesday, October 9th, Section 3.1 of *Simulation*.

For Friday, October 13th, Section 3.2 of *Simulation*.

For Monday, October 16th, Section 4.1 of *Simulation*.

For Wednesday, October 18th, Sections 4.1 and 4.2 of *Probability Models*.

For Friday, October 20th, Section 4.3 of *Probability Models*.

For Monday, October 23rd, Section 4.4 of *Probability Models*.

Assignment exercises to hand in

1. Using $m = 2^{31} - 1$, $a = 7^5$ and x_0 set to your student id, generate ten $[0, 1]$ numbers via a linear congruential generator. Round to the nearest thousandth.
2. Student ids begin with the digits 301. Do you expect that you answer to question 1 is that same as for other students? Why or why not?
3. Repeat question 1 using x_0 set to the last six digits of your student number.
4. Assuming that the last six digits of student numbers are uniformly random (unlikely), explain how you expect this sequence to compare to other students' sequences?
5. Repeat question 3, but use the value of x_{10} generated from the last six digits of your student number, rather than the digits themselves.
6. Repeat question 4 for the new sequences generated.
7. Explain how to use the numbers you generated in questions 1, 3 and 5 and the inverse transform method to generate a "random" graph on 10 vertices per the random graph model presented in *Probability Models* Section 3.6.2.
8. Generate three such graphs, one corresponding to each of the sequences you generated.
9. Explain how to use a random number generator and the inverse transform method to generate a random graph on 5 vertices per the Erdős-Rényi random graph model with edge probability p.
10. Using your answer to question 5, generate such random graphs for $p = 0.2$, $p = 0.5$ and $p = 0.8$.

11. *Simulation*, Chapter 3, Exercise 3. Try computations using each of 10, 100, 1000 and 10000 pairs of points. (Note this will require a small amount of computer programming.)

12. Take the digits of your student id, in order: a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8 and a_9. Your *personal Markov Chain* tracks your frame of mind from hour to hour. You have three states: relaxed (state 0), busy (state 1) and asleep (state 2). You personal transition matrix is given by:

$$\begin{pmatrix} \frac{a_1}{a_1+a_2+a_3} & \frac{a_2}{a_1+a_2+a_3} & \frac{a_3}{a_1+a_2+a_3} \\ \frac{a_4}{a_4+a_5+a_6} & \frac{a_5}{a_4+a_5+a_6} & \frac{a_6}{a_4+a_5+a_6} \\ \frac{a_7}{a_7+a_8+a_9} & \frac{a_8}{a_7+a_8+a_9} & \frac{a_9}{a_7+a_8+a_9} \end{pmatrix}$$

Verify that this a transition probability matrix.

13. Suppose that you are relaxed at a given moment. What are the probabilities that you are relaxed, busy and asleep, respectively, three hours later?

14. In the long run, what fraction of the time are you asleep?

15. Which of your states are recurrent, and which are transient?

16. Is your Markov chain reducible or irreducible?

17. Suppose that Lady MacBeth is not able to sleep. What do you expect is true of her student number?

18. Suppose that Rip Van Winkle sleeps for 20 years, after which SFU retires his student number, releasing him from his personal Markov chain. What do you expect was true of his student number?