CHAPTER 11 Logical Form and Sentential Logic

Abstracting from the content of an argument reveals the logical form of the argument. The initial sections of this chapter show that logical form is the key to the validity of deductively valid arguments. The chapter then explores sentential logic, the logic of sentences. Finally, the chapter investigates the tricky terms *only*, *only if* and *unless*. To understand reasoning that uses these terms, many persons are forced to hire a lawyer. You, however, will be one step closer to becoming their lawyer.

Logical Equivalence

If you were told, "John stepped on the camera by accident," you wouldn't have learned anything different from having been told "The camera was accidentally stepped on by John." These sentences say the same thing — they make the same statements — even though they are grammatically different. Because the two say the same thing logically, they are said to be *equivalent*, or, more technically, *logically equivalent*. Logical equivalence is somewhat like synonymy except that it is for sentences, not words. That phrase *say the same thing* is a bit imprecise, so here is a definition using more precise terminology:

Definition Statement P is **logically equivalent** to statement Q provided P follows from Q with certainty and Q also follows from P with certainty.

The certainty mentioned here is not a psychological notion; it is a logical notion. That is, the certainty is not about feeling sure but instead about the solidity of the logical relationship of support among statements.

Alternative Definition Statement P is **logically equivalent** to statement Q provided P logically implies Q and also Q logically implies P.

Here is a pair of logically equivalent statements:

Tiffany is so sincere, you can't doubt her.
The sincerity of Tiffany cannot be doubted.

Yet these two are not logically equivalent:

Tiffany got married and got pregnant.
Tiffany got pregnant and got married.

Time order is the problem.

Here is a much less obvious example of logical equivalence. Suppose P is the sentence "Not all mammals are land dwellers," and Q is the sentence "Some mammals are not creatures that live on land." Does Q follow from P with certainty? Yes. How about vice versa? Yes. So P and Q are logically equivalent. This relationship between the two sentences would hold even if the word *mammal* were replaced by the phrase *fish in the Indian Ocean*. Consequently, logical equivalence between two sentences can be a matter of the **form** of the two sentences, not just what they are about.

———CONCEPT CHECK———

Does the definition of logical equivalence permit a true sentence and a false sentence to be logically equivalent to each other?

———[250]

Deciding whether two phrases are logically equivalent[251] can be critical in assessing the quality of an argument. Here is an example involving an argument in which the conclusion follows from the premises with certainty:

1. If the attraction that baseball has will persist in America over the next decade, then our income from concessions will also remain steady over the next decade.

250 Don't rush to look at the answer before thinking seriously about the question. The answer is in the next footnote.

251 No, if one is true and the other is false in the same circumstances, they must be saying something different from each other and thus cannot be logically equivalent.

2. I know the attraction that baseball has will in fact persist in America over the next decade.

3. So, our income from concessions will remain steady over the next decade.

Would the conclusion still follow if premise 2 were replaced with the following statement?

2′. Baseball will continue to flourish in America over the next ten years.

It depends. If statement 2' is logically equivalent to statement 2, then the conclusion would still follow with certainty. However, if you cannot be sure they are equivalent, you cannot be sure the conclusion of the argument with 2′ follows with certainty. To decide whether 2 and 2' are equivalent, you should be sensitive to context and use the principle of charity. If, after doing all this, you still cannot tell whether 2 and 2' are equivalent, and if you need to be sure, you will have to ask the speaker or author to be clearer.

———CONCEPT CHECK———

Does the conclusion follow with certainty from the premises in this argument? Explain why a simple "yes" or "no" answer is unacceptable because of logical equivalence.

> If the latest version of the word processing program Word is warmly received on presentation, its owners and programmers are going to be happy about what they created. All reports indicate Word did hit the market with a splash and got many good reviews. So we can conclude that WordPerfect's creators felt a sense of accomplishment.

———[252]

The concept of logical equivalence is useful in other ways. This usefulness arises from the fact that the deductive validity or invalidity of an argument usually depends on the logical forms of its sentences, as we will see later in this chapter. In turn, the ability to identify the logical forms of sentences requires the ability to translate the sentence into a logically equivalent one.

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252 The argument's conclusion follows with certainty from its premises if the principle of charity permits us to say that "warmly received on presentation" means the same as "hit the market with a splash," and if it also permits us to say "happiness" here is the same as "feeling a sense of accomplishment" and if the "owners and programmers" include the "creators." It is likely that these equivalences hold, but you cannot be sure and therefore cannot definitely say, "Yes, the conclusion follows with certainty." If you needed to be sure, you should ask the author to be clearer about all this.

Logical Forms of Statements and Arguments

The logical form of an argument is composed from the logical forms of its component statements or sentences. These logical forms are especially helpful for assessing the validity of *deductive* arguments. For instance, consider the following argument, which is in standard form:

> If all crystals are hard, then diamond crystals are hard.
> Diamond crystals are hard.
> ______________________________
> All crystals are hard.

This is a deductively invalid argument, but it can be difficult to see that this is the case. The difficulty arises from the fact that the conclusion is true and all the argument's premises are true. One way to detect the invalidity is to abstract away from the content of the argument and to focus at a more general level on the form of the argument. The argument has this **logical form**:

> If Cryst, then Diam.
> Diam.
> ______________
> Cryst.

The term *Cryst* abbreviates the clause "All crystals are hard." The term *Diam* abbreviates the clause "Diamond crystals are hard." It is easier to see that the form is invalid than it is to see that the original argument is invalid. The form is invalid because other invalid arguments have the same form. For example, suppose Cryst were instead to abbreviate "You are a Nazi" and Diam were to abbreviate "You breathe air." The resulting argument would have the same form as the one about diamonds:

> If you are a Nazi, then you breathe air.
> You do breathe air.
> ______________________________
> You are a Nazi.

Nobody would accept this argument. Yet it is just like the argument about diamonds, as far as form is concerned. That is, the two are logically analogous. So if one is bad, both are bad. The two arguments are logically analogous because both have the following logical form:

> If P, then Q.
> Q.
> ________
> P.

It is really the logical form of the diamond argument that makes it be invalid not that it is about diamonds. If someone were to say of the argument about diamonds, "Hey, I can't tell whether the argument is valid or not; I'm no expert on diamonds," you could point out that the person doesn't have to know anything about diamonds, but just pay attention to the pattern of the reasoning.

Standard form is not a kind of logical form; it is merely a way of writing down arguments.

Just as valid patterns are a sign of valid arguments, so invalid arguments have invalid patterns, as we shall see. All arguments have patterns (logical forms). The first person to notice that arguments can be deductively valid or invalid because of their pattern or logical form was the ancient Greek philosopher Aristotle. He described several patterns of good reasoning in his book *Organon*, in about 350 B.C. As a result, he is called "the father of logic." He started the whole subject with this first and yet deep insight into the nature of argumentation.

In our example, the terms *Diam* and *Cryst* served as logical symbols that abbreviated sentences. We will be introducing more and more logical symbolism as this chapter progresses. The reason for paying attention to logical symbols is that when arguments get complicated, a look at the symbolic logical form can show the important heart of the argument. The reason for using symbolism is much like that for translating mathematical word problems into mathematical symbols: the translation makes the mathematics within the statements more visible for those who have a feeling for the symbols. The purpose of introducing symbols and logical forms is to aid in evaluating reasoning that is too complicated to handle directly in the original English.

However, this chapter hasn't yet spelled out how to determine the logical form of a sentence. Unfortunately, determining the logical form of a sentence is tricky because the same sentence can have more than one logical form depending on how one treats it. It can have a sentential form in an area of logic called sentential logic, a class logic form in class logic, or even other forms. This difficulty will be clearer after the following introduction to sentential form and sentential logic.

The argument about diamonds was analyzed using sentential logic. Sentential logic is the branch of logic that focuses on how the logical forms of complex sentences and arguments are composed from the logical forms of their sub-sentences. The sub-sentences must be connected by one of the following sentence connectors or their synonyms: and, or, not, if-then. The technical symbols for these connectors are &, v, ~, → but we will deal with these later.

Strictly speaking, sentential logic is about statements, but this chapter won't be careful about this. Also, it is arguments, not forms, that are invalid. However, that distinction will not be observed in this chapter either. Nor will the distinction between a statement constant and a statement variable; that is, sometimes the letter P will stand for a specific statement, and at other times it will stand for any particular statement.

The Logic of *Not*

In sentential logic, an inconsistent group of sentences is defined via their logical form. By definition, a sentence group is inconsistent if it implies some complex statement whose logical form is "P and not -P." This complex statement is composed of two sub-statements, the statement P and its opposite, not -P. The two sub-statements are joined by the connector *and*. The statement form "P and not-P" is said to be the logical form of a contradiction. Note that the statement form "not-P" does not stand for any statement that isn't P; rather, it stands for any statement that negates P—that says something that must be false when P is true and that must be true when P is false. Not-P is the negation of P. This information about negation can be summarized in the following **truth table for negation**:

P	not-*P*
T	F
F	T

Here the capital letter *T* represents the possibility of the sentence at the top of its column being true, and *F* represents the possibility of being false.

———CONCEPT CHECK———

The negation of "She's moral" is

a. She's immoral.
b. It's not the case that she's immoral.
c. She's amoral.
d. None of the above.

———[253]

In sentential logic, there are two ways a pair of sentences can be inconsistent. They are **contradictory** if they are inconsistent and if, in addition, one sentence must be true while the other must be false. However, two sentences are **contrary** if they are inconsistent yet could both be false. Out on the street you have to be on the alert since so many people use the one word "contradiction" for both terms. But here we will use these technical terms properly.

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253 Answer (d). Here are three, equivalent negations of "She's moral:"

No, she's not.
She's not moral.
It's not the case that she's moral.

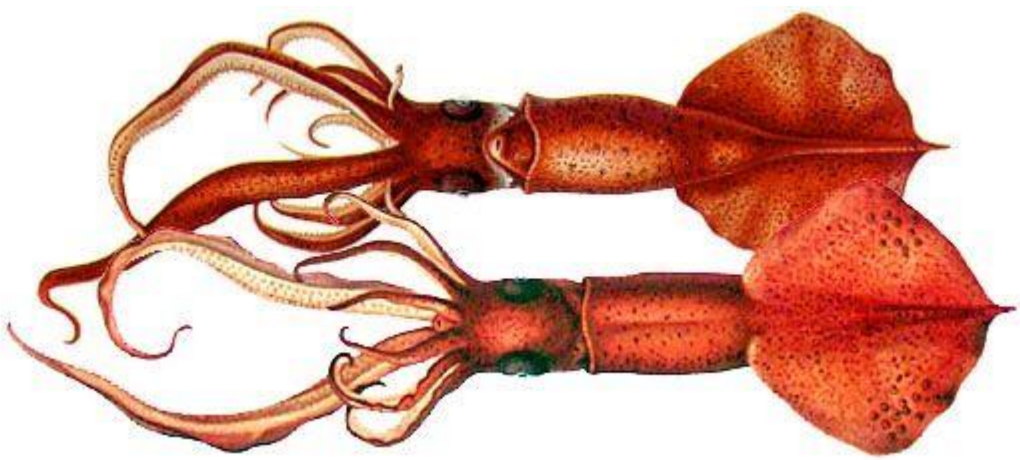
Sentences A and B below are contradictory, whereas sentences A and C are contrary, and sentences B and C are consistent.

A. The house is all red.

B. The house is not all red.

C. The house is all green.

Another term that occurs when people are thinking about inconsistent is “opposite.” When a squid is hiding at the bottom of the ocean in camouflage it is not signaling but hiding. A biologist might say that when a squid is in camouflage, that is the *opposite* of the squid’s signaling. The kind of opposite meant here, is being contrary, not being contradictory. The squid might be neither signaling nor in camouflage if it is just swimming along peacefully.



——CONCEPT CHECK——

Create a sentence that is contrary to the claim that it's 12:26 p.m. but that does not contradict that claim.

——[254]

254 "It is noon." Both sentences could be false if it is really 2 p.m., but both cannot be true (in the same sense at the same place and time without equivocating).

———CONCEPT CHECK———

Which statement below serves best as the negation of the statement that Lloyd Connelly is an assemblyman who lives in the capital?

a. Lloyd Connelly lives in the capital and also is an assemblyman.

b. No one from Lloyd Connelly's capital fails to be an assemblyman.

c. It isn't true that Lloyd Connelly is an assemblyman who lives in the capital.

d. Lloyd Connelly doesn't live in the capital.

a. Lloyd Connelly is not an assemblyman.

———[255]

If you were to learn that x = 8, would it be reasonable for you to conclude that it isn't true that x is unequal to 8? Yes. The valid form of your reasoning is

255 Answer (c). Answer (d) is incorrect because both the original statement and (d) could be false together. A statement and its negation cannot both be false; one of them must be true.

$$\frac{P}{\text{not-not-}P} \quad \text{[deductively valid]}$$

One could infer the other way, too, because any statement is logically equivalent to its **double negation**.

The Logic of *And*

Sentential logic explores not only the patterns of sentences but also the patterns of arguments. In this section we will explore arguments whose validity or invalidity turns crucially on the use of the word *and*. The truth table for "**and**," or "**&**," or **conjunction** as it is called by logicians, has four rows for all the possibilities:

P	*Q*	*P* and *Q*
T	T	T
T	F	F
F	T	F
F	F	F

Each row of the truth table represents a possible situation or way the world could be. If you were to learn that x = 5 and that y < 7, would it be valid for you to infer that y < 7? Yes. It would also be trivial. The general point of that example of reasoning is that the following is its deductively valid form:

$$\frac{P \text{ and } Q}{Q} \quad \text{[deductively valid]}$$

If you were to learn that x = 105, would it be valid for you to infer both that x = 105 and that y = 14? No. The general point is this:

$$\frac{P}{P \text{ and } Q} \quad \text{[deductively invalid]}$$

The truth table for conjunction can be used to demonstrate that in the previous argument form there are possibilities in which the premise is true while the conclusion is false. [Just left P be true and Q be false.] There are no such counterexample possibilities for deductively valid forms. In this way, the tables provide a general method of assessing the validity of arguments in sentential logic.

> Any instance of a valid argument form must produce a valid argument.

The Logic of *Or*

In any statement of the form "P or Q" the statement P is called "the left disjunct," and the Q is called "the right disjunct." The operation of "or" or "**v**" is called **disjunction**. Consider the statement "x = 5 or y < 19." If you were told that either the left or the right disjunct is true, but not which is true, could you be sure that the right one is true? No. The moral is the following:

P or *Q*
———
Q   [deductively invalid]

However, if you knew that the left disjunct is not true, you could infer that the right one is. The general form is

P or *Q*
not-*P*
———
Q   [deductively valid]

———CONCEPT CHECK———

State whether the main argument below is a deductively valid inference, then describe the logical form of the inference.

Assuming x = 4 and y < 7, as you have said, then x is not unequal to 4.

———[256]

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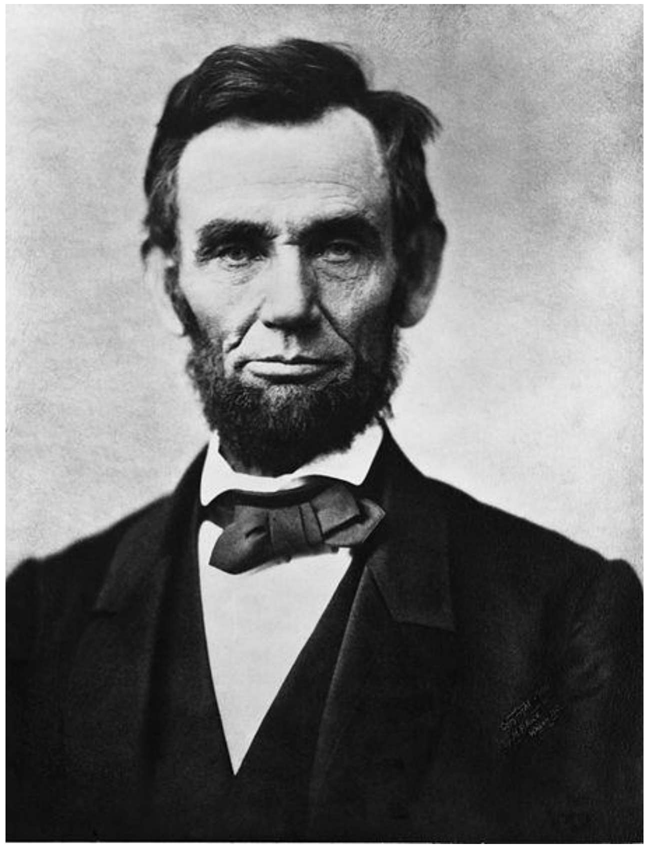
256 Yes, it is a deductively valid inference. Its logical form is



CONCEPT CHECK

Does this argument have valid logical form?

> Your information establishes that President Abraham Lincoln was assassinated by Ulysses Grant. Now we can be sure that John was right when he said, "Abraham Lincoln was assassinated by Grant or Booth."

In trying to assess whether the argument is deductively valid, you can abstract the main argument to produce the following form:

$$\frac{P \text{ and } Q}{\text{not-not-}P}$$ [deductively valid]

P is $x = 4$
Q is $y < 7$

P
______
P or *Q*

where we have defined

P = President Abraham Lincoln was assassinated by Ulysses Grant.

Q = Abraham Lincoln was assassinated either by Grant or else by Booth.

The first premise happens to be false, yet the conclusion is true. The problem, however, is not to decide which sentences are true but to decide whether the logical form is deductively valid.

[257]

The truth (and falsehood) possibilities for *or* can be summarized in this truth table:

P	*Q*	*P* or *Q*
T	T	T
T	F	T
F	T	T
F	F	F

The Logic of *If-Then*

Some argument patterns are so common that they have been given names. The most common of all is **modus ponens**:

If *P* then *Q*
P
______ [deductively valid]
Q

The validity of this form can be checked by using the truth table for implication (that is, the conditional) and noticing that there is no possibility of a counterexample, namely a situation where the premises are truth and the conclusion is false.

P | Q | If P then Q
----

257 This is a valid form. Any argument with that form has to be valid.

T | T | T
T | F | F
F | T | T
F | F | T

The Greek logician Chrysippus discovered the modus ponens form in 200 B.C.E. Here is an example of an argument whose sentential logical form is modus ponens:

> If they bought that much aluminum stock right before Chile had its general strike, then they will be wiped out.
>
> They did buy that much aluminum stock just before Chile had its general strike.
>
> They will be wiped out.

To show that this argument does have the modus ponens logical form, we could use this dictionary of abbreviations:

> P = They bought that much aluminum stock right before Chile had its general strike.
>
> Q = They will be wiped out.

If we symbolize the argument, then we get this logical form:

If *P* then *Q*
P
__________ [deductively valid]
Q

Choosing the letters P and Q was arbitrary. We could have used A and B. The following form is also called *modus ponens*:

If A then B
A
So, B

In addition to modus ponens, there are other forms of deductively valid reasoning in sentential logic. **Modus tollens** is another common one, and it has this form:

If *A* then *B*
not-*B*
__________ [deductively valid]
not-*A*

Examples of this form of valid reasoning were examined in earlier chapters without mentioning the Latin term for it. Here is an example:

> If he is a lawyer, then he knows what the word *tort* means. Surprisingly, he doesn't know what the word means. So, he's not a lawyer.

In this example of modus tollens, the letters A and B represent the following:

> A = He is a lawyer.
> B = He knows what the word tort means.

In daily life, you wouldn't be apt to detect the modus tollens form so clearly because the above argument might be embedded in a passage like this:

> If he were really a lawyer as he claims, he'd know the word *tort*. So, he's some sort of imposter.

This passage contains the same modus tollens argument that was in the previous passage. However, the premise not-B is now implicit. The conclusion of the modus tollens is also implicit. In addition, a second conclusion has been drawn — namely, that he's an imposter. It takes a lot more logical detective work to uncover the modus tollens argument.

Although the modus tollens form is a valid form — that is, any argument with that form is a valid argument—there are apparently similar arguments that are invalid yet are often mistaken for valid ones. Here is an example of a common one:

> If she's Brazilian, then she speaks Portuguese. She's no Brazilian, so she doesn't speak the language.

The reasoning is deductively invalid. In standard form it might be written this way:

> If she's Brazilian, then she speaks Portuguese.
> She is not Brazilian.
> ---
> She does not speak Portuguese.

The conclusion does not follow with certainty. To suppose it does is to commit the **fallacy of denying the antecedent**. The fallacy also occurs more transparently in this argument: "If you are a Nazi, then you breathe air, but you obviously are not a Nazi, so you don't breathe air." This invalid argument is logically analogous to the one about speaking Portuguese.

In sentential logic, the logical form of the fallacy is

If *P* then *Q*
Not-*P* [deductively invalid]
———————
Not-*Q*

The form is what defines the fallacy. The if-part of a conditional, the P, is called its **antecedent**. Then then-part is called the **consequent**. The second premise, "not-P," denies (negates) the antecedent. The arguer asserts the conditional, denies the antecedent, and draws an invalid inference. That is why the fallacy has the name it does.

——CONCEPT CHECK——

The singularity is the moment when artificial intelligence rapidly escalates. “To analyze the argument for a singularity in a more rigorous form, it is helpful to introduce some terminology. Let us say that AI is artificial intelligence of human level or greater (that is, at least as intelligent as an average human). Let us say that AI+ is artificial intelligence of greater than human level (that is, more intelligent than the most intelligent human). Let us say that AI++ (or super-intelligence) is AI of far greater than human level (say, at least as far beyond the most intelligent human as the most intelligent human is beyond a mouse). Then we can put the argument for an intelligence explosion as follows:

> 1. There will be AI+.
> 2. If there is AI+, there will be AI++.
> ———————————————
> 3. There will be AI++.”

Is the indented argument valid?

——[258]

Here is an invalid argument that is often mistaken for a modus ponens argument.

> If she's Brazilian, then she speaks Portuguese. She does speak Portuguese. So she is Brazilian.

The premises of this argument do give a weak reason to believe the woman is Brazilian. However, if the arguer believes that the premises establish with certainty that she is Brazilian, then the arguer is committing the fallacy of **affirming the consequent**. Rewriting the argument in standard form yields

> If she's Brazilian, then she speaks Portuguese.

258 Yes, it is valid. The passage is from “The Singularity: A Philosophical Analysis,” by David J. Chalmers, *Journal of Consciousness Studies* 17:7-65, 2010.

She does speak Portuguese.

She is Brazilian.

The logical form, in sentential logic, of the fallacy is

If *P* then *Q*
Q ________ [deductively invalid]
P

The fallacy has its name because then *then* part of any conditional is called its consequent and because affirming the second premise Q affirms the consequent of the conditional.

———CONCEPT CHECK———

Which passage commits the fallacy of denying the antecedent?

a. If pork prices continued to drop in Japan from 1789 to 1889, then pork would have been eaten regularly by the average citizen of Tokyo in 1890. Pork prices continued to drop in Japan during that time. Consequently, the average citizen of Tokyo in 1890 did eat pork regularly.

b. If pork prices continued to drop in Japan from 1789 to 1889, then pork would have been eaten regularly by the average citizen of Tokyo in 1890. The average citizen of Tokyo in 1890 did not eat pork regularly. So pork prices did not continue to drop in Japan from 1789 to 1889.

c. If pork prices continued to drop in Japan from 1789 to 1889, then pork would have been eaten regularly by the average citizen of Tokyo in 1890. So the average citizen of Tokyo in 1890 did not eat pork regularly, because pork prices did not continue to drop in Japan from 1789 to 1889.

d. If pork prices continued to drop in Japan from 1789 to 1889, then pork would have been eaten regularly by the average citizen of Tokyo in 1890. The average citizen of Tokyo in 1890 did not eat pork regularly. So pork prices did continue to drop in Japan from 1789 to 1889.

e. If pork prices continued to drop in Japan from 1789 to 1889, then pork would have been eaten regularly by the average citizen of Tokyo in 1890. The average citizen of Tokyo in 1890 did eat pork regularly. So pork prices did continue to drop in Japan from 1789 to 1889.

———[259]

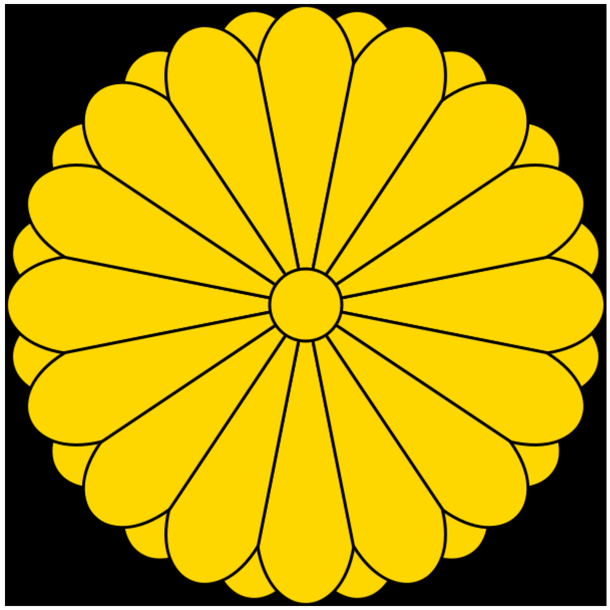
259 Answer (c).

———CONCEPT CHECK———

In the previous Concept Check, which passage commits the fallacy of affirming the consequent?

———[260]

260 Answer (e).



———CONCEPT CHECK———

In the previous Concept Check, which passage is an example of modus ponens?

————[261]

Logicians working in sentential logic have discovered many other deductively valid and invalid argument forms. For example:

> A conditional statement of the form "If P, then Q" implies with certainty the statement whose form is "If not-Q, then not-P."

In other words, the following argument form is deductively valid:

261 Answer (a).

P implies *Q* [deductively valid]
___
not-*Q* implies not-*P*

If someone says to you, "If it snows today, you should seek shelter at David's cabin," is the person implying that it is snowing today? No, and the point behind why the person is not is captured by the following invalid argument form:

If *P* then *Q* [deductively invalid]
___
P

The following inference is also deductively invalid:

If *P* then *Q* [deductively invalid]
___
Q

The last three argument forms represent points about logical reasoning that were made informally when the notion of a conditional was first introduced. With the tools of logical symbolism, the points can be made more briefly, as we've just seen.

The techniques of sentential logic can often be helpful in analyzing deductive argumentation, but there are many deductive arguments for which sentential methods do not apply. The recognition of this fact has led logicians to create new and more powerful logics to handle these other arguments. These logics are studied in college courses called "symbolic logic" and "deductive logic" and "formal logic."

The Logic of *Only*, *Only-If*, and *Unless*

The word *only* is an important one for logical purposes. To explore its intricacies, suppose that to get an A grade in Math 101 you need to do two things: get an A- or better average on the homework, and get an A average on the tests. It would then be correct to say, "You get an A grade in Math 101 *only if* you get an A- or better average on all the homework." Now drop *only* from the sentence. Does it make a difference? Yes, because now you are left with the false statement "You get an A grade in Math 101 if you get an A- or better average on the homework." Speaking more generally, dropping the *only* from *only if* usually makes a significant difference to the logic of what is said. Unfortunately, many people are careless about using these terms. Let's display the logical forms of the two phrases in sentential logic, using these abbreviations:

A = You get an A in Math 101.

B = You get an A- or better average on all the homework.

Now, “A only if B” is true but “A if B” is false. So "A only if B" and "A if B" are not equivalent; they must be saying something different. They have a different "logic."

Here is a summary of the different logical behavior of *if* as opposed to *only if*. The following three statement patterns are logically equivalent:

(1) P only if Q

(2) P implies Q

(3) If P, then Q

but none of the above three is equivalent to any of the following three:

(4) P if Q.

(5) Q implies P.

(6) If Q then P.

Yet (4), (5), and (6) are all logically equivalent to each other.

The phrase *if and only if* is a combination of *if* plus *only if*. For example, saying, "You're getting in here if and only if you get the manager's OK" means "You're getting in here if you get the manager's OK, and you're getting in here only if you get the manager's OK."

——CONCEPT CHECK——

Which of the following are true?

For all x, x = 4 only if x is even.
For all x, x is even only if x = 4.
For all x, x is even if and only if x = 4.

——[262]

262 Just the first one.

——CONCEPT CHECK——

Are all three of these sentences logically equivalent to each other? If not, which two are equivalent to each other? Watch out. This is tricky because your background knowledge about geography is useless here.

a. If you're from the USA, then you're from North Dakota.

b. You're from the USA only if you're from North Dakota.

c. You're from North Dakota if you're from the USA.

———[263]

The logical form of sentences containing the word *unless* is important to examine because so many errors and tricks occur with the word. It usually means the same as *or*, although many people, on first hearing this, swear it is wrong. You're going to go to jail unless you pay your taxes, right? So, either you pay your taxes or you go to jail.

Consider a more complicated situation. Suppose you will not get an A in this course unless you are registered. Does it follow with certainty that if you are registered, you will get an A? No. Does it follow that you will not get an A? No, that doesn't follow either. Does it follow instead that if you do not get an A, you are not registered? No. What would, instead, be valid is this:

> You will not get an A in this course unless you are registered.
>
> So, if you get an A, then you are registered.

---

263 All three are equivalent to each other, and all three are false.

The logical form of the reasoning is

Not-A unless REG.
So, if A, then REG.

Does this really look valid? It is.

Sentential Logic

When we created logical forms for arguments we sometimes abbreviated clauses or simple sentences with words and at other times with capital letters. If we were always to use capital letters and were to use following special symbols for the connective phrases, then we'd be expressing logical forms in the language of **Sentential Logic**. The connective symbol 'v' abbreviates the English connective word 'or' and the phrase "either…or." The symbol '&' replaces the English conjunction word 'and' and also 'but.' The '~' symbol represents negation. The arrow '→' represents 'if-then.'

Here is a table of the connective symbols of Sentential Logic with the English phrases they replace. We will use the symbols with two simple sentences A and B:

~A	not-A (it's not true that A)
A v B	A or B
A & B	A and B (A but B)
A → B	if A then B (B if A) (A only if B)
A ↔ B	A if and only if B (A just when B)

The valid argument form we called *modus ponens*, namely "If P then Q; P; so Q" becomes

P → Q
P

Q

in Sentential Logic. 'P' and 'Q' might be any statement having any truth value. If we write out the argument form horizontally this way

P → Q, P ⊢ Q

then it is called a **sequent**. In a sequent, commas separate all the premises, and the turnstile symbol "⊢" is used instead of the line.

———CONCEPT CHECK———

Is this a valid sequent in Sentential Logic?

~A, C → A ⊢ ~C

———[264]

Equivalent names for Sentential Logic are *Statement Logic* and *Propositional Logic*.

———CONCEPT CHECK———

Is this a valid sequent in Sentential Logic?

A → B, B ⊢ A

That is, if we obey the rules of the truth tables, can we be confident that there is no way to assign truth values to the simple statement letters that will produce a counterexample?

———[265]

Review of Major Points

This chapter introduced the concept of logical equivalence between sentences and the concept of a sentence's logical form. Arguments have logical forms composed of the forms of their component sentences. Logical form in turn is the key to assessing whether a deductive argument is valid. Once you have identified the logical form of a deductive argument, the hardest part of assessing validity or invalidity is over, because with the form you can spot logical analogies, determine whether the argument has a counterexample, or demonstrate the validity if it is valid. In this chapter we concentrated on logical forms in Sentential Logic: forms involving *and*, *or*, *not*, and *if-then*. We also examined three terms that are likely to cause logical confusion: *only*, *only if* and *unless*. You have now developed an arsenal of powerful logical tools to use in attacking cases of complex reasoning.

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264 Yes, it is valid. It has the form of *modus tollens*. There is no way for the premises to be true while the conclusion is false without violating the truth tables.

265 No. This argument has the invalid form called *affirming the consequent*. In a situation where A is false and B is true, we have a counterexample because then the argument has true premises and a false conclusion. For example, here is an argument that obviously commits the fallacy of affirming the consequent: If that boy you've never heard of is your father, then that boy is a male. That boy is a male. So, that boy you've never heard of is your father. This has true premises and presumably a false conclusion.

Glossary

antecedent The if-part of a conditional.

conditional An if-then statement.

consequent The then-part of a conditional.

contradictory A logical inconsistency between two statements in which one must be true while the other is false.

contrary A logical inconsistency between two statements when both could be simultaneously false.

fallacy of affirming the consequent A deductive argument of the form "If P then Q; Q; so, P."

fallacy of denying the antecedent A deductive argument of the form "If P then Q; not-P; so not-Q."

logical form of a contradiction The statement form "P and not-P."

logically analogous Having the same logical form.

logically equivalent Logically implying each other. Alternatively we can say two statements are logically equivalent if they are true in the same situations and false in the same situations.

modus ponens A deductive argument of the form "If P then Q; P; so Q."

modus tollens A deductive argument of the form "If P then Q; not-Q; so not-P."

negation The negation of statement P is a statement of the form "not-P" that is true when P is false and that is false when P is true.

Sentential Logic The branch of logic that focuses on how the logical forms of complex sentences and arguments are composed from the logical forms of their sub-sentences or clauses. The clauses will be connected by one of the following sentence connectors or their synonyms: *and*, *or*, *not*, or *if-then*. Sentential Logic is also called *Propositional Logic* and *Statement Logic*.

Exercises

Logical Equivalence

■ 1. Is every statement logically equivalent to itself?[266]

■ 2. Does the definition of logical equivalence ever permit false sentences to be logically equivalent to each other?[267]

3. Let A = "x is 4," let B = "x is an even number," and let C = "x is 8/2." Then are A and B logically equivalent?

4. Let A = "x is 4," let B = "x is an even number," and let C = "x is 8/2." Then are A and C logically equivalent?

5. If no items from column C are nondeductible, can we infer with certainty that all items for column C are deductible? How about vice versa?

The Logic of Not, And, Or, and If-Then

■ 1. Show, by appeal to its logical form in sentential logic, why the following argument is valid or is invalid, and be sure to say which it is.

> If politicians are corrupt, their friends are also corrupt. Thus, if politicians are not corrupt, they don't have peculiar friends because they have peculiar friends if their friends are corrupt.[268]

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266 Yes.

267 Here is an example: "Churchill was the first prime minister" and "The first prime minister was Churchill."

268 The argument can be treated in sentential logic using the following definitions of the sentences
(clauses):

PC = politicians are corrupt
FC = the friends of politicians are corrupt
PF = politicians have peculiar friends

Here is the logical form (noticing that the conclusion is not the last statement in English):

If PC, then FC.
If FC, then PF.

■ 2.The following argument is

a. deductively valid
b. deductively invalid.

I don't know whether polyvinyls are corrosive or not, but I do know that if they are, then ferrophenyls are also corrosive. Therefore, if polyvinyls are not corrosive, they don't have picoferrous properties because they have picoferrous properties if ferrophenyls are corrosive.[269]

Defend your answer by appeal to logical form.

3. The following argument is

a. deductively valid   b. deductively invalid

I don't know whether polyvinyls are corrosive or not, but I do know that if they are corrosive, then ferrophenyls are also corrosive. Isn't it reasonable, then, to suppose that, if polyvinyls are corrosive, they have picoferrous properties because, as you've already said, they have picoferrous properties if ferrophenyls are corrosive?

Defend your answer by appeal to logical form.

4. Consider this amusing argument: "If God had wanted people to fly, He would not have given us bicycles." Here is its implicit premise: But He has given them to us; so it's clear what God wants.

   a. State the implicit conclusion.

   b. State the logical form of the argument in sentential logic.

   c. Is the argument deductively valid? Why or why not?

If not-PC, Then not-PF.

This form is invalid. If you can't tell whether this form is valid, maybe the invalidity is easier to see by using a logical analogy. Here is an analogous argument that is also invalid:

If it's a house cat, then it's a feline.
If it's a feline, then it's a mammal.
So, if it's not a house cat, then it's not a mammal.

269 It is invalid. Here is the form: If PVC then FC. If FC then PF. So, If not-PVC then not-PF.

d. Is the arguer assuming that God is not evil?

5. The following passage contains one or more deductive sub-arguments, all of whose premises are stated explicitly. For each one, (a) identify the sub-argument by rewriting it in standard form, (b) give its logical form, and (c) say whether it is valid.

   You've got to give up this sort of behavior. God frowns on homosexuality. Besides, all the community standards oppose it, and this is hurting your father's business. He has to serve the public, remember. If your homosexuality is illegal, you should give it up, and your homosexuality is illegal, as you well know. You say it's OK because it feels right, but there is more to this world than your feelings. I love you, but you must quit this nonsense.

■ 6. Valid or invalid?

If *A*, then *B*
A
___
A and *B* [270]

■ 7. Is this a deductively valid argument pattern?

P or not-*P*
If *P*, then *R*
not-*R*
___
not-*P* [271]

■ 8. Is the following argument form deductively valid in sentential logic?[272]

A
If *A*, then *B*
___
B

9. Here is the logical form of an argument in sentential logic. Is it valid or invalid?

If *A*, then *B*
A
___
A or *B*

10. Which statement patterns below would be inconsistent with the pattern "P or Q"?

a. not-P   b. not-Q

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270 Valid.

271 Yes.

272 The form is valid, and any specific argument with that form is also valid.

c. not-P and not-Q d. not-P or not-Q

e. If P, then not-Q

11. Which of the following statement forms has the logical form "If A, then B"?

 a. A, from which it follows that B.

 b. A, which follows from B.

■ 12. Is this argument deductively valid?

The report writing was not difficult. Since report writing is either difficult or pleasant, the report writing must have been pleasant.[273]

13. Identify the lowercase letter preceding any passage below that contains an argument or a sub-argument that has the following logical form:

 not -B
 <u>A implies B</u>
 not-A

 a. X is the family of all open-closed intervals together with the null set. If X is the family of all open-closed intervals together with the null set, then X is closed under the operation of intersection. Consequently, X is not closed under the operation of intersection.

 b. X is not the family of all open-closed intervals together with the null set. If X is the family of all open-closed intervals together with the null set, then X is closed under the operation of intersection. Consequently, X is not closed under the operation of intersection.

 c. X is not the family of all open-closed intervals together with the null set. But X is the family of all open-closed intervals together with the null set if X is closed under the operation of intersection. Consequently, X is not closed under the operation of intersection.

 d. If X is the family of all open-closed intervals together with the null set, then X is closed under the operation of intersection. X is the family of all open-closed intervals together with the null set. Consequently, X is closed under the operation of intersection.

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273 This is an example of valid reasoning, and it remains valid even if you were to learnt hat one of the premises is false.

e. X is the family of all open-closed intervals together with the null set. If X is the family of all open-closed intervals together with the null set, then X is closed under the operation of intersection. Consequently, X is closed under the operation of intersection.

f. If X is closed under the operation of intersection, then X is the family of all open-closed intervals together with the null set. X is not the family of all open-closed intervals together with the null set. Consequently, X is not closed under the operation of intersection.

14. In regard to the previous question, identify the letters of the passages that are deductively invalid.

15. Is this argument deductively valid? Defend your answer by appeal to sentential logic.

If state senators are corrupt, their staff members are corrupt. The staff members of state senators are indeed corrupt, so state senators are corrupt.

■ 16. Is this argument deductively valid? Defend your answer by appeal to sentential logic.

If Einstein were alive today, the physics department at Princeton University in New Jersey would be affected by his presence. So, if you look at the department you'll see he's one dead duck.[274]

■ 17. Is this a valid sequent in Sentential Logic?

$A \rightarrow B, \sim A \vdash \sim B$

That is, if we obey the rules of the truth tables, can we be confident that there is no way to assign truth values to the simple statement letters that will produce a counterexample?[275]

The Logic of Only, Only If, and Unless

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274 Valid because its sentential form is modus tollens. The argument is superficially invalid but is actually valid when the principle of charity is used in these two ways: (1) to say that the conclusion is logically equivalent to "Einstein is not alive," and (2) to add the implicit premise that the physics department at Princeton University in New Jersey is not affected by the presence of Einstein.

275 No. This argument has the invalid form called *denying the antecedent*. In a situation where A is false and B is true, we have a counterexample because then the argument has true premises and a false conclusion.

■ 1. You can usually get from the bottom floor to the top floor of a building that has an elevator _________ you walk up the stairs.

a. only if b. if and only if

c. just when d. unless

e. none of the above[276]

2. You are president of the United States ________________ you are a U.S. citizen.

a. only if b. if and only if

c. provided that d. if e. unless

■ 3. You are president of the United States _________________ you are not president of the United States.

a. only if b. if and only if

c. just when d. if e. unless[277]

4. For any whole number x, x is even _____________ x is odd. (Which ones cannot be used to fill in the blank and still leave a true statement?)

a. only if b. if and only if
c. provided that d. if
e. unless

■ 5. For any whole number x, x is even _____________ x is not odd. (Which ones cannot be used to fill in the blank and still leave a true statement?)

a. only if b. if and only if
c. provided that d. if
e. unless[278]

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276 Answer (e)

277 Answer (e).

278 Answer (e). It would be true to say "x is even unless x is odd." Adding the *not* makes (e) not fit in the blank.

6. If it were the case that only people favoring cost-cutting techniques in the administration are advocates of decreasing the number of administrative positions, would it follow that you have got to be an advocate of decreasing the number of administrative positions to be a person favoring cost-cutting techniques in the administration?

7. A sign says, "Only adults may view this film." Does it follow with certainty that if you're an adult, you may view this film?

8. Joseph will not graduate in cosmetology unless he passes either the developmental cosmetology course or the course in experimental design. So, if Joseph passes experimental design, he will graduate in cosmetology.

   a. deductively valid b. not deductively valid

9. Carlucci calls us only if the war room is in condition orange, but the war room *is* in condition orange. So, Carlucci will call.

   In analyzing this argument, let the word *Orange* stand for the statement "The war room is in condition orange," and let the word *Call* stand for "Carlucci calls us." Rewriting the first premise as a conditional and then generalizing to the argument pattern yields which one of the following?

   a. If Orange, then Call.
      Orange.
      Call.

   b. If Call, then Orange.
      Orange
      Call.

   c. Call only if Orange.
      Orange.
      Call.

10. The argument in the previous question commits the fallacy of denying the antecedent when the first premise is rewritten as a logically equivalent conditional statement and then the argument is translated into its form in sentential logic.

    a. true b. false

11. Consider this memo from an employer:

    Employees must be given the opportunity to give or withhold their consent before the private aspects of their lives are investigated. The firm is justified in inquiring into the employee's life only if the employee has a clear understanding that the inquiry is being made. The means used to gain this information are also important; extraordinary methods would include hidden microphones, lie detector tests, spies, and personality inventory tests.

    ■ i. If the quotation is correct, then if the employee has a clear understanding that the inquiry is being made, the firm is justified in inquiring into the employee's life.

a. follows b. does not follow[279]

ii. If the quotation is correct, the firm is justified in inquiring into the employee's life if the employee has a clear understanding that the inquiry is being made.

a. follows b. does not follow

iii. If the quotation is correct, then if the firm is justified in inquiring into the employee's life, the employee has a clear understanding that the inquiry is being made.

a. follows b. does not follow

12. Suppose $x = 4$ if and only if $y < 22$. From this fact, which of these follow with certainty?

a. $x = 4$ provided that $y < 22$.

b. $x = 4$ unless $y < 22$.

c. $y < 22$ if $x = 4$.

d. $x = 4$ or $y < 22$.

e. ($y = 22$ or $y > 22$) just when x is not equal to 4.

13. Are these two arguments logically analogous? Is either of them deductively valid ?

Carlucci will call us only if the war room is in condition orange, but the war room is in condition orange. So Carlucci will call.

Carlucci will call us only if he is alive. Carlucci is alive, so he will call.

■ 14. Are these two sentence forms logically equivalent?

Not-A unless B.
A only if B.[280]

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279 Answer (b). The phrase *only if* works like *if-then* in the sense that "Inquiry is justified only if employee has understanding" is logically equivalent to "If inquiry is justified, then employee has understanding." Note that statement (i) is the converse of this —namely, "If employee has understanding, then inquiry is justified." Consequently, (i) does not follow from the statement containing the *only if*, which is why the answer is (b).

280 Yes. The first is equivalent to "not-A or B." The second is equivalent to "A implies B." These two are equivalent to each other.