

Mechatronic Systems Engineering School of Engineering Science SIMON FRASER UNIVERSITY

# Autonomous Navigation Optimization Project Report

MSE 426 – Design Optimization

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# **List of Figures and Tables**



## **Abstract**

This report covers the three problem formulations from each team member, they are the autonomous navigation problem, Weiss Schwarz card game problem, and POÄNG Armchair problem. The autonomous navigation problem was chosen for further studies. Optimization methods fmincon and genetic algorithm are selected to solve the navigation problem. Results showed that fmincon is faster but genetic algorithm provides better solutions.

# **Introduction**

Decision making can be difficult when there are multiple solutions to a problem, nevertheless, optimal solutions can be identified that best suit the objective through optimization. In order to obtain the best solution, formulating the problem mathematically and correctly is a crucial step, in fact, majority of the project time was spent on generating problem statement, collecting data, defining design variables, identifying criteria to be optimized and identifying constraints.

In this project, 3 problems are formulated with objective functions, design variables and constraints, respectively. The first problem is to minimize total distance travelled by a robot, in which the robot is to move from point (0,0) to point (10,10) autonomously while avoiding obstacles. The second problem is to maximize damage at center stage of the card game, Weiss Schwarz, for the Psycho Pass deck. The third problem is to minimize materials cost in an attempt to increase structural strength of POÄNG Armchair from IKEA by building the leg chair out of composite materials using wood and steel.

For this project, the autonomous navigation problem is chosen for further analysis since it has the most implications in the mechatronic engineering field. Fmincon and genetic algorithm are selected as the optimization tool for minimizing the distance of the navigation, results obtained from fmincon differ based on the starting points, average value of the solutions is 15.3 whereas solutions obtained from genetic algorithm have an average value of 14.9. Nevertheless, the calculation time for fmincon is almost instant whereas for genetic algorithm is about 2 minutes. Hence to obtain quick solutions knowing the general navigation path, one can specify starting points to fmincon for optimization. On the other hand, the shortest navigation route can be obtained through genetic algorithm in compromise of long wait time.

# **Problem Formulation 1: Autonomous Navigation**

## Problem description

The most common problem in the field of autonomous navigation is obstacle avoidance and planning navigation. Various algorithms are used to find solution depending on the scenario, most common of which are iterative and exhaustive algorithms which require large memory reserves to process all viable options to present the most optimal path.

The goal for this project is to use optimization routines to decrease memory consumption of navigation algorithms with computational cost, and assess the

viability of the approach. The problem is a simplistic 2D planar navigation problem in which the robot needs to move from one corner of a square area (0,0) to another (10,10) while avoiding any obstacles; represented by grey circulars with varying radii show in Figure 1: The Robot Shall Travel from Start Point (0,0) to End Point (10,10) without Colliding into Obstacles Represented by the Grey Circles. The robot would attempt to move between start and end point by following 4 way points. The robot will move from way-point to way-point in a straight path while avoiding crossing through any of the obstacles. The objective is to minimize the net distance travelled from start to end point.



*Figure 1: The Robot Shall Travel from Start Point (0,0) to End Point (10,10) without Colliding into Obstacles Represented by the Grey Circles*

Before presenting the problem in standard optimization objective form we will discuss the functions and components required to describe the problem.

#### Variable selection

In order to realize the net travel distance from start to end point, x and y coordinates for the 4 waypoints are selected as the design variables. Since the waypoints should not coincide with the obstacles, location and radii of each obstacle shall be identified for constraint variable formulation. Overall there are 8 design variables (4 waypoints) to be optimized. The design variable coordinates below are extracted from Figure 1.

1) Design variables: Waypoints Coordinates  $(x_i, y_i)$ ,  $i = 1, 2, 3, 4$ 

a.  $(x_1, y_1) = (2.5)$ b.  $(x_2, y_2) = (4.5)$ c.  $(x_3, y_3) = (7.5)$ d.  $(x_4, y_4) = (9,5)$ 

## 2) Fixed parameters: Obstacle n centered at  $(x_n, y_n)$  and  $radii r_n, n = 1,2,3,4$

- a.  $(x_1, y_1) = (7.8), r_1 = 1.5$
- b.  $(x_2, y_2) = (4, 3), r_2 = 1.5$
- c.  $(x_3, y_3) = (2, 7), r_3 = 0.25$
- d.  $(x_4, y_4) = (8,2), r_4 = 0.25$

Connecting the starting and end point with the 4 waypoints forms the purple path shown in Figure 1 above. This purple line has a net distance of 17.48-unit length; it is not optimized but serve as the starting point for fmincon to evaluate a shorter navigation route. In this case, the red line with way points (2.8,4), (4.1,4.7), (6.9, 5.3), (8,6) is found to the be local minimum with a net distance of 15.04-unit length.

#### Objective function formulation

In order to be energy efficient, the objective function for autonomous navigation is to **minimize the net distance required in travelling from start to end point** through 4 waypoints. Mathematically, the objective function is formulated as

minimize 
$$
f(x) = \sum_{i=1}^{e-1} |(x_i, y_i) - (x_{i+1}, y_{i+1})|
$$

where  $(x_i, y_i)$ , are the waypoints including the end and start points,  $i = 1 ... e$ ,  $(x_i, y_i)$  is the coordinate of the current waypoint, and  $(x_{i+1}, y_{i+1})$  is the coordinate of the next waypoint. In this report, 4 waypoints are implemented for study ( $e =$ 6), nevertheless; the number of waypoints can be adjusted in the code such that more waypoints can be implemented in the navigation for smooth change in direction.

#### Constraint formulation

In order to avoid the robot from hitting the obstacles, circle line intersection formula from Wolfram MathWorld [1] as well as projection of vectors for intercepts projected on vector along length of path are formulated for the constraint.



*Figure 2: Circle Line Intersection Illustration, courtesy to [1]*

1) Circle line intersection [1]- length of robot path overlapping with an obstacle,  $l(x_i, y_i, x_{i+1}, y_{i+1}, x_n, y_n, r_n)$ 

Given a certain obstacle and two adjacent waypoints, the function computes the length of path interconnecting the waypoints overlapping with the circle. A value of zero would indicate no collisions whereas higher values indicate higher collision. The basis of the algorithm is defined as follows:

Given

 $(x_i, y_i)$ , and  $(x_{i+1}, y_{i+1})$  as the adjacent waypoints

Obstacle *n* centered at  $(x_n, y_n)$  and radii  $r_n$ 

$$
(x_1, y_1) = (x_i, y_i) \quad (x_n, y_n)
$$

$$
(x_2, y_2) = (x_{i+1}, y_{i+1}) \quad (x_n, y_n)
$$

$$
d_x = x_2 \quad x_1
$$

$$
d_y = y_2 \quad y_1
$$

$$
d_r = \sqrt{d_x^2 + d_y^2}
$$

$$
D = \begin{vmatrix} x_1 & x_2 \\ y_1 & y_2 \end{vmatrix}
$$

The discriminant

$$
= r_n^2 d_r^2 D^2
$$

Determines incidence as

 $\leq$  0, no intersection or tangential, thus overlapping length is 0

> 0, intersection

The points of intersection between line and circle are calculated as:

$$
x = \frac{Dd_y \pm sign(d_y)d_x\sqrt{r^2d_r^2} \quad D^2}{d_r^2}
$$

$$
y = \frac{Dd_x \pm |d_y|\sqrt{r^2d_r^2} \quad D^2}{d_r^2}
$$

And denoted as  $(x_{i1}, y_{i1})$ ,  $(x_{i2}, y_{i2})$ 



$$
V_{2-1} = (d_x, d_y)
$$
  
\n
$$
V_{i1-1} = (x_{i1}, y_{i1}) \quad (x_1, y_1)
$$
  
\n
$$
V_{i2-1} = (x_{i2}, y_{i2}) \quad (x_1, y_1)
$$

And the projection of vectors for intercepts projected on vector along length of path are given as:

$$
l_{i1} = \frac{V_{2-1} \cdot V_{i1-1}}{dr}
$$

$$
l_{i2} = \frac{V_{2-1} \cdot V_{i2-1}}{dr}
$$

And overlap is found as follows

 $l = |l_{i1} \quad l_{i2}|$ , for  $0 \leq l_{i1}, l_{i2} \leq d_{r}$  $l = |V_{i1-1}|$ , for  $0 \le l_{i1} \le d_r$ , and  $l_{i2} \le 0$ 

$$
l = |(x_{i2}, y_{i2}) \quad (x_2, y_2)|, \text{ for } 0 \le l_{i1} \le d_r, \text{ and } l_{i2} \ge d_r
$$
\n
$$
l = |V_{i2-1}|, \text{ for } 0 \le l_{i2} \le d_r, \text{ and } l_{i1} \le 0
$$
\n
$$
l = |(x_{i2}, y_{i2}) \quad (x_2, y_2)|, \text{ for } 0 \le l_{i2} \le d_r, \text{ and } l_{i1} \ge d_r
$$

Otherwise both points intersect with the projection of line connecting the waypoint but do not lie in the path between the waypoints and do not count towards the overlap length.

#### **2)** Non Linear Constraint Function,  $g(x)$

Standard formulation of optimization problems requires the definition of a function which holds  $g(x) \leq 0$  inequality true. Similar effect is achieved by considering the length of overlap between the path and the obstacles as no collision would reflect a value of 0 while more collisions would result in value progressively larger than 0.

Given:

 $(x_i, y_i)$ , as the waypoints including the end and start points,  $i = 1 ... e$ 

 $(x_n, y_n, r_n)$ , are the obstacles,  $n = 1 ... o$ 

$$
g(x) = \sum_{i=1}^{e-1} \sum_{n=1}^{o} l(x_i, y_i, x_{i+1}, y_{i+1}, x_n, y_n, r_n)
$$

Where,

 $x = (x_2, y_2, x_3, y_3, ..., x_{e-1}, y_{e-1})$ , is the variable vector

# **Problem Formulation 2: Weiss Schwarz Card Game**

#### Problem description

Weiss Schwarz is a collectable card game by Bushiroad. This card game is played between 2 people; each player has a deck of 50 cards. The winning conditions of Weiss Schwarz includes forcing the opponent to Level 4 or opposing player has no cards left in hand and deck. Details on how to play the game can be found in Heart of the Cards website [2]. In this problem formulation, the only the 50 cards(belongs

to one of the write) in Figure 4: Weiss Schwarz Psycho Pass Deck for Attack Power Optimization at Level 3 will be taken into consideration for problem formulation.



*Figure 4: Weiss Schwarz Psycho Pass Deck for Attack Power Optimization at Level 3*

In general, the higher level the card the stronger the attack power, and the more damage can be inflicted on the opponent so as to force the opponent to level up. In Weiss Schwarz there are 3 types of card: character card, event card, and climax card. Character cards are the damage dealer, whereas event and climax cards work in combination to increase attack power of character cards. The objective is to maximize the attack power of cards in hand so as to deal as much damage as you can to the opponent.

#### Variable selection

Note from Figure 4 above all the cards have duplicates except one card. We have 36 distinct cards and the quantity of each kind of card shall be represented as the design variables shown in the appendix. Context and images in the appendix are referenced from Heart of the Cards translation page [3].

- 1) Design variables: quantity of each kind of card
	- a.  $x_i$ ,  $i = 1, ..., 36$

#### Objective function formulation

Since the objective is to **maximize the attack power of cards in hand,** we need to look at the souls of each character card, the combination effects of climax and event cards, as well as the trigger effect of each card.

1) Average attack power from character souls:

$$
S_{avg} = \frac{1}{50} (x_{01} + x_{02} + x_{03} + x_{04} + 2x_{05} + x_{06} + x_{07} + x_{08} + x_{09} + x_{10} + x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} + x_{19} + x_{23} + x_{24} + x_{25} + x_{26} + 2x_{27} + x_{28} + x_{29} + x_{30} + x_{31} + 2x_{33})
$$

2) Average souls at center stage:

$$
S_{cs} = CS \times S_{avg}
$$

3) Average souls from trigger:

$$
S_{trigger} = \frac{CS}{LB}(x_{03} + x_{04} + x_{05} + x_{17} + x_{18} + x_{19} + x_{20} + 2x_{21} + 2x_{22} + x_{27} + x_{32} + x_{33} + x_{35} + x_{36})
$$

4) Average souls from climax:

$$
S_{climax} = \frac{1}{LB} [CS(x_{20}) + x_{21} + CS(x_{22}) + CS(x_{35}) + x_{36}]
$$

$$
minimize f(x_i) = (S_{cs} \times S_{trigger} \times S_{climax}), i = 1, ..., 36
$$

The objective function is taking into account the average attack power of the whole deck, the average attack power from the trigger and the average attack power from climax. Multiplying the product of the 3 sources of attack power (souls). We can then formulate the problem mathematically as a function of quantity of each kind of card.

#### Constraint formulation

Weiss Schwarz has limitations on the total number of cards in a deck, total number of climax card in a deck and some recommendation for first time card builder. Both the limitations and recommendations from Weiss Schwarz wiki [4] are translated into constraints below.

1) Number of cards in a deck = 50

$$
g_1: \sum_{i=1}^{36} x_i = 50
$$

2) Number of cards at center stage = CS, a player can have up to 3 cards at the center stage to attack:

$$
g_2\text{: }0
$$

3) Number of cards in library (deck) = LB, maximum number of card in library is 47 since we have 3 cards in level zone:

$$
g_3: 0 < LB \le 47
$$

4) Each card can only have up to 4 duplicates:

$$
g_4: 0 < x_i \leq 4, i = 1, \ldots, 36
$$

5) Recommended Maximum Number of Level 0 Cards:

 $g_5: 0 < x_{01} + x_{06} + x_{07} + x_{08} + x_{09} + x_{10} + x_{11} + x_{23} + x_{24} + x_{28} + x_{29} \le 18$ 6) Recommended Number of Level 1 Cards:

$$
g_6: 2 < x_{02} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} + x_{25} + x_{26} + x_{30} + x_{31} \le 14
$$

7) Recommended Number of Level 2 Cards:

$$
g_7: 7 < x_{03} + x_{04} + x_{17} + x_{18} + x_{19} + x_{32} + x_{33} + x_{34} \le 14
$$

8) Recommended Number of Level 3 Cards:

$$
g_8: 3 < x_{05} + x_{27} \le 7
$$

9) Recommended Number for Climax Cards:

$$
g_9: x_{20} + x_{21} + x_{22} + x_{35} + x_{36} = 8
$$

# **Problem Formulation 3: POÄNG Armchair** Problem description



*Figure 5: POÄNG Armchair cushion, Robust Glose off-white, courtesy to [5]*

We have this nice fancy chair that's selling quite well in Europe from IKEA [5]. Therefore, management has decided they'd like to start selling this in North America, namely the United States, believing that the chair would meet the same amount of success. However, the first few months since the start of sales in North America have produced nothing but alarming reports of chairs snapping or breaking due to overweight or grossly obese people taking undue liberties with the chair. Thus, the task has fallen to your engineering department to re-engineer the chair to withstand heavier loads while maintaining the same outer aesthetics.

The engineering manager has magically arrived to the conclusion that embedding a length of curved metal shown in Figure 7 into the legs is the best solution to strengthening the legs of the chair without ruining the outer aesthetics. Feeling particularly please at his "ingenious" solution, your engineering manager has now passed the project down to you, the engineering department grunt worker, coop student, to complete the task of ensuring the damn reinforcement actually works as the engineering manager intends.

Therefore, the cross section of the chair will look approximately like Figure 6 below. We'll assume that the materials wood and steel are used to simplify the problem. We'll also simplify the geometry of the chair legs in Figure 7.



*Figure 6: Cross Sectional View of the Chair Legs*



*Figure 7: Simplified Geomotry of Chair Legs – Side View*

## Objective function formulation

We want to be able to **minimize cost of materials used**. Using the price ratio of wood(pine) vs steel [6] is used in the objective function. Therefore, the objective function will look like the following in which the design variables are described in the section below:

$$
min f(x) = (4_{2}w_{2} + [({}_{2} + {}_{1})w_{1} - {}_{2}w_{2}]) L
$$



*Figure 8: Cross Section Area of Chair Leg,* , *are the Dimensions for Wood Portion whereas are for Steel Portion*

#### Design Variable selection

Since the legs of the chair essentially consists of two curved beams, the design variables can be chosen as such

> $_1 = t$  ickness of wood beam  $w_1 = \text{width}$  of wood beam  $z = t$  ickness of steel beam  $w_2 = \text{width}$  of steel beam  $L = length$  of c air legs

#### Constraint formulation

In order to ensure that the chair legs are capable of supporting a certain limit on the minimum amount of mass, we'll place some constraints on the geometry of the chair.

Given Hooke's Law for stresses in curved beams [6]:

$$
\sigma_{\theta\theta} = E e_{\theta\theta} = E w (\frac{R_n}{r} \quad 1)
$$

We can derive the equation for hoop stress. This will allow us to calculate what kind of force a curved beam can endure.

$$
\sigma_{\theta\theta} = \frac{N}{A} + \frac{M(A - rA_m)}{A_r(RA_m - A)}
$$

Using the equation for hoop stress, we can write the force requirement as follows

$$
(1) \frac{\sigma_{\theta\theta\text{wood}}}{\left[A_{wood}^{-1} + \frac{(100 + y_{wood})(A_{wood} - rA_{mwood})}{(A_{wood})r(R_{wood}A_{mwood} - A_{wood})}\right]} + \frac{\sigma_{\theta\theta\text{steel}}}{\left[A_{steel}^{-1} + \frac{(100 + y_{steel})(A_{steel} - rA_{msteel})}{(A_{steel})r(R_{steel}A_{msteel} - A_{steel})}\right]} \leq N
$$

The following equations are used in the above constraint, to simplify presentation:

$$
A_{wood} = 1W_1 \t 2W_2
$$
  

$$
A_{steel} = 2W_2
$$

$$
R_{wood} = \frac{100 + 1 + 2}{2}
$$
  
\n
$$
R_{steel} = \frac{100 + 2}{2}
$$
  
\n
$$
A_{mwood} = (1 + 2) \ln[\frac{100 + 1 + 2}{100}]
$$
  
\n
$$
A_{msteel} = (2) \ln[\frac{100 + 2}{100}]
$$
  
\n
$$
y_{wood} = \frac{[1 + 2 + (w_2 - w_1)(2)(\frac{2}{2})]}{[(1 + 2)w_1]}
$$

```
y_{steel} = \frac{2}{2}N = 250 lbs = -1200 N
\sigma_{\theta\theta wood} = ~40 MPa\sigma_{\theta\theta steel} = {\sim}505\,MPa
```
We'll also place a constraint on the minimum length of the chair leg.

(2)  $400 < L$ The thicknesses of the two materials should not exceed these values.

(3)  $0 \leq 1 \leq 25$ mm (4)  $0 \leq z \leq 20$ mm

The width of the chair legs should not exceed 50 mm, and the width of the steel bar should not exceed the width of the wooden chair. Additionally, since we want wood to be the primary cosmetic material, we will also place a minimum limit on its width.

(5)  $30mm \leq w_1 \leq 50mm$ (6)  $0 \leq w_2 \leq w_1$ 

# **Optimization Method Selection**

Various optimization functions are available in MATLAB, however each optimization function have its pros and cons. Three main optimization methods were introduced in the MSE 426 optimization labs, they were fminocn, genetic algorithm and Oasis. The writers chose **fmincon** and **genetic algorithm** to optimize the autonomous navigation problem due to ease of accessibility of the two methods in MATLAB. Moreover, the former provides local minimums whereas the latter attempts for solve for global optimum. Results obtained and post-simulation analysis will be discussed in the following section in details.

# **Solutions and Results Analysis**

Running fmincon to solve the autonomous navigation problem generates results shown in Figure 9: Solving Autonomous Navigation using Fmincon – Lower Path Example and Figure 10: Solving Autonomous Navigation using Fmincon –Upper

Path Example. Note there are a purple line and a red line show in each figure. The purple line is the un-optimized navigation route, the initial guesses of the waypoints are fed into fmincon as the starting point; these 4 waypoints are linked together with the start and end point to form the purple path.

On the other hand, the red line is the optimized navigation route, 4 waypoints are outputted from fmincon and it is evident that the net distance of the purple line is longer than that of the red line.



*Figure 9: Solving Autonomous Navigation using Fmincon – Lower Path Example*



*Figure 10: Solving Autonomous Navigation using Fmincon –Upper Path Example*

The navigation path was also optimized by genetic algorithm (GA) in an attempt to find the global optimum, in other words, the travel route with the shortest net distance while avoiding all the obstacles. Figure 11 : Solving Autonomous Navigation using Genetic Algorithm is one of the optimum obtained using GA.



*Figure 11 : Solving Autonomous Navigation using Genetic Algorithm*

Comparing the two algorithms, we see that in Table 1: Fmincon Starting Points, Optimum Obtained, Number of Iterations, Function Evaluations, Function Values and Average Function Values and Table 2: Genetic Algorithm Starting Points, Optimum Obtained, Number of Iterations, Function Evaluations, Function Values and Average Function Values; in terms of the optimized function values, fmincon and GA are similar, however comparing the average function value, fmincon has mean navigation distance of 15.35-unit length whereas GA has a mean of 14.89 unit length.

Although both algorithm converge, calculation time for fmincon is instant whereas GA has a long wait time of 2 minutes per run. The algorithm selection depends on the need of the users; if there is a general path that the robot can follow, input the 4 waypoints as initial guess into fmincon for obtaining a quick solution. If computation time is not a concern, utilize GA to solve for a shorter path instead.

<b>Fmincon</b>						
<b>Starting Point</b>	<b>Optimum Obtained</b>	No. of	No.	Fval		
		<b>Iterations</b>	οf			
			<b>Func</b>			
			Eval			
$(2,2)$ , $(6,0)$ , $(9,0)$ ,	$(2.6, 0.5)$ , $(4.9, 1.8)$ ,	16	242	14.98		
(10, 10)	$(5.1, 1.9)$ , $(9.6, 9.4)$					
$(0,6)$ , $(2,9)$ , $(5,10)$ , $(8,10)$	$(0.7, 4.8)$ , $(3.2, 7.5)$ , $(5.6, 1.5)$	13	216	16.02		
	$(9.4)$ , $(7.8, 9.6)$					
$(2,5)$ , $(4,5)$ , $(7,5)$ , $(9,5)$	$(2.8,4)$ , $(4.1,4.7)$ , $(6.9, 5.3)$ ,	20	326	15.04		
	(8,6)					
				15.35		

*Table 1: Fmincon Starting Points, Optimum Obtained, Number of Iterations, Function Evaluations, Function Values and Average Function Values*

GA					
<b>Optimum Obtained</b>	Generations   Funccount   Fval				
$(3,4.9)$ , $(3.8,4.9)$ , $(4.8, 7.8)$ , $(6.2, 9.4)$	6	79400	15.11		
$(5,1.7)$ , $(6.9,4.7)$ , $(7,5)$ , $(7.2,5.4)$	6	62600	14.94		
$(3.2, 4.5)$ , $(4.6, 5.1)$ , $(4.9, 5.2)$ , $(8, 6.8)$	-6	69800	14.62		
			14.89		

*Table 2: Genetic Algorithm Starting Points, Optimum Obtained, Number of Iterations, Function Evaluations, Function Values and Average Function Values*

## **Conclusion**

In this project, 3 problems were formulated with objective functions, design variables and constraints, respectively. The first problem was to minimize total distance travelled by a robot, in which the robot is to move from start to end autonomously while avoiding obstacles. The second problem was to maximize damage at center stage of the card game, Weiss Schwarz, for the Psycho Pass deck. The third problem was to minimize materials cost in an attempt to increase structural strength of POÄNG Armchair from IKEA by building the leg chair out of composite materials with wood and steel.

The autonomous navigation problem was chosen for further studies since it has a lot of applications in the mechatronic engineering field. Fmincon and genetic algorithm were utilized for minimizing the distance of the navigation. Results obtained from fmincon differ based on the starting points, average value of the solutions was 15.3 whereas solutions obtained from genetic algorithm had an average value of 14.9. Nevertheless, the calculation time for fmincon was instant whereas for genetic algorithm was about 2 minutes. Hence to obtain quick solutions knowing the general navigation path, one could specify starting points to fmincon for optimization. On the other hand, the shortest navigation route can be obtained through genetic algorithm in compromise of wait time.

# **Appendix**

For MATLAB code please refer to the attached zip file Context and Images for the Weiss Schwarz Card Game Problem are referenced from Heart of the Cards [3]







