ENSC 281 Group Project

Truss and Frames Analysis of Manitowoc 31000
with Luffing Jib Boom Configuration

Courtesy of Manitowoc Inc.

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Section 1: Introduction

“In architecture, a truss is a structure that consists of one or more triangular units constructed with straight members whose ends are connected at joints” [1]. Usually, the members connected to each joint are assumed to be welded, and the joints do not provide moments (torques) to prevent the members connected to them from rotation, since they are considered pins, or revolutes. The entire structure thus consists only of two-force and zero-force members.

As second-year students, who just learned how to calculate the external and internal forces using various techniques from this course, working on a project related to truss analysis is definitely challenging in itself. Yet since we have four group members, we decided to take on the challenge and work with a even more complicated model.

During the brainstorming and topic selection process, we came up with a few general or specific topics including the following:

- ***Manitowoc 31000***
- Liebherr RMG Rail Mounted Gantry
- Human Frame
- Robots
- Liebherr Ship to Shore STS
- Crawler Crane LR 13000
- 255KL P12
- 7070 TCK
- Terminator and bipedal robot frame — by Sohail

- ThyssenKrupp Robins Bagger 288
- ThyssenKrupp Robins In-Pit Crushing Plants
- ThyssenKrupp Robins In-Pit Mobile Transfer Conveyors
- ThyssenKrupp Robins Spreader Systems
- Frame of Camping Tent — by Vincent

- World's Largest Crane
- Saipen 7000
- Asian Hercules 2
- Yantai Raffles
- Crawler crane SCC86000TM — by Evan

- RX-78GP03 Gundam Dendrobium
- Truss Building
- 3D Mechanical Microstructures
- Trans-siberian railway — by Michael

After a discussion about the complexity of each potential topic and a assessment regarding the availability of their corresponding specifications, we chose to go with the Manitowoc 31000 model provided by Sohail. Manitowoc 31000 is one of the biggest crawler cranes of its kind. It is composed of several booms and hence has a relative complex truss structure.

Section 2: External Forces and Free Body Diagram

In the external free body diagram, we are going to study the luffing jib boom configuration as it offers us the most structural complexity. The unknowns in this case will be:

1. The angle of the main boom with horizontal
2. The angle of the luffing jib with horizontal
3. Counterweight
4. Loading
5. Reaction forces at the base of the crane

For the sake of simplicity, we are going to fix the angles of the main boom and the luffing jib with respect to horizontal at 70° and 45°, respectively. Now we are going to calculate the location of the centroid of the whole crane in the given configuration. Most of the parts in the crane are symmetrical and it is thus easy to find the centroids for the individual parts while for some the centroids have been assumed keeping in mind the weight distribution in such pieces. To calculate the centroid of the whole crane, we consider a top view of the crane so that all the weights are perpendicular and pointing into the plane, and the locations of the centroids from the origin are projections onto the x-y plane. The weight distribution are shown in Fig 2.1, succeeded by the corresponding tabulated data.

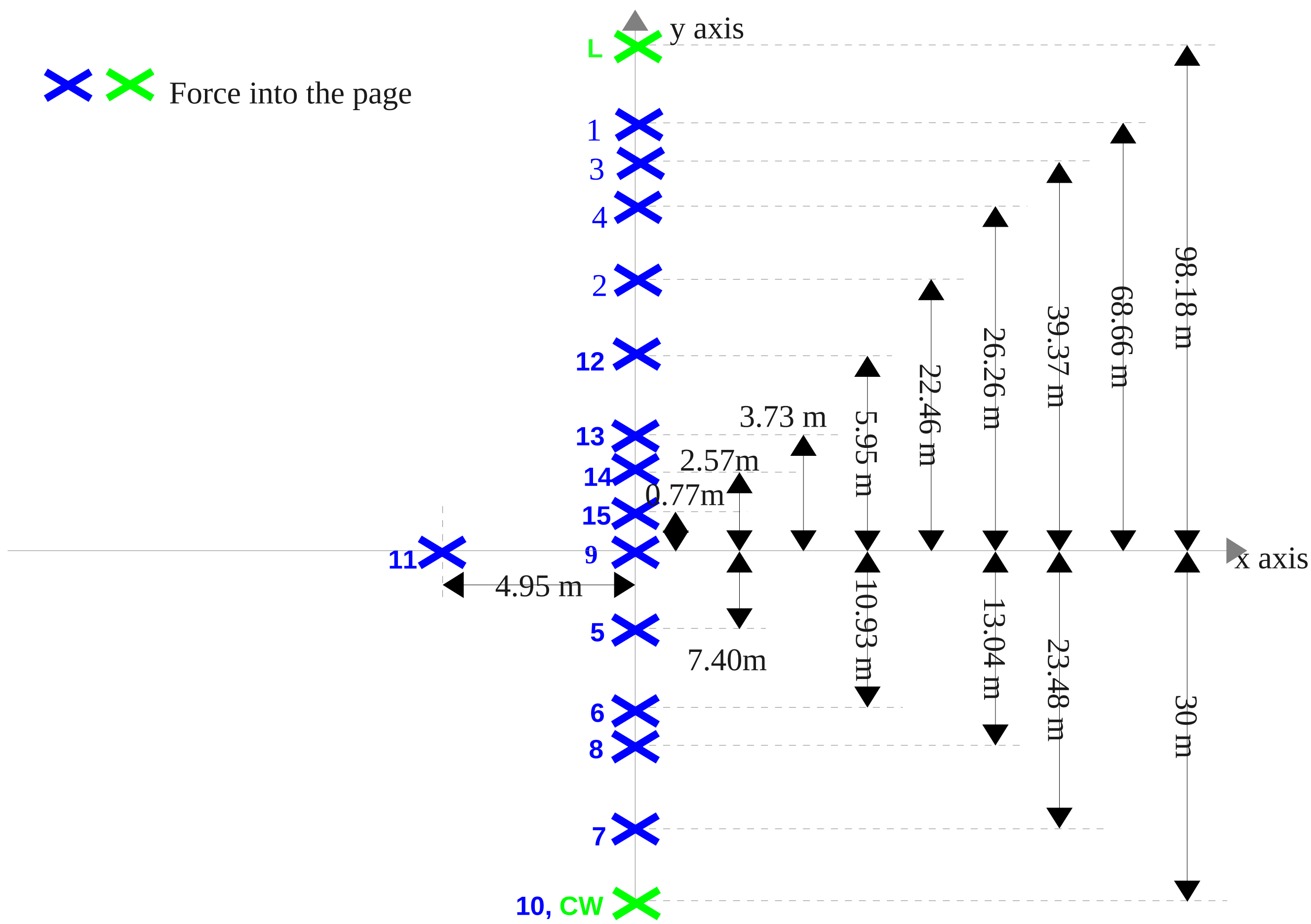

Fig 2.1: Top view of external free body dig of crane

1	Luffing Jib	132.70	0.00	68.66	0.00	9111.18
2	Main Boom	277.60	0.00	22.46	0.00	6234.90
3	Boom I	77.00	0.00	29.45	0.00	2267.65
4	Boom II	77.00	0.00	26.26	0.00	2022.02
5	Mast	65.30	0.00	-7.40	0.00	-483.22
6	Back Hitch	58.40	0.00	-10.93	0.00	-638.31
7	CW Frame-Part I	19.00	0.00	-23.48	0.00	-446.12
8	CW Frame-Part II	19.00	0.00	-13.04	0.00	-247.76
9	Base	155.40	0.00	0.00	0.00	0.00
10	CW Tray	102.56	0.00	-30.00	0.00	-3076.80
11	Cabin	36.70	-4.95	0.00	-181.67	0.00
12	Front Roller	10.20	0.00	5.95	0.00	60.69
13	Whip Hoist Drum	31.00	0.00	3.73	0.00	115.63
14	Main Drum 1	41.28	0.00	2.57	0.00	106.09
15	Main Drum 2	38.10	0.00	0.77	0.00	29.34
	Total weight=	1141.24		total=	-181.67	15055.28

Table 2.1: Calculation of centroid of crane body.

From Table 2.1,

$$\bar{x}=\frac{total\ xW}{total\ weight}=-0.16\,m\,,\ \bar{y}=\frac{total\ yW}{total\ weight}=13.19\,m$$

It is clear that the offset due to cabin is not much and the weight of the whole body can be considered to lie only on the y axis i.e. Along its symmetry as seen from top. Hence the problem for solving the unknown forces can be reduced to 2d, as shown in Fig.2.2.

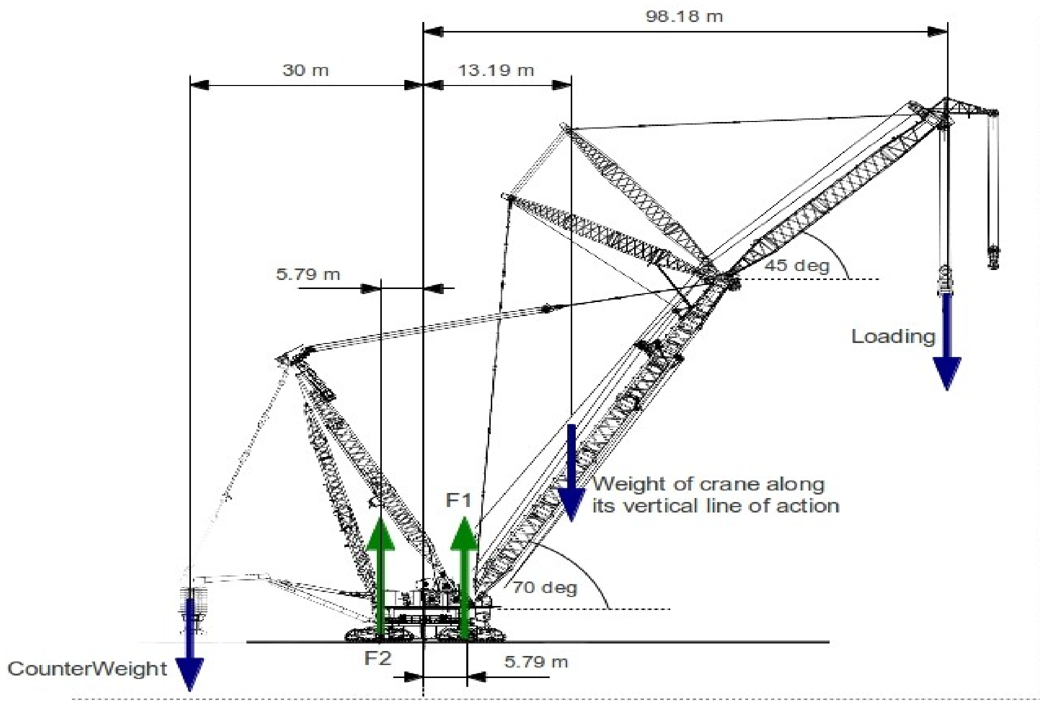

Fig 2.2: Side view of external free body diagram of crane – with crane components' weights reduced to a single force

The next step is to calculate the maximum counterweight we can load on the crane without making it tip over. The safe limit of the counterweight will be such that the crane is perfectly balanced on its backside tracks while it does not carry any loading.
Hence, from Fig 2.2, we have

$$For\ L=F_1=0$$
$$F_y=-CW_{max}+F_2-1141.24=0 \quad (2.1)$$
$$M_{(0,0)}=30*CW_{max}-1141.24*13.19-5.79*F_2=0 \quad (2.2)$$

Solving eq(2.1) and eq(2.2), we get $CW_{max}=894.84\ kgt$

For our case we are going to consider a loading less than the maximum loading capable. To calcuate L_{max}, we are going to assume that the crane is carrying CW_{max} as the counterweight and it is perfectly balanced on its front tracks. From dig 2.1

$$For\ CW_{max}=894.84\ kgt,\ F_2=0$$
$$F_y=-L_{max}+F_1-(1141.24+894.84)=0 \quad (2.3)$$
$$M_{(0,0)}=-98.18*L_{max}-1141.24*13.19+926.29*30+5.79*F_1=0 \quad (2.4)$$

Solving eq(2.3) and eq(2.4), we get $L_{max}=255.20\ kgt$

For our case study let us assume that L < L_{max} , Hence let L = 200 kgt. From Dig2.1

$$F_y=-200+F_1+F_2-(1141.24+894.84)=0 \quad (2.5)$$
$$M_{(0,0)}=-98.18*200-1141.24*13.86+894.84*30+5.79*F_1-5.79*F_2=0 \quad (2.6)$$

Solving eq(2.5) and eq(2.6), we get $F_1=1795.69\ kgt,\ F_2=440.39\ kgt$.

Name	**Value (kgt)**	**Value (Mega Newton)**
Loading	200	1962
Counterweight	894.84	8778.38
Force on each front track	897.85	8807.91
Force on each back track	220.2	2160.16

Table 2.2: Summary of the forces assumed and calculated which are external to the crane body.

Section 3: FBDs of Frames and Trusses Comprising the Crane

In this section we are calculating the forces acting on each member and the trusses as a whole, inside the structure of Manitowoc 31000. The fbd's are simplified models of the actual members and some of the forces due to struts and wires have been neglected, if it had been found that their presence made the member statically indeterminate and the member was able to achieve equilibrium even without the presence of forces from the neglected componenets. The main components and their names are indicated in Fig. 3.1 below.

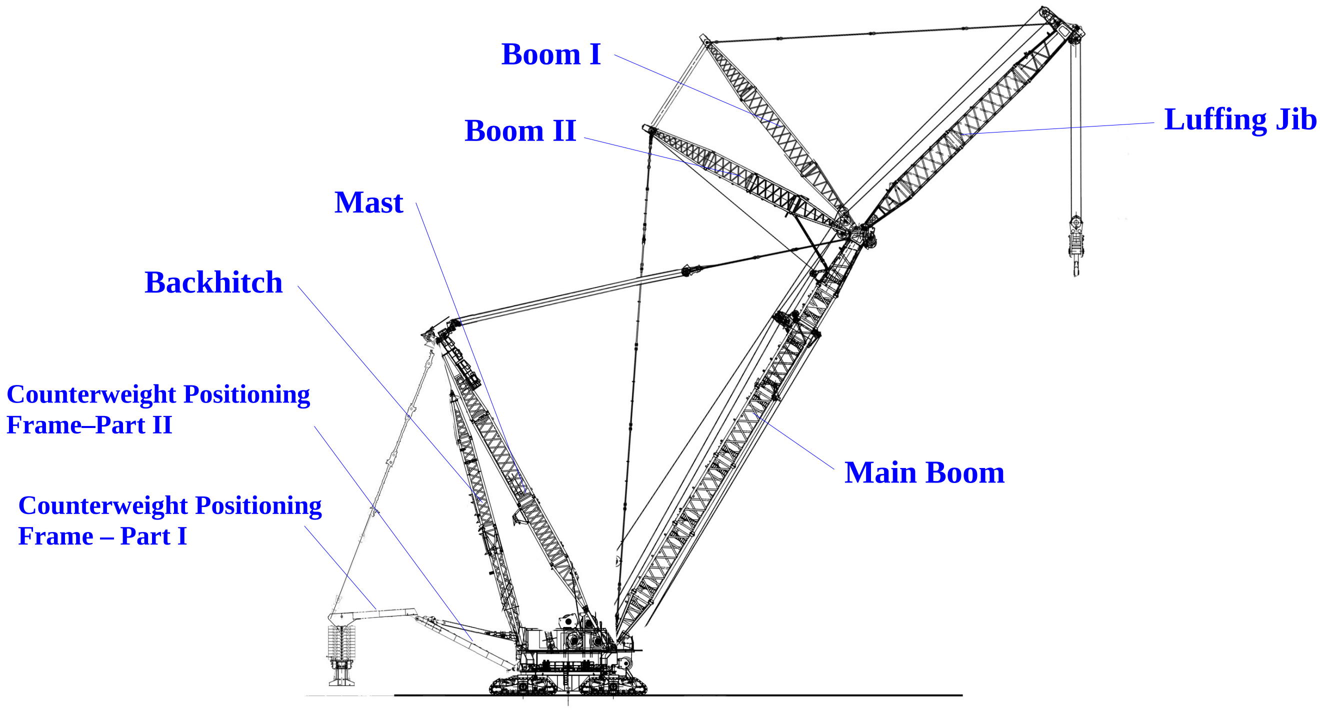

Fig. 3.1 Main components

3.1. Luffing Jib

The luffing jib is part of the entire boom configuration - with one end of it connected to the main boom, and the other being where the loading is suspended.

The luffing jib makes an angle of 45 degrees with the horizontal, according to our assumptions. All measurements use the manitowoc product guide as reference but are approximate. Here the centroid is assumed to be located at the geometric center of the jib. The angle the luffing jib makes with the horizontal can lie somewhere between 10-75 deg.

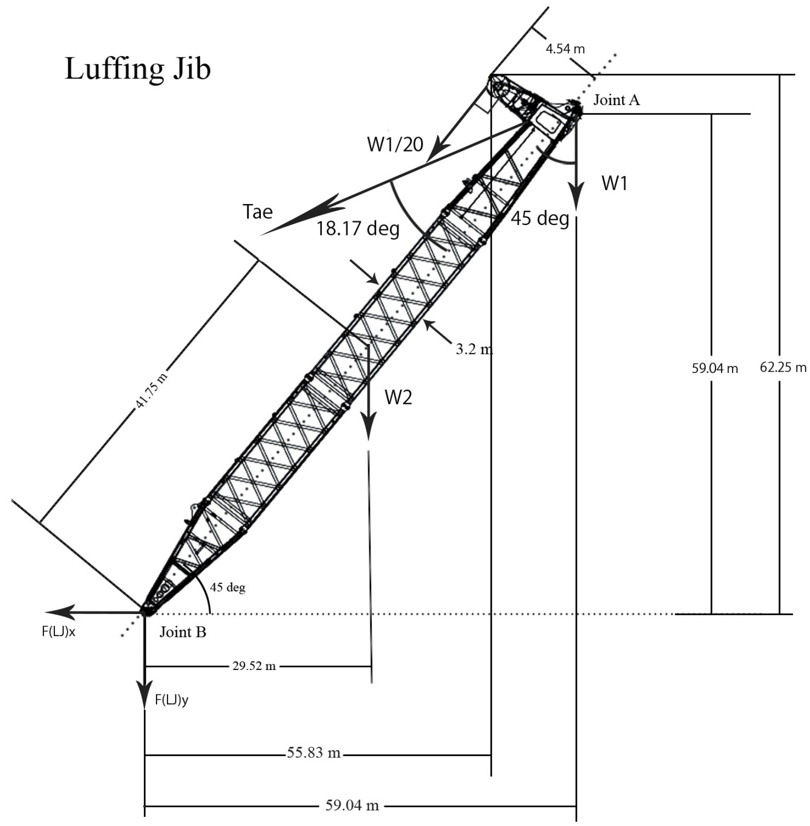

Fig. 3.2 Luffing Jib

From Fig. 3.2,

$$F_x = F_{(JL)x} - \frac{W_1}{20}\cos(45^o) - T_{ae}\cos(26.83^o) = 0 \quad (3.1)$$

$$F_y = F_{(JL)y} - W_2 - W_1 - \frac{W_1}{20}\sin(45^o) - T_{ae}\sin(26.83^o) = 0 \quad (3.2)$$

$$M_j = -42.5\mathrm{W}_2 - 59.04\mathrm{W}_1 + 59.04\mathrm{T}_{ae}(\cos(26.83^o) - \sin(26.83^o)) + \frac{W_1}{20}(62.25\cos(45^o) - 55.83\sin(45^0)) = 0 \quad (3.3)$$

Unknown Name	Value (t)	Value (MN)
T_{ae}	668.14	6554.48
$F_{(JL)x}$	603.29	5918.26
$F_{(JL)y}$	640.63	6284.62

Table 3.1: Values of unknowns from equations 3.1, 3.2, and 3.3

3.2. Boom I

Boom I is assumed to form a 90 deg triangle with the luffing jib. All other angles can be derived from that assumption. Here the centroid is also assumed to be located at the geometric center of the structure

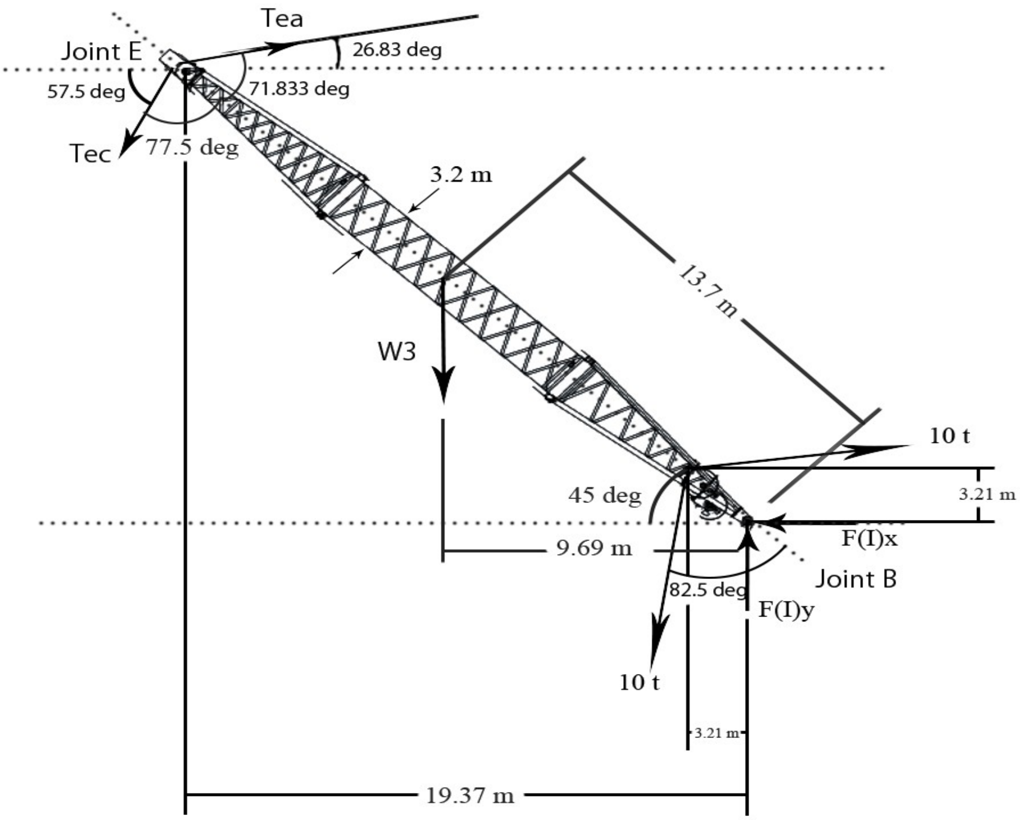

Fig. 3.3 Boom I

From Fig. 3.3,

$$F_x = T_{ae}\cos(26.83^o) - T_{ec}\cos(57.5^o) + 10(\cos(45^o) - \cos(57.5^o)) - F_{Ix} = 0 \quad (3.4)$$

$$F_y = T_{ae}\sin(26.83^o) - T_{ec}\sin(57.5^o) - W_3 + \frac{W_1}{20}(\sin(45^o) - \sin(57.5^o)) + F_{Iy} = 0 \quad (3.5)$$

$$M_j = -19.37\mathrm{T}_{ae}(\sin(26.83^o) + \cos(26.83^o)) + 19.37\mathrm{T}_{ec}(\sin(57.5^o) + \cos(57.5^o)) + 19*9.69 + \frac{W_1}{20}*3.21*(\sin(57.5^o) + \cos(57.5^o) - \sin(45^o) - \cos(45^o)) = 0 \quad (3.6)$$

Unknown Name	Value (t)	Value (MN)
T_{ec}	644.15	6319.10
$F_{(I)x}$	251.81	2470.30
$F_{(I)y}$	262.07	2570.90

Table 3.2: Values of unknowns from equations 3.4, 3.5, and 3.6

3.3. Boom II

Boom II is also assumed to form a 90 deg triangle with the main boom. Unlike Boom I above, this boom has an extra wire,Tec/20, that controls the distance between the tips of boom I and II. In other words, this wire controls the angle that the luffing jib makes with the horizontal.

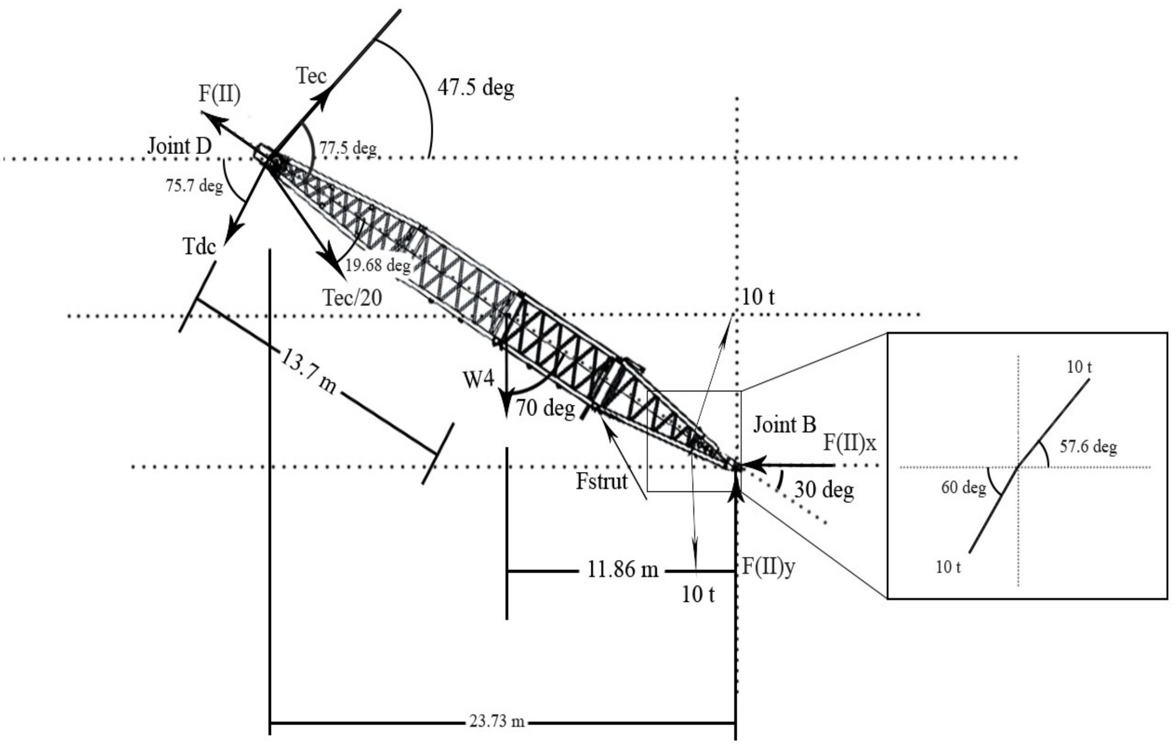

Fig. 3.4. Boom II

From Fig. 3.4,

$$F_x = T_{ec}\cos(47.5^o) + \frac{T_{ec}}{20}\cos(49.68^o) - T_{dc}\cos(75.7^o) + \frac{W_1}{20}(\cos(57.5^o) - \cos(60^o)) - F_{IIx} = 0 \qquad (3.7)$$

$$F_y = T_{ec}\sin(47.5^o) - \frac{T_{ec}}{20}\sin(49.68^o) - T_{dc}\sin(75.7^o)$$
$$-W_4 + \frac{W_1}{20}(\sin(57.5^o) - \sin(60^o)) - F_{IIy} = 0 \qquad (3.8)$$

$$M_j = -23.72\mathrm{T}_{ec}(\cos(47.5^o) + \sin(47.5^o)) + 23.73\frac{T_{ec}}{20}(\sin(49.68^o) - \cos(49.68^o))$$
$$+23.73\mathrm{T}_{dc}(\sin(75.7^o) + \cos(75.7^o)) + 23.73\frac{W_4}{2} \qquad (3.9)$$
$$\frac{+W_1}{20} * 3.93(\cos(60^o) + \sin(60^o) - \cos(57.5^o) - \sin(57.5^o)) - F_{IIx} = 0$$

Unknown Name	Value (t)	Value (MN)
T_{dc}	737.58	7235.64
$F_{(II)x}$	282.60	2772.29

$F_{(II)y}$	281.99	2766.35

Table 3.3: Values of unknowns from equations 3.7, 3.8, and 3.9

3.4. Main Boom

The main boom is the longest boom in the structure and is also the base which supports boom I, boom II and the luffing jib. All wires are invariably connected to the main boom is some way. Of notes is the wire(Fwire(ac)) which connects the main boom with the mast and backhitch. This wire controls the angle at which the main boom makes with the horizontal. This angle can lie somewhere between 70-83 deg.

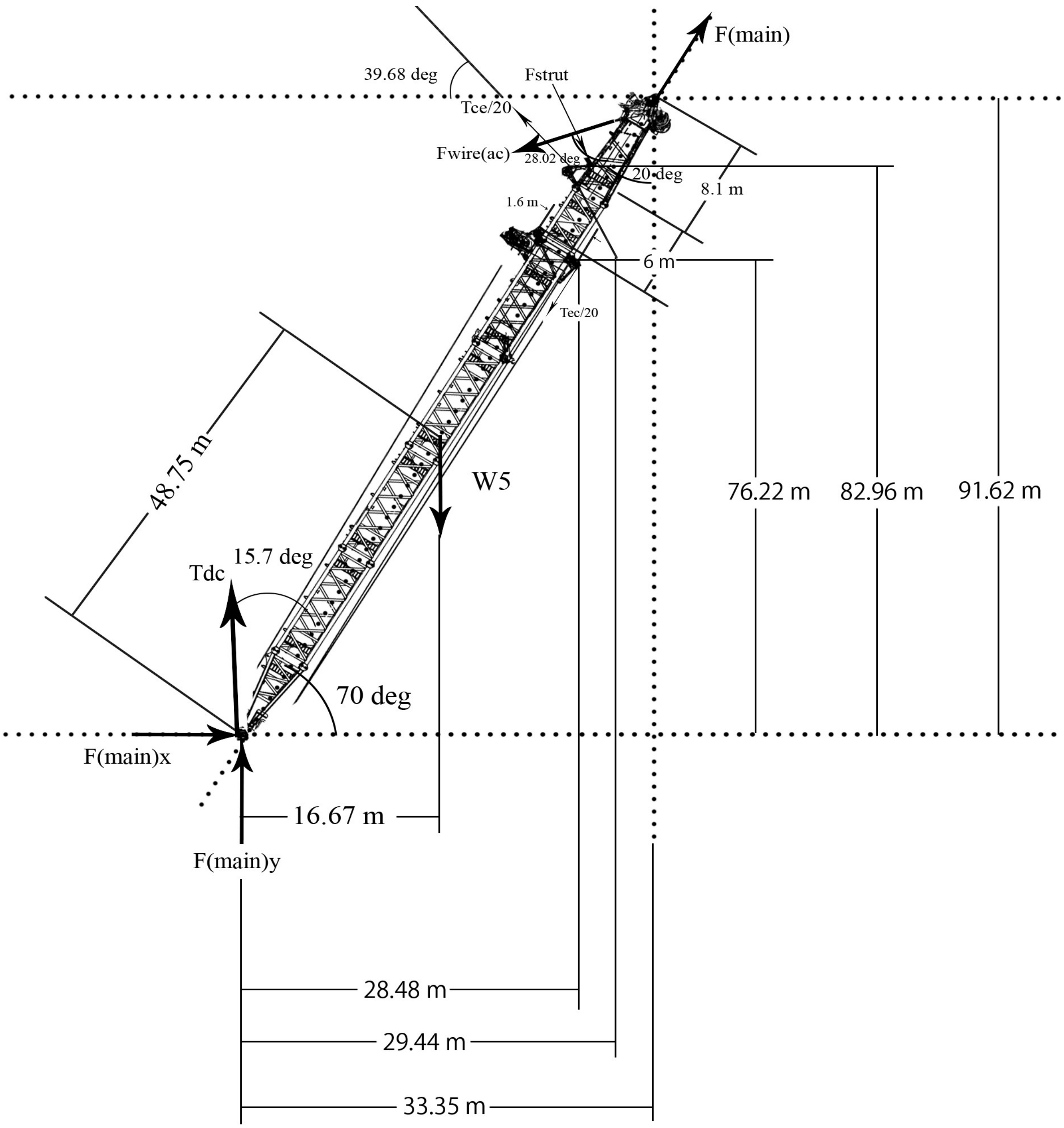

Fig. 3.5 (a) Main Boom

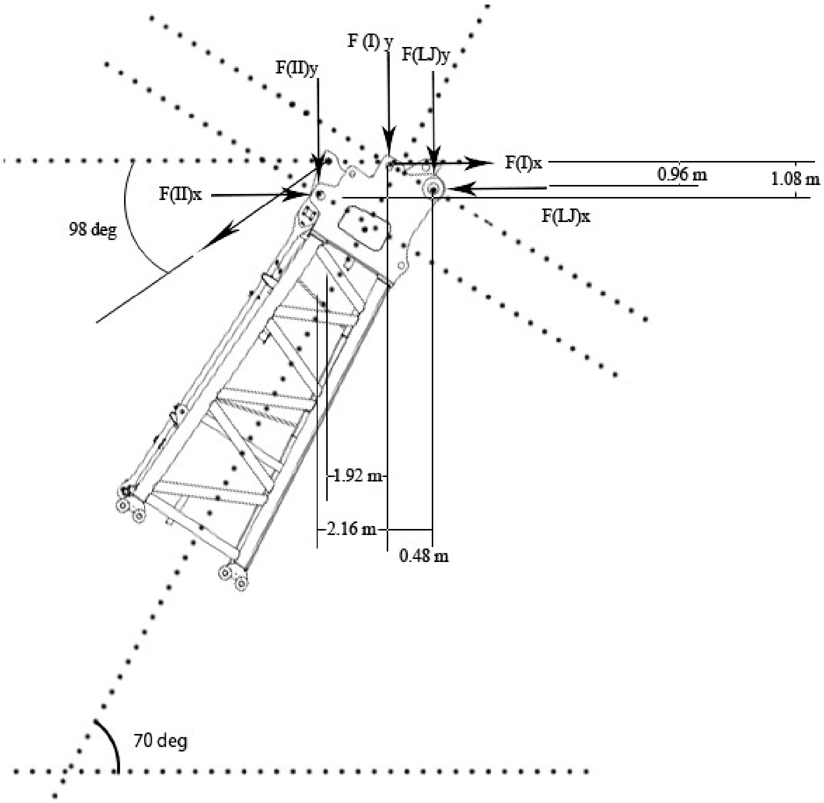

Fig. 3.5 (b) Main Boom - Magnified

From Fig. 3.5 (a) and (b),

$$F_x = F_{(main)x} + T_{dc}\cos(85.7^o) - \frac{T_{ec}}{20}(\cos(70^o) + \cos(39.68^o)) - T\cos(49.18^0) + F_{(II)x} + F_{(II)x} - F_{(JL)x} = 0 \quad (3.10)$$

$$F_y = F_{(main)y} + T_{dc}\sin(85.7^o) - W_5 - \frac{T_{ec}}{20}(\sin(70^o) - \sin(39.68^o)) - T\sin(49.18^0) - F_{(II)y} - F_{(II)y} - F_{(JL)y} = 0 \quad (3.11)$$

$$M_j = \frac{T_{ec}}{20}(-1.6 + 28.48\cos(39.68^o) + 82.46\sin(39.68^o)) - 16.93W_5 + T(91.62\cos(41.98^o) - 31.43\sin(41.98^o)) - 31.19F_{(II)y} - 90.54F_{(II)x} - 33.82F_{(JL)y} + 90.66F_{(JL)x} - 91.62F_{(I)x} - 33.35F_{(I)y} = 0 \quad (3.12)$$

Unknown Name	Value (t)	Value (MN)
T	754.26	7399.29
$F_{(main)x}$	492.85	4834.86
$F_{(main)y}$	1307.29	12824.51

Table 3.4: Values of unknowns from equations 3.10, 3.11, and 3.12

3.5. Counterweight Positioning Frame – Part I

//something

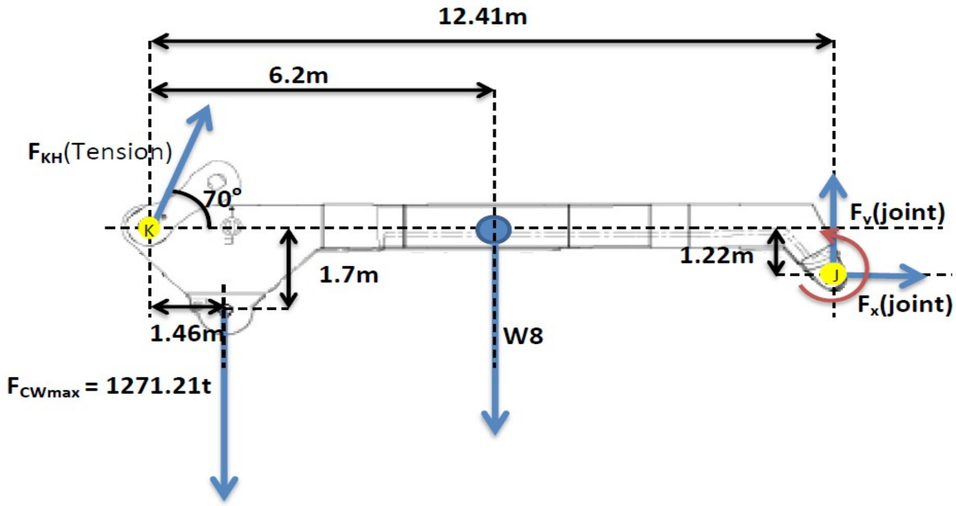

Fig. 3.6 Counterweight Positioning Frame – Part I

From Fig. 3.6, //something

$$F_x = F_{kh}\cos(70^o) + F_{x(joint)} = 0 \qquad (3.13)$$

$$F_y = F_{kh}\sin(70^o) - CW - W_8 + F_{y(joint)} = 0 \qquad (3.14)$$

$$M_j = -12.41\mathrm{F}_{kh}\sin(70^o) - 1.22\mathrm{F}_{kh}\cos(70^o) + \frac{12.41}{2}W_8 + 10.95\mathrm{CW} = 0 \qquad (3.15)$$

Unknown Name	Value (t)	Value (MN)
F_{kh}	754.26	7399.29
$F_{x(joint)}$	492.85	4834.86
$F_{y(joint)}$	1307.29	12824.51

Table 3.5: Values of unknowns from equations 3.10, 3.11, and 3.12

6. Counterweight Positioning Frame – Part II

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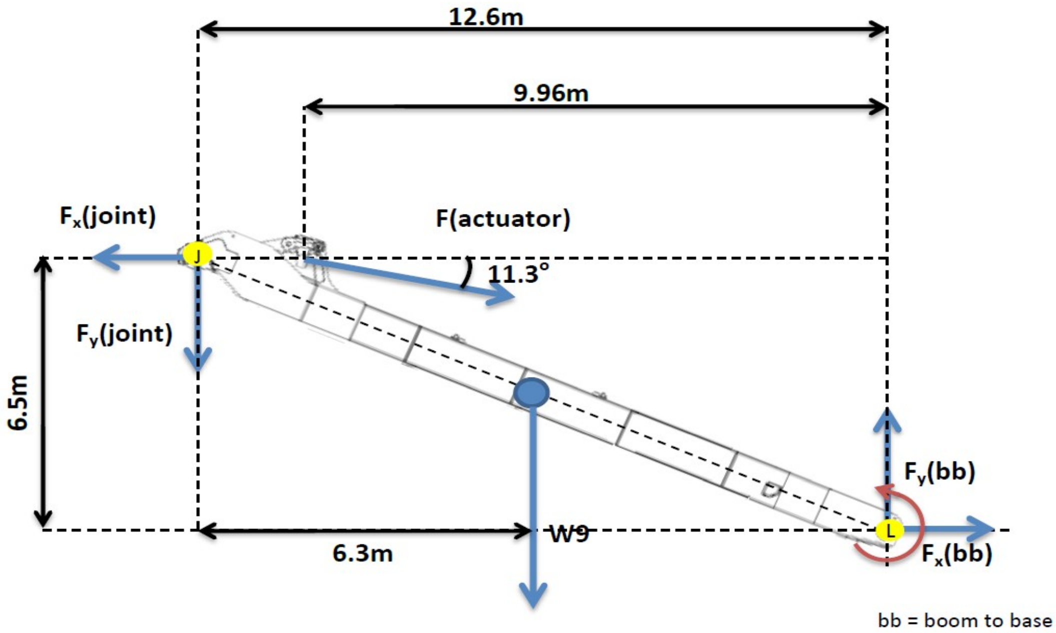

Fig. 3.7 Counterweight Positioning Frame – Part II

From Fig. 3.7, //something

$$F_x = -F_{x(joint)} + F_{act}\cos(11.3^o) + F_{x(bb)} = 0 \quad (3.16)$$
$$F_y = F_{y(joint)} + F_{act}\sin(11.3^o) + W_9 - F_{y(bb)} = 0 \quad (3.17)$$
$$M_j = -6.5\text{F}_{act}\cos(11.3^o) + 9.96\text{F}_{act}\sin(11.3^o) + 6.3\text{W}_9 + 12.6\text{F}_{y(joint)} + 6.5\text{F}_{x(joint)} = 0 \quad (3.18)$$

Unknown Name	Value (t)	Value (MN)
F_{act}	754.26	7399.29
$F_{x(bb)}$	492.85	4834.86
$F_{y(bb)}$	1307.29	12824.51

Table 3.6: Values of unknowns from equations 3.10, 3.11, and 3.12

3.7. Mast

//something

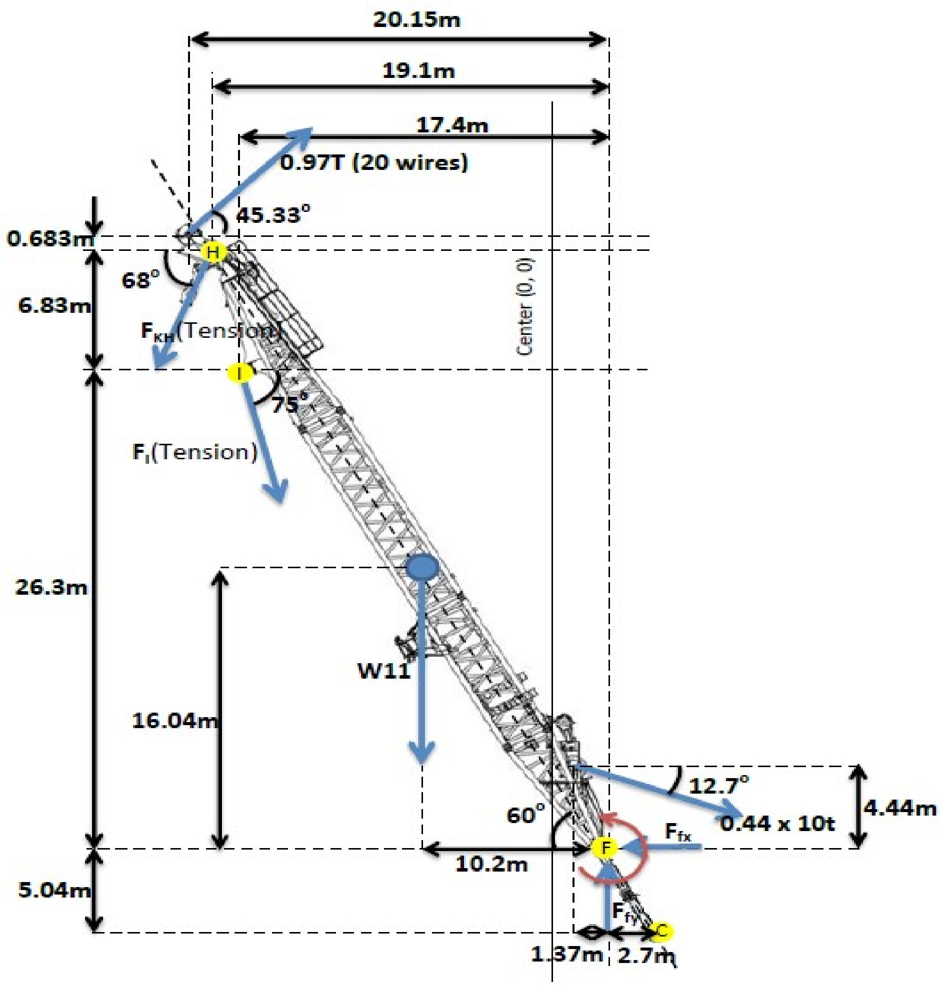

Fig. 3.8 Mast

From Fig. 3.8, //something

$$F_x = 0.97\text{T}\cos(45.33^o) - F_{kh}\cos(68^o) + F_1\cos(75^0) - F_{fx} + 0.44\frac{W_1}{20}\cos(12.7^o) = 0 \quad (3.19)$$

$$F_y = 0.97\text{T}\sin(45.33^o) - F_{kh}\sin(68^o) - F_1\sin(75^0) - W_{11} + F_{fy} - 0.44\frac{W_1}{20}\sin(12.7^o) = 0 \quad (3.20)$$

$$\begin{aligned} M_j = & -0.97\text{T}\left(33.81\cos(45.33^o) + 20.15\sin(45.33^o)\right) \\ & + F_{kh}\left(33.13\cos(68^o) + 19.1\sin(68^o)\right) + F_1\left(17.4\sin(75^o) - 26.3\cos(75^o)\right) \\ & + 10.2\text{W}_{11} + 0.44\frac{W_1}{20}\left(1.3\sin(12.7^o) - 4.44\cos(12.7^o)\right) = 0 \end{aligned} \quad (3.21)$$

Unknown Name	Value (t)	Value (MN)
F_1	250.02	2452.7
F_{fx}	275.82	2705.79
F_{fy}	548.65	5382.26

Table 3.7: Values of unknowns from equations 3.19, 3.20, and 3.21

Section 4: Structural Analysis of the Main Boom

4.1. Overview

While doing the structural analysis of the main boom, it has been assumed that all the inserts and components are attached by the means of single pins located at their edges. Also the components are assumed to be entirely composed of 2 force members, which allows us to study the components using the method of sections which otherwise was not possible as there were frames with 15 to 20 different components attached to it directly leaving the structure statistically indeterminate.

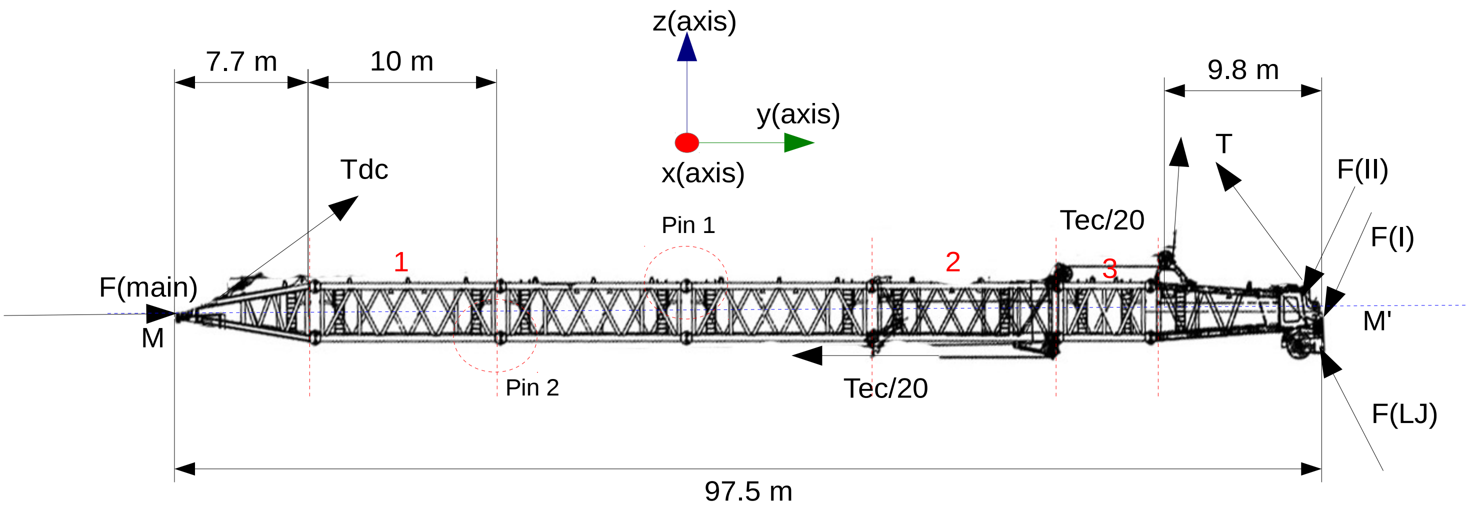

Fig 4.1. Main Boom

The main boom is composed of three main type of components:
1. Butt
2. Insert
3. Top

Our key interest is how this trusses behave under the loading conditions presented by this situation. Our primary focus will be on finding the loading difference on the inserts inside the main boom. For this purpose we shall study three different inserts, marked by 1, 2, and 3 in fig4.1. The reason for this selection is that all the inserts in from 1 to 2 are acting as load carriers across the main boom and are not being acted upon by any external forces. hence if we study one of the inserts positioned at the extreme(1 or 2) then the behavior of the remaining inserts from 1 to 2 can be simply understood by linearity. While insert 3 is being subjected to completely different loading thus it deserves separate study.

4.2. Inserts

To study the inserts we want to know the forces acting external to them. The problem which arises is that as all the components are attached to each other by means of 4 pins. This leaves the whole structure statistically indeterminate. The second problem is that we cannot directly study the cross section of a component itself as in the 3d structure there are at least 7 members passing through any cross section.

The other possibility is that we solve along the butt of the main boom, using the joints methods to obtain the loading. The problem with this calculation is that as we reach pins marked 3 and 4 in fig 4.1 (in the 3d structure total of four pins bind the insert 1 to the butt) we encounter at least 4 unknowns including 3 unknown forces exerted by the pins along the three axis and other unknown forces arising due to the members connecting the joints together. Also as we will see later in the study of the inserts, that some of the loadings cannot be determined by the joints method but have to be specified instead to calculate the forces acting on the members inside the insert.

To circumnavigate the problem of unknown forces we shall make some simplistic assumptions regarding this problem.

1. Due to the symmetry of the whole 3d structure of main boom about MM' in fig 4.1, we assume that the forces acting on the two pins attaching components along the top(ex pin 1 fig 4.1) will be same in magnitude and direction in a given axis. The same can be said about the pins at the bottom(ex pin 2 fig 4.1)

2. As the forces acting on the main boom do not have any components acting along the x axis as given in fig 4.1, and act the plane of symmetry of the main boom, we assume that the pins do not have to provide any axial force.

Hence from the above two assumptions the calculation of forces on pins attaching each component inside the main boom is reduced to 4 unknowns.

Calculation of forces on the pins.

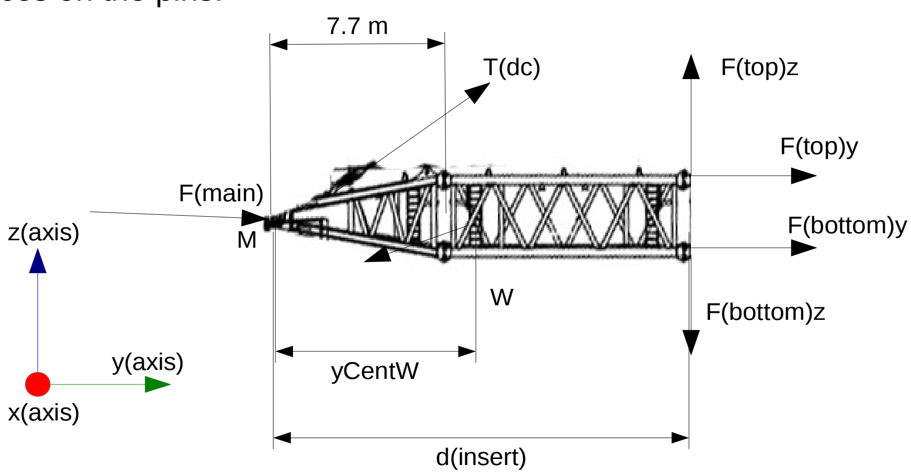

Fig. 4.2. Insert

Suppose that the pins of the inserts are located at a y distance of d_{insert} , with weight W located at d_W from fig 4.2

$$F_y = 2\mathrm{F}_{(top)y} + 2\mathrm{F}_{(bottom)y} + T_{dc}\cos(15.7^o) + F_{main}\cos(0.66^o) = 0 \qquad (4.1)$$
$$F_z = 2\mathrm{F}_{(top)z} - 2\mathrm{F}_{(bottom)z} + T_{dc}\sin(15.7^o) - F_{main}\sin(0.66^o) = 0 \qquad (4.2)$$
$$M_M = (2, d_{insert}, 1.6) \times (0, F_{(top)y}, F_{(top)z}) + (-2, d_{insert}, 1.6) \times (0, F_{(top)y}, F_{(top)z})$$
$$(2, d_{insert}, -1.6) \times (0, F_{(bottom)y}, -F_{(bottom)z}) + (-2, d_{insert}, -1.6) \times (0, F_{(bottom)y}, -F_{(bottom)z}) = 0$$
$$M_M = (-3.2\mathrm{F}_{(top)y} + 2\mathrm{d}_{insert} F_{(top)z} + 3.2\mathrm{F}_{(bottom)y} - 2\mathrm{d}_{insert} F_{(bottom)z})\,\hat{i} = 0 \qquad \text{eq 4.3}$$

From eq 4.1, 4.2, and 4.3 it is clear that the assumption 1 and 2 are not sufficient to determine the unknown forces in the pins as we have only three equations. Hence in equation 4.2 and 4.3 let

$F_{(top)z} - F_{(bottom)z} = F_{(net)z}$ eq 4.4

therefore

$F_y = 2\mathrm{F}_{(top)y} + 2\mathrm{F}_{(bottom)y} + T_{dc}\cos(15.7^o) + F_{main}\cos(0.66^o) = 0$ eq 4.5

$F_z = 2\mathrm{F}_{(net)z} + T_{dc}\sin(15.7^o) - F_{main}\sin(0.66^o) = 0$ eq 4.6

$M_M = (-3.2\mathrm{F}_{(top)y} + 2\mathrm{d}_{insert}F_{(net)z} + 3.2\mathrm{F}_{(bottom)y})\hat{i} = 0$ eq 4.7

Hence the above three equations model the forces on the sections of main boom taken with respect to inserts. Solving the above equations we get the values given in the Table 4.1

d (m)	F(net)z (MN)	F(top)y (MN)	F(bottom)y (MN)
7.70	-900.48	-7334.38	-3000.83
17.72	-900.48	-10154.01	-181.21
27.70	-900.48	-12962.37	2627.16
37.70	-900.48	-15776.37	5441.15
47.70	-900.48	-18590.36	8255.14
57.70	-900.48	-21404.36	11069.14
67.70	-900.48	-24218.35	13883.13
77.70	-900.48	-27032.34	16697.13
87.70	-900.48	-29846.34	19511.12

Table 4.1: Forces acting on the cross sections of the main boom

Regarding the calculation of the remaining unknowns let us consider the butt of the main boom Fig 4.3:

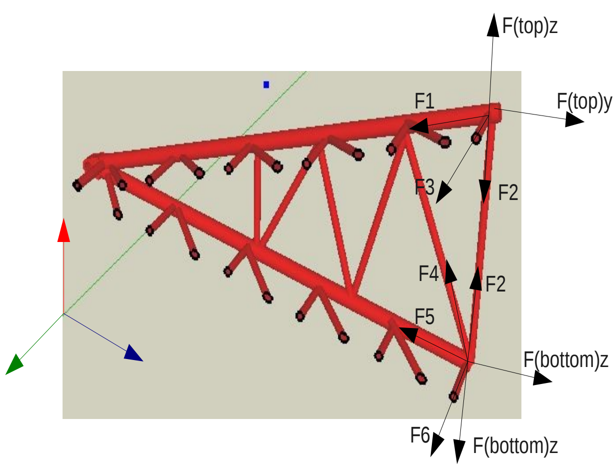

Fig. 4.3. Butt

From fig 4.3, considering the forces lying in the y-z plane

$\vec{F}_1 + \vec{F}_2 + \vec{F}_{(top)z} + \vec{F}_{(top)y} = 0$ eq 4.8

$\vec{F}_4 + \vec{F}_5 + \vec{F}_2 + \vec{F}_{(bottom)z} + \vec{F}_{(bottom)y} = 0$ eq 4.9

Combining above two equations using $\vec{F}_2$, we get

$$\vec{F}_{(top)z} - \vec{F}_{(bottom)z} = \vec{F}_4 + \vec{F}_5 + \vec{F}_{(bottom)y} - \vec{F}_1 - \vec{F}_{(top)y} \quad \text{eq 4.10}$$

It is clear that the joint method will not be sufficient to compute $F_{(bottom)z}$ and $F_{(top)z}$ as they yield equation which is similar to eq 4.4. This problem arises in each and every truss composing the main boom as the top joint and bottom joints are connected directly by a two force member.

The second option to this problem is to analyze the structure by considering the deformations in order to calculate the unknowns. The structure is far too complex for this method. Hence to circumnavigate this problem we shall make one more assumptions

3. The pins at the bottom provide force along the y axis only hence $F_{(bottom)z}$ for both bottom pins = 0. Thus $F_{(top)z}$ = $F_{(net)z}$

From table 4.1, we can obtain the forces acting on the inserts inside the main boom. For ex. Taking insert 1, the forces on the insert will be as given in Fig 4.4.

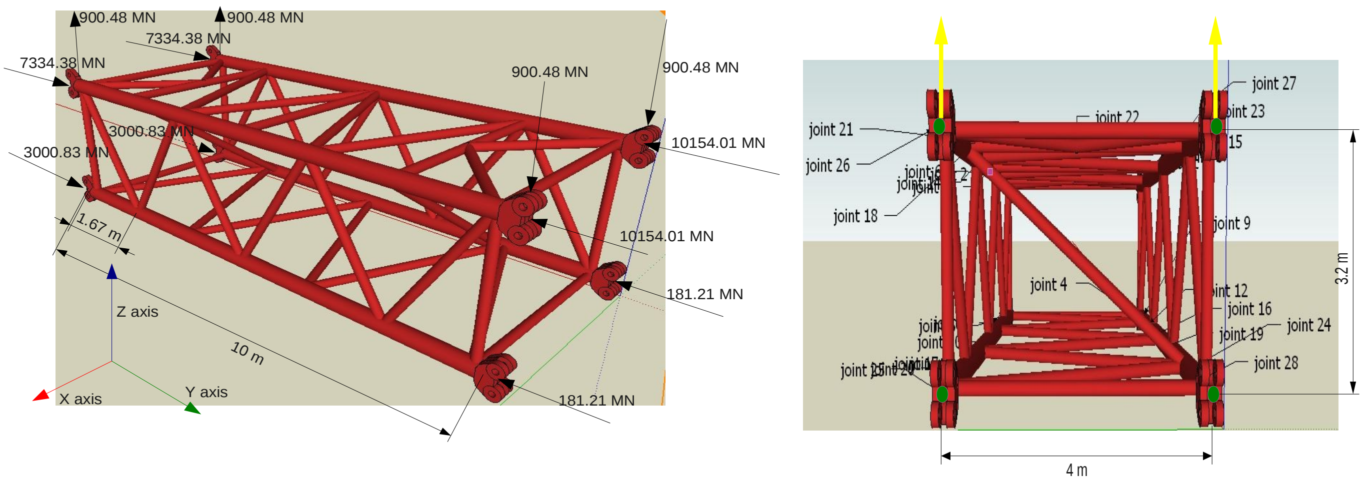

Fig. 4.4.

By referring to table 4.1, we can see that except insert 1 and its adjacent insert, every insert up till insert 2 suffers from compression on the top while shear force in the y direction on the bottom(with orientation as in the Fig 4.3).
For further study we will focus exclusively on insert 1 and insert 2

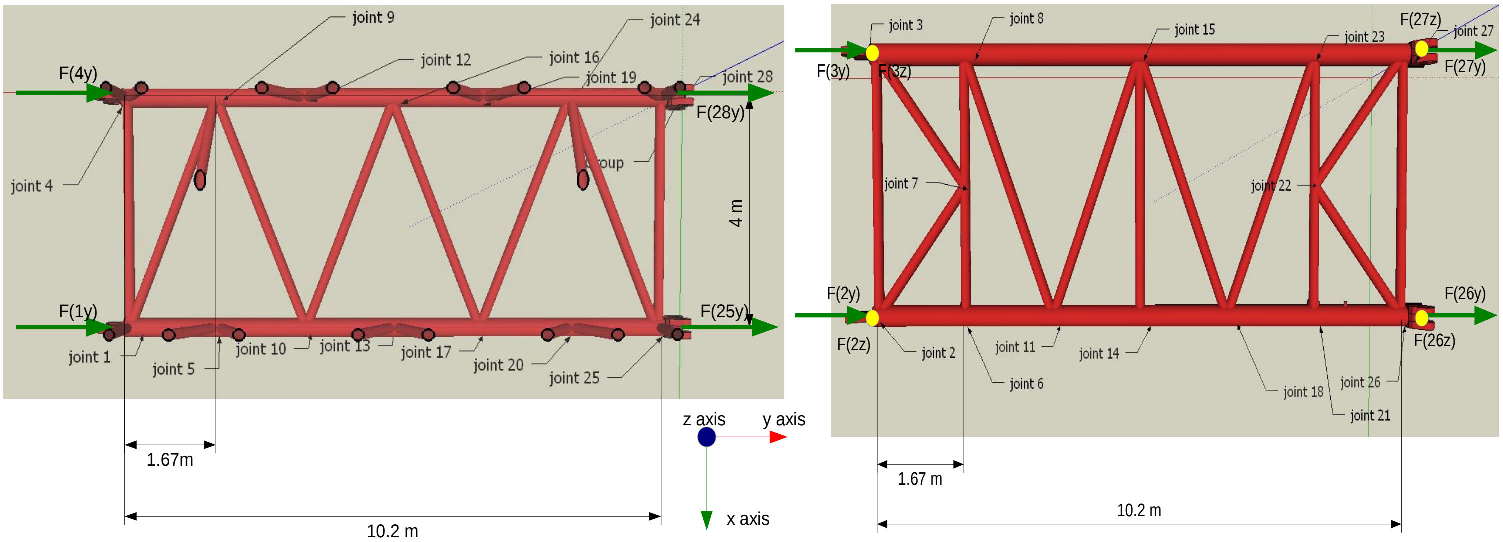

Fig. 4.5.

To establish nomenclature regarding the forces acting on members:

1. All the forces are supposed to be compressive
2. F_{1-5} means a force acting on joint 1 and points to joint 5. The force is tensile in nature on the member connecting joint 1 to 5. The equal and opposite force will F_{5-1} .

3. Members will be named for example 1-5 meaning the member connects joint 1 and joint 5, with smaller joint number in the lead.

Equations for joint-based study of inserts

Joint 1

$$F_x = -F_{1-4} - F_{1-9}\cos(22.66^o) = 0$$
$$F_y = F_{1-9}\sin(22.66^o) + F_{1-5} + 1_y = 0$$
$$F_z = F_{1-2} = 0$$

Joint 2

$$F_x = -F_{2-3} - F_{2-7}\cos(39.86^o) = 0$$
$$F_y = F_{2-6} + F_{2-7}\sin(39.86^o) + F_{2-5}\cos(62.44^o) + 2_y = 0$$
$$F_z = -F_{2-1} - F_{2-5}\sin(62.99^o) + 2_z = 0$$

Joint 3

$$F_x = F_{3-2} + F_{3-7}\cos(39.86^o) = 0$$
$$F_y = F_{3-7}\sin(39.86^o) + F_{3-8} + 3_y = 0$$
$$F_z = -F_{3-4} + 3_z = 0$$

Joint 4

$$F_x = F_{4-1} = 0$$
$$F_y = F_{4-9} + F_{4-8}\cos(62.44^o) + 4_y = 0$$
$$F_z = F_{4-8}\sin(62.44^o) + F_{4-3} = 0$$

Joint 5

$$F_x = 0$$
$$F_y = F_{5-10} - F_{5-1} + F_{5-11}\cos(62.44^O) - F_{5-2}\cos(62.44^o) = 0$$

$F_z = F_{5-11}\sin(62.44^o) + F_{5-2}\sin(62.44^o) = 0$

Joint 6

$F_x = -F_{6-7} - F_{6-9}\cos(36.66^o) = 0$

$F_y = F_{6-11} - F_{6-2} = 0$

$F_z = -F_{6-9}\sin(36.66^o) = 0$

Joint 7

$F_x = F_{6-7} - F_{7-8} + F_{7-2}\cos(39.86^o) - F_{7-3}\cos(39.86^o) = 0$

$F_y = -F_{7-2}\sin(39.86^o) - F_{7-3}\sin(39.86^o) = 0$

Joint 8

$F_x = F_{8-7} + F_{8-11}\cos(22.66^o) = 0$

$F_y = F_{8-11}\sin(22.66^o) + F_{8-15} - F_{8-3} + F_{8-12}\cos(62.44^o) - F_{8-4}\cos(62.44^o) = 0$

$F_z = -F_{8-12}\sin(62.44^o) - F_{8-4}\sin(62.44^o) = 0$

Joint 9

$F_x = F_{9-10}\cos(22.66^o) + F_{9-1}\cos(22.66^o) + F_{9-6}\cos(38.66^o) = 0$

$F_y = F_{9-12} - F_{9-4} + F_{9-10}\sin(22.66^o) - F_{9-1}\sin(22.66^o) = 0$

$F_z = F_{9-6}\sin(36.66^o) = 0$

Joint 10

$F_x = -F_{10-16}\cos(22.66^o) - F_{10-9}\cos(22.66^o) = 0$

$F_y = F_{10-13} - F_{10-5} + F_{10-16}\sin(22.66^o) - F_{10-9}\sin(22.66^o)$

Joint 11

$F_x = -F_{11-15}\cos(22.66^o) - F_{11-8}\cos(22.66^o) = 0$

$F_y = F_{11-14} - F_{11-6} + F_{11-15}\sin(22.66^o) - F_{11-8}\sin(22.66^o) + F_{11-13}\sin(27.56^o) - F_{11-5}\cos(27.56^o) = 0$

$F_z = -F_{11-13}\cos(27.66^o) - F_{11-5}\cos(27.56^o) = 0$

Joint 12

$F_y = F_{12-16} - F_{12-9} + F_{12-15}\cos(62.44^o) - F_{12-8}\cos(62.44^o) = 0$

$F_z = F_{12-15}\sin(62.44^o) + F_{12-8}\sin(62.44^o) = 0$

Joint 13

$F_x = 0$

$F_y = F_{13-17} - F_{13-10} + F_{13-18}\cos(62.44^o) - F_{13-11}\cos(62.44^o) = 0$

$F_z = F_{13-18}\sin(62.44^o) + F_{13-11}\sin(62.44^o) = 0$

Joint 14

$F_y = F_{14-18} - F_{14-11} = 0$

$F_z = -F_{14-15} = 0$

Joint 15

$F_x = F_{15-14} + F_{15-18}\cos(22.66^o) + F_{15-11}\cos(22.66^o) = 0$

$F_y = F_{15-18}\sin(22.66^o) + F_{15-23} - F_{15-11}\sin(22.66^o) - F_{15-8} + F_{15-19}\cos(62.44^o) - F_{15-12}\cos(62.44^o) = 0$

$F_z = -F_{15-19}\sin(62.44^o) - F_{15-12}\sin(62.44^o) = 0$

Joint 16

$F_x = F_{16-17}\cos(22.66^o) + F_{16-10}\cos(22.66^o) = 0$

$F_y = F_{16-17}\sin(22.66^o) + F_{16-19} - F_{16-12} - F_{16-10}\sin(22.66^o) = 0$

Joint 17

$F_x = -F_{17-24}\cos(22.66^o) - F_{17-16}\cos(22.66^o) = 0$

$F_y = F_{17-24}\sin(22.66^o) - F_{17-16}\sin(22.66^o) + F_{17-20} - F_{17-13} = 0$

Joint 18

$F_x = -F_{18-23}\cos(22.66^o) - F_{18-15}\cos(22.66^o) = 0$

$F_y = F_{18-21} - F_{18-14} + F_{18-23}\sin(22.66^o) - F_{18-15}\sin(22.66^o) + F_{18-20}\cos(62.44^o) - F_{18-13}\cos(62.44^o) = 0$

$F_z = -F_{18-20}\sin(62.44^o) - F_{18-13}\sin(62.44^o) = 0$

Joint 19

$F_y = F_{19-24} - F_{19-16} + F_{19-23}\cos(62.44^o) - F_{19-15}\cos(62.44^o) = 0$

$F_z = F_{19-23}\sin(62.44^o) + F_{19-15}\sin(62.44^o) = 0$

Joint 20

$F_y = F_{20-25} - F_{20-17} + F_{20-26}\cos(62.44^o) - F_{20-18}\cos(62.44^o) = 0$

$F_z = F_{20-26}\sin(62.44^o) + F_{20-18}\sin(62.44^o) = 0$

Joint 21

$F_x = -F_{21-22} - F_{21-24}\cos(38.66^o) = 0$

$F_y = F_{21-26} - F_{21-18} = 0$

$F_z = -F_{21-24}\sin(38.66^o) = 0$

Joint 22

$F_x = F_{22-21} - F_{22-23} + F_{22-26}\cos(39.86^o) - F_{22-27}\cos(39.86^o) = 0$

$F_y = F_{22-26}\sin(39.86^o) + F_{22-27}\sin(39.86^o) = 0$

Joint 23

$F_x = F_{23-22} + F_{23-18}\cos(22.66^o) = 0$

$F_y = F_{23-27} - F_{23-15} - F_{23-18}\sin(22.66^o) + F_{23-28}\cos(62.44^o) - F_{23-19}\cos(62.44^o) = 0$

$F_z = -F_{23-28}\sin(62.44^o) - F_{23-19}\sin(62.44^o) = 0$

Joint 24

$F_x = F_{24-21}\cos(38.66^o) + F_{24-25}\cos(22.66^o) + F_{24-17}\cos(22.66^o) = 0$

$F_y = F_{24-25}\sin(22.66^o) - F_{24-17}\sin(22.66^o) + F_{29-28} - F_{24-19} = 0$

$F_z = F_{24-21}\sin(38.66^o) = 0$

Joint 25

$F_x = -F_{25-28} - F_{25-24}\cos(22.66^o) = 0$

$F_y = -F_{25-20} + 25_y - F_{25-24}\sin(22.66^o) = 0$

$F_z = F_{25-26} = 0$

Joint 26

$F_x = -F_{26-27} - F_{26-22}\cos(39.86^o) = 0$

$F_y = -F_{26-21} - F_{26-22}\sin(39.86^o) - F_{26-20}\cos(62.44^o) + 26_y = 0$

$F_z = -F_{26-25} - F_{26-20}\sin(62.44^o) + 26_z = 0$

Joint 27

$F_x = F_{27-26} + F_{27-22}\cos(39.86^o) = 0$

$F_y = -F_{27-23} - F_{27-22}\sin(39.86^o) + 27_y = 0$

$F_z = -F_{27-28} + 27_z = 0$

Joint 28

$F_x = F_{28-25} = 0$

$F_y = -F_{28-23}\cos(62.44) - F_{28-24} + 28_y = 0$

$F_z = F_{28-27} + F_{28-23}\sin(62.44^o) = 0$

Iteration 1

In the first iteration all the listed equations for joints were evaluated in a plug and play from, where the values of the known were used to find the unknown.

Member name	Force (MN)	Member name	Force (MN)	Member name	Force (MN)	Member name	Force (MN)
1-2	0.00	6-9	0.00	14-15	0.00	21-22	0.00
1-4	0.00	8-12	1015.74	15-19	1015.74	21-24	0.00
1-5	-3000.83	9-10	0.00	16-17	0.00	23-28	1015.74
1-9	0.00	9-12	-2530.87	16-19	-1590.95	24-25	0.00
2-5	1015.74	10-13	-2060.91	17-20	-1120.99	24-28	-651.03
3-4	900.48	10-16	0.00	17-24	0.00	25-26	0.00
4-8	-1015.74	11-13	1015.74	18-20	1015.74	25-28	0.00
4-9	-2530.87	12-15	-1015.74	19-23	-1015.74	27-28	-900.48
5-10	-2060.91	12-16	-1590.95	19-24	-651.03		
5-11	-1015.74	13-17	-1120.99	20-25	-181.07		
6-7	0.00	13-18	-1015.74	20-26	-1015.74		

Table 4.2: Values obtained in first iteration. +ve value means force is tensile, -ve means it is compressive. Values corresponding to insert 1

Iteration 2

Using F_x equation for joint 2 and joint 3, combining using $F_{3-2}=F_{2-3}$ we get
$-F_{2-7}\cos(39.86^o)+F_{3-7}\cos(39.86^o)=0$
using the above equation and F_y for joint 7, and solving as a linear system. $F_{2-7}=F_{3-7}=0$, all the remaining values were obtained by plug and play.

Member name	Force (MN)	Member name	Force (MN)	Member name	Force (MN)
2-3	0.00	8-15	-27972.26	21-26	-29382.14
2-6	-27502.30	11-14	-28442.22	22-23	0.00
2-7	0.00	11-15	0.00	22-26	0.00
3-7	0.00	14-18	-28442.22	22-27	0.00
3-8	-27032.34	15-18	0.00	23-27	-29852.10
6-11	-27502.30	15-23	-28912.18	26-27	0.00
7-8	0.00	18-21	-29382.14		
8-11	0.00	18-23	0.00		

Table 4.3: Values obtained in iteration 2. +ve values are tensile, -ve are compressive. Values corresponding to insert 1.

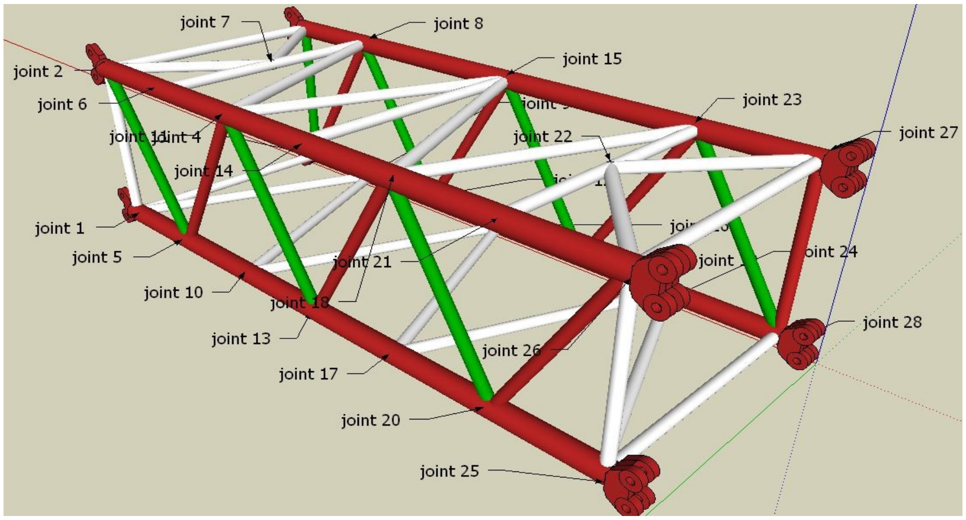

Fig 4.6. Insert (White: Zero Force Member, Green: Tension, Red: Compression)

Discussion:

Because of our assumption that the inserts are entirely composed of two force members, we see that the behaviour of inserts change from a 3-d to a 2-d structure. In the model of the inserts, the main horizontal load carrying beams along the top and bottom suffer extensive compressive forces(range ~30,000 to ~2500 MN-compressive), hence justifying their larger diameter as compared to that of members connecting these main beams together. Among the cross connecting, nearly half suffer from compression while others suffer from tension. With regrads to material suitable for selection, the truss has to be composed of material which performs well under both tension and compression.

Our simplified model behaves separately to the forces acting on it in the x or y direction. Any force in y-z plane is not transferred to the members lying in the x-y plane and vice versa. The reason for this is that our assumption, of two force member truss, renders the diagonal members inside the truss useless(member 6-9 and 21-24). The purpose of these members is force distribution. If we are able to analyze the structure using more powerful mathematical tools we will see that any deformation of the insert along y-z will deform the diagonal bars as well. Hence they will induce forces on the members lying along the x-y plane. The same can be said about members in y-z plane if deformation of insert occurs in x-y plane initially.

Insert 3 (With Guiding Wire)

Insert 3 is not much different from the regular insert previously introduced other than the loading – the forces exerted by the guiding wire on one end of the insert in opposite directions parallel to the insert. A free body diagram is presented in Fig. 4.7 below.

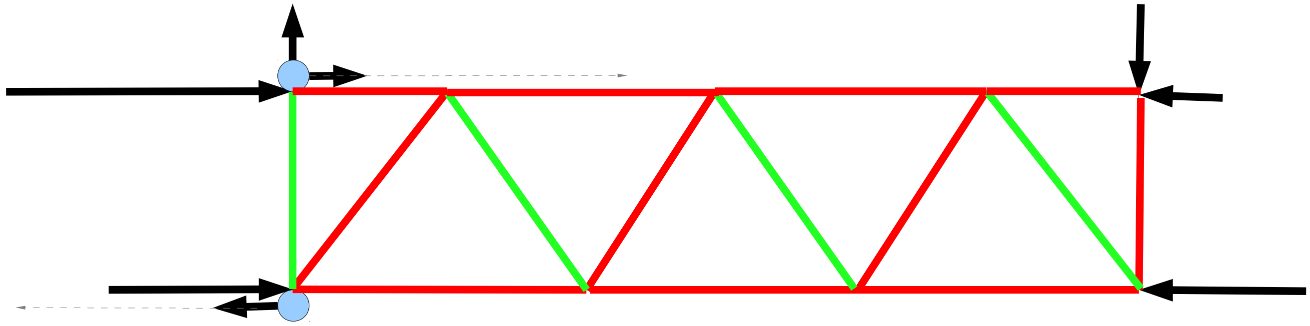

Fig. 4.7. Insert 3 (With Guiding Wire)
(Green: Tension, Red: Compression)

Since the forces exerted by the wire are insignificant compared to the main external forces acting on the two ends of the insert, the general behaviour of the truss is the same. We therefore do not put extra emphasis and perform further qualitative analysis on this component. In this simplified model built for qualitative analysis, the two segments of the wire are parallel to the main axis of the insert, the pulleys are set to be mounted on the left end of the insert, and the tensile forces in the opposite directions exerted by the wire are assumed to be acting exactly on the top and the bottom joints on the left end of the insert.

4.3. Main boom top

External FBD:

We have 5 external forces acting on main boom top, F(I), F(II), F(LJ), T, and Tce, since the point of application of Tce is close to the top left end; an assumption is made so that force Tce is distributed evenly in the two joints illustrated in Figure 4.8 below:

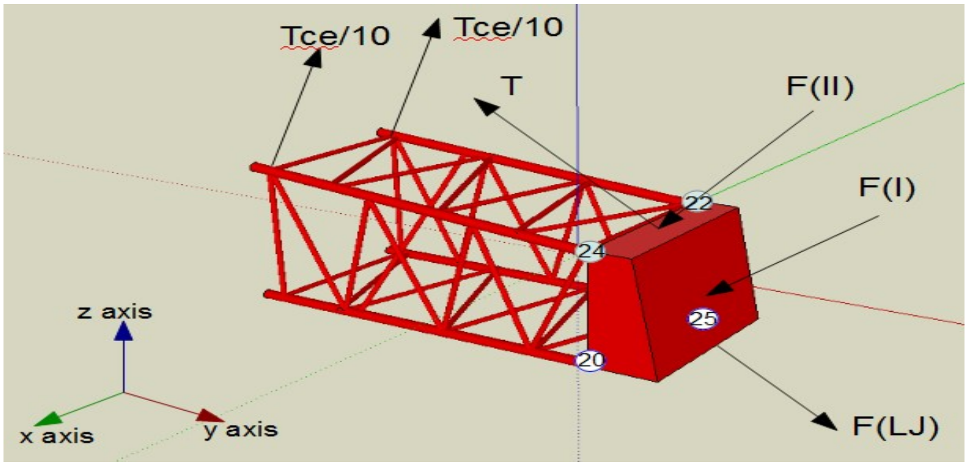

Fig. 4.8. Main Boom Top

End Joints FBD:

Now we only look at the red box where the four forces T, F(II), F(I), and F(LJ) are acting. Note that those forces are in the yz-plane (x = 0), we assume those four forces to be distributed evenly to left (+ve x) and right (-ve x) joints. T and F(II) are assumed to only affect the joints 24 and 22, F(LJ) are assumed to affect only joints 20 and 25. Since F(I) is acting in the middle of the red box, equal distribution of F(I) into all top and bottom joints (20,24,22,25) seems reasonable.

$$F_{(top)}=\sqrt{\left(T_X+F_{IIx}+\frac{F_{Ix}}{2}\right)^2+\left(T_y+F_{IIy}+\frac{F_{Iy}}{2}\right)^2} \quad \text{eq 4.11}$$

$$F_{(bottom)}=\sqrt{\left(F_{LJx}+\frac{F_{Ix}}{2}\right)^2+\left(F_{LJy}+\frac{F_{Iy}}{2}\right)^2} \quad \text{eq 4.12}$$

F(top) = 10597.1MN and F(bottom) = 10415.24MN, by breaking the forces into y and z components we obtain Fig. 4.9:

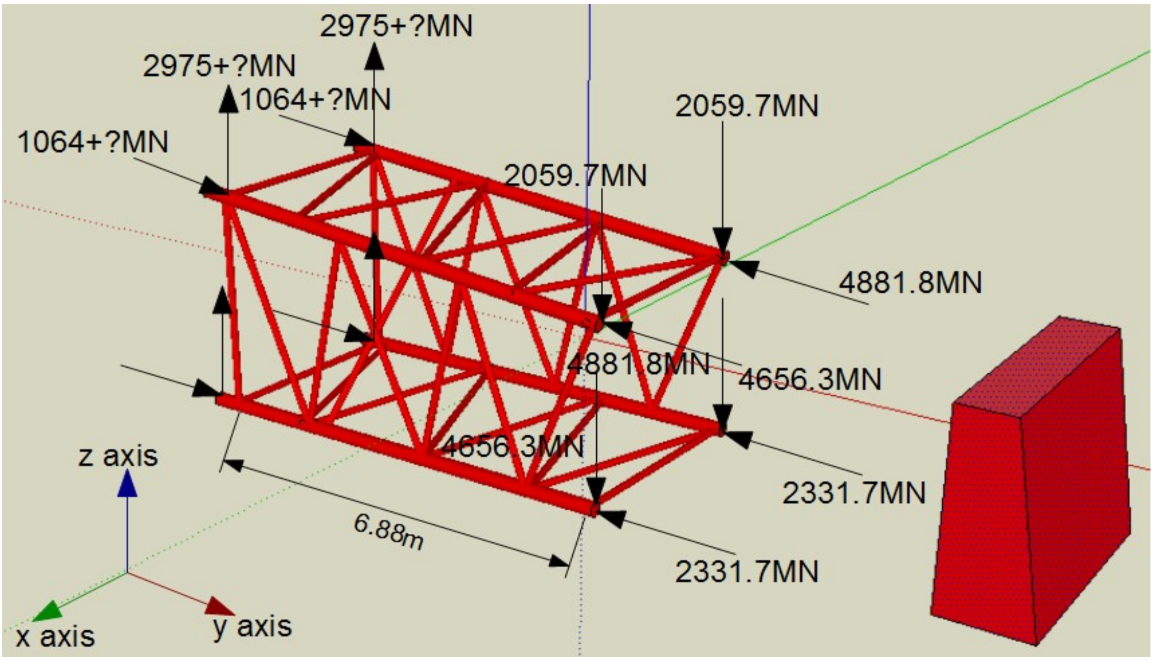

Fig. 4.9.

Joint method:

All the external forces are now known, hence we use method of joints to calculate all the internal forces in the two force members. Fig. 4.10 labels all joints with numbers from 1 to 24:

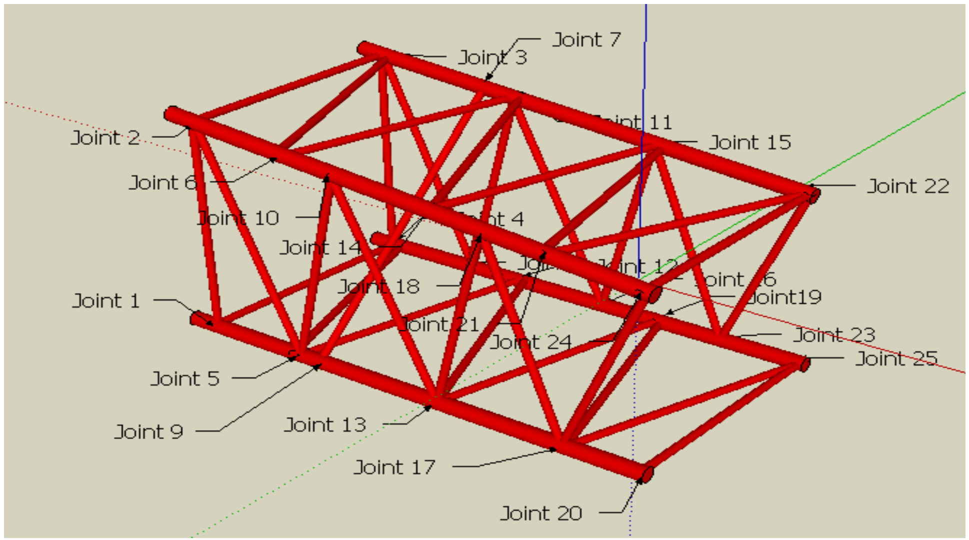

Fig. 4.10.

Equations of Joint Based Study of Main Boom Top:

Joint1:

$F_x = F_{(1-4)} = 0$

$F_y = ? + F_{(1-5)}$

$F_z = F_{(1-2)} = 0$

Joint2:

$F_x = -F_{(2-3)} = 0$

$F_y = 1064.03 + ? + F_{(2-6)}$

$F_z = -F_{(2-1)} - F_{(2-5)} \cos(66.3) + 2975 + ?$

Joint3:

$F_x = F_{(3-2)} = 0$

$F_y = 1064.03 + ? + F_{(3-7)}$

$F_z = -F_{(3-4)} - F_{(3-8)} \cos(66.3) + 2975 + ?$

Joint4:

$F_x = F_{(4-1)} = 0$

$F_y = ? + F_{(4-8)} + F_{(4-5)} \cos(66.3)$

$F_z = F_{(4-3)}$

Joint5:

$F_x = -F_{(5-4)} \sin(66.3) - F_{(5-12)} \sin(66.3)$

$F_y = -F_{(5-1)} - F_{(5-2)} \cos(66.3) - F_{(5-4)} \cos(66.3) + F_{(5-12)} \cos(66.3) + F_{(5-10)} \cos(66.3) + F_{(5-9)}$

$F_z = F_{(5-2)} \sin(66.3) + F_{(5-10)} \sin(66.3)$

Joint6:

$F_x = -F_{(6-3)} \sin(66.3) - F_{(6-11)} \sin(66.3)$

$F_y = -F_{(6-2)} - F_{(6-3)} \cos(66.3) + F_{(6-11)} \cos(66.3) + F_{(6-14)}$

$F_z = 0$

Joint7:

$F_x = F_{(7-9)} \cos(45)\cos(8.9)$
$F_y = -F_{(7-3)} + F_{(7-9)} \cos(45)\sin(8.9) + F_{(7-11)}$
$F_z = -F_{(7-9)} \sin(45)\cos(8.9)$

Joint8:

$F_X = 0$
$F_y = -F_{(8-4)} - F_{(8-3)} \cos(66.3) + F_{(8-11)} \cos(66.3) + F_{(8-12)}$
$F_z = F_{(8-3)} \sin(66.3) + F_{(8-11)} \sin(66.3)$

Joint9:

$F_x = -F_{(9-7)} \cos(45)\cos(8.9)$
$F_y = -F_{(9-5)} - F_{(9-7)} \cos(45)\sin(8.9) + F_{(9-13)}$
$F_z = F_{(9-7)} \sin(45)\cos(8.9)$

Joint10:

$F_x = 0$
$F_y = -F_{(10-6)} - F_{(10-5)} \cos(66.3) + F_{(10-13)} \cos(66.3) + F_{(10-14)}$
$F_z = -F_{(10-5)} \sin(66.3) - F_{(10-13)} \sin(66.3)$

Joint11:

$F_x = F_{(11-6)} \sin(66.3) + F_{(11-14)} \sin(66.3)$
$F_y = -F_{(11-7)} - F_{(11-6)} \cos(66.3) - F_{(11-8)} \cos(66.3) + F_{(11-15)} + F_{(11-14)} \cos(66.3) + F_{(11-16)} \cos(66.3)$
$F_z = -F_{(11-8)} \sin(66.3) - F_{(11-16)} \sin(66.3)$

Joint12:

$F_x = F_{(12-5)} \sin(66.3) + F_{(12-13)} \sin(66.3)$
$F_y = -F_{(12-5)} \cos(66.3) - F_{(12-8)} + F_{(12-13)} \cos(66.3) + F_{(12-16)}$
$F_z = 0$

Joint13:

$F_x = -F_{(13-12)} \sin(66.3) - F_{(13-19)} \sin(66.3)$
$F_y = -F_{(13-9)} - F_{(13-12)} \cos(66.3) - F_{(13-10)} \cos(66.3) + F_{(13-18)} \cos(66.3) + F_{(13-19)} \cos(66.3) + F_{(13-17)}$
$F_z = F_{(13-10)} \sin(66.3) + F_{(13-18)} \sin(66.3)$

Joint14:

$F_x = -F_{(14-11)} \sin(66.3) - F_{(14-15)} \sin(66.3)$
$F_y = -F_{(14-10)} - F_{(14-11)} \cos(66.3) + F_{(14-15)} \cos(66.3) + F_{(14-18)}$
$F_z = 0$

Joint15:

$F_x = F_{(15-14)} \sin(66.3) + F_{(15-21)} \sin(66.3)$
$F_y = -F_{(15-11)} - F_{(15-16)} \cos(66.3) - F_{(15-14)} \cos(66.3) + F_{(15-21)} \cos(66.3) + F_{(15-23)} \cos(66.3) + F_{(15-22)}$

$F_z = -F_{(15-16)}\sin(66.3) - F_{(15-23)}\sin(66.3)$

Joint16:

$F_x = 0$

$F_y = -F_{(16-12)} - F_{(16-11)}\cos(66.3) + F_{(16-15)}\cos(66.3) + F_{(16-19)}$

$F_z = F_{(16-11)}\sin(66.3) + F_{(16-15)}\sin(66.3)$

Joint17:

$F_x = -F_{(17-19)}\sin(66.3) - F_{(17-25)}\sin(66.3)$

$F_y = -F_{(17-13)} - F_{(17-18)}\cos(66.3) - F_{(17-19)}\cos(66.3) + F_{(17-24)}\cos(66.3) + F_{(17-25)}\cos(66.3) + F_{(17-20)}$

$F_z = F_{(17-18)}\sin(66.3) + F_{(17-24)}\sin(66.3)$

Joint18:

$F_x = 0$

$F_y = -F_{(18-14)} - F_{(18-13)}\cos(66.3) + F_{(18-17)}\cos(66.3) + F_{(18-21)}$

$F_z = -F_{(18-13)}\sin(66.3) - F_{(18-17)}\sin(66.3)$

Joint19:

$F_x = F_{(19-13)}\sin(66.3) + F_{(19-17)}\sin(66.3)$

$F_y = -F_{(19-16)} - F_{(19-13)}\cos(66.3) + F_{(19-17)}\cos(66.3) + F_{(19-23)}$

$F_z = 0$

Joint20:

$F_x = -F_{(20-25)} = 0$

$F_y = -F_{(20-17)} - 2331.7$

$F_z = -4656.3$

Joint21:

$F_x = -F_{(21-15)}\sin(66.3) - F_{(21-22)}\sin(66.3)$

$F_y = -F_{(21-18)} - F_{(21-15)}\cos(66.3) + F_{(21-22)}\cos(66.3) + F_{(21-24)}$

$F_z = 0$

Joint22:

$F_x = F_{(22-24)} + F_{(22-21)}\sin(66.3)$

$F_y = -F_{(22-15)} - F_{(22-23)}\cos(66.3) - F_{(22-21)}\cos(66.3) - 4881.8$

$F_z = -F_{(22-23)}\sin(66.3) - 2059.7$

Joint23:

$F_x = 0$

$F_y = -F(23-19) - F_{(23-15)}\cos(66.3) + F_{(23-22)}\cos(66.3) + F_{(23-25)}$

$F_z = F_{(23-15)}\sin(66.3) + F_{(23-22)}\sin(66.3)$

Joint24:

$F_x = -F_{(24-22)} = 0$

$F_y = -F_{(24-21)} - F_{(24-17)}\cos(66.3) - 4881.8$

$F_z = -F_{(24-17)}\sin(66.3) - 2059.7$

Joint25:

$F_x = F_{(25-17)}\sin(66.3) + F_{(25-20)}$

$F_y = -F_{(25-23)} - F_{(25-17)}\cos(66.3) - 2331.7$

$F_z = -4656.3$

Now problems arise at Joint 25 and 20 where Fz is not equal to zero. We can bypass these issues by assuming that Fz are acting on Joint 24 and 22 instead, which is reasonable since Joint 25 and 20 do not have two force member supports in the z-direction and hence all the vertical loading must be taken by members 24-17 and 22-23.

Therefore, Fz = 0 for Joint 25 and 20, and the Fz equations for Joints 22 and 24 becomes

Joint22:

$F_z = -F_{(22-23)} \sin(66.3) - 6716$

Joint24:

$F_z = -F_{(24-17)} \sin(66.3) - 6176$

Calculate forces in each member; values are listed in table 4.3:

Member	Force(MN)	Member	Force(MN)	Member	Force(MN)	Member	Force(MN)
1-4	0	15-16	-6744.84	11-14	0	5-4	0
1-2	0	11-16	6744.84	14-15	0	25-23	-2331.7
2-3	0	17-18	6744.84	15-21	0	17-13	-6877.8
2-5	6744.84	13-18	-6744.84	21-22	0	13-9	-11423.96
3-8	6744.84	17-20	-2331.7	22-24	0	9-5	-11423.96
3-4	0	22-23	-6744.84	17-25	0	5-1	-11423.96
7-9	0	15-23	6744.84	17-19	0	19-16	-7753.64
5-10	-6744.84	17-24	-6744.84	13-19	0	16-12	-13175.99
8-11	-6744.84	3-6	0	12-13	0	12-8	-13175.99
10-13	6744.84	6-11	0	5-12	0	14-10	-8030.88
24-21	-2170.72	21-18	-2170.72	18-14	-8030.88	10-6	-2608.65
23-19	-7753.84	6-2	-2608.65	8-4	-18598.1		
2-6	-2608.65	3-7	-2608.65	4-8	-11423.96		

Table 4.3: Member Forces of Main Boom Top (+ve tension / -ve compression)

Pay attention to Joint 25, in Fig. 4.9 we clearly see a force polygon consists of -2331MN, F(25-17),F(25-20),and F(25-23), however since F(25-20) is zero from Fx equation of Joint 20, the force polygon reduces to just a line of force along the y-axis.

$$F_{(25-17)} = F_{(17-19)} = F_{(19-13)} = F_{(13-12)} = F_{(12-5)} = F_{(5-4)} = 0 \quad \text{eq 4.13}$$

$$F_{(24-17)} = -F_{(17-18)} = F_{(18-13)} = -F_{(13-10)} = F_{(10-5)} = -F_{(5-2)} = -5655.13 \quad \text{eq 4.14}$$

$$F_{(22-13)} = -F_{(23-15)} = F_{(15-16)} = -F_{(16-11)} = F_{(11-8)} = -F_{(8-3)} = 5655.13 \quad \text{eq 4.15}$$

$$F_{(24-22)}=F_{(22-21)}=F_{(21-15)}=F_{(15-14)}=F_{(14-11)}=F_{(11-6)}=F_{(6-3)}=F_{(3-2)}=0 \qquad \text{eq 4.16}$$

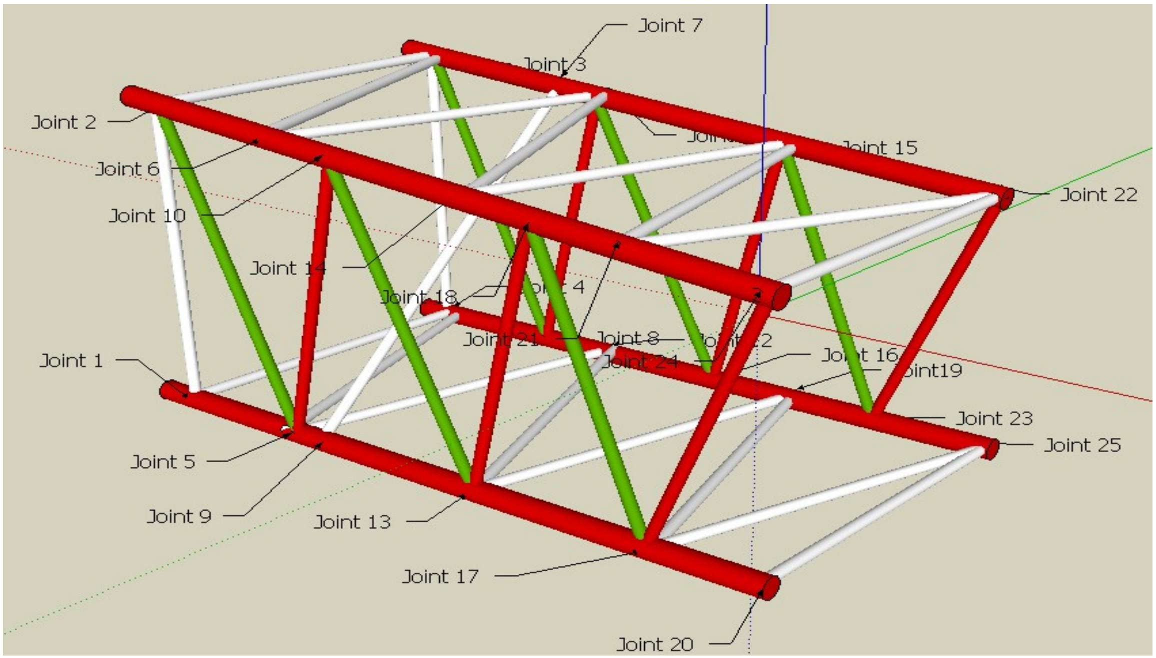

Fig. 4.11. Main Boom Top – Coloured (White: Zero Force Member, Green: Tension, Red: Compression)

Conclusions:

Now we have obtained a tension/compression coloured model of the main boom top shown in Figure 4.4, where green represents members in tension and red represents members in compression. Members in the xy-plane are all zero force members, this phenomenon occurs due to the assumption of two force members for all the rods. On the other hand, the external forces acting on joint 1,2,3,4,20,22,24, and 25 do not sum to zero in the z-direction. This problem occurs due to moving the forces from the pulley to joint 1 and 2, and the simplifying of forces on the red box. We could use force couple technique to move the forces, nevertheless the problem will become that the sum of moments will not be zero. As a result, based on the limited level of mathematical knowledge and truss analysis techniques, we have tried our best to make assumptions to simplify this truss to achieve an analyzable state.

Discussion:

Disregard our assumption of moving forces on the pulley to joints 2 and 3, and the assumption of making the whole boom top composed of two force members, it is reasonable to say that the top xy-plane with vertices joints 2,3,22,24 is under compression due to the four legs structure being pulled by the pulley (Tce/20). The diagonal rod (joint 7 to 9) will transfer the force Tce/20 into the front and back yz-planes with vertices joints 1,2,20,24 and joints 3,4,22,25, and the bottom xy-plane with vertices joints 1,4,20,25 will be in tension.

4.4. Main Boom Butt

In the previous sections, we've seen that all the 3D structures eventually reduce to 2D as a result of our assumption – all the structures are trusses, that is, the main beams in each boom are broken down into pieces of either two-force members or zero-force members. Therefore, we now analyze our trusses using plain 2D models.

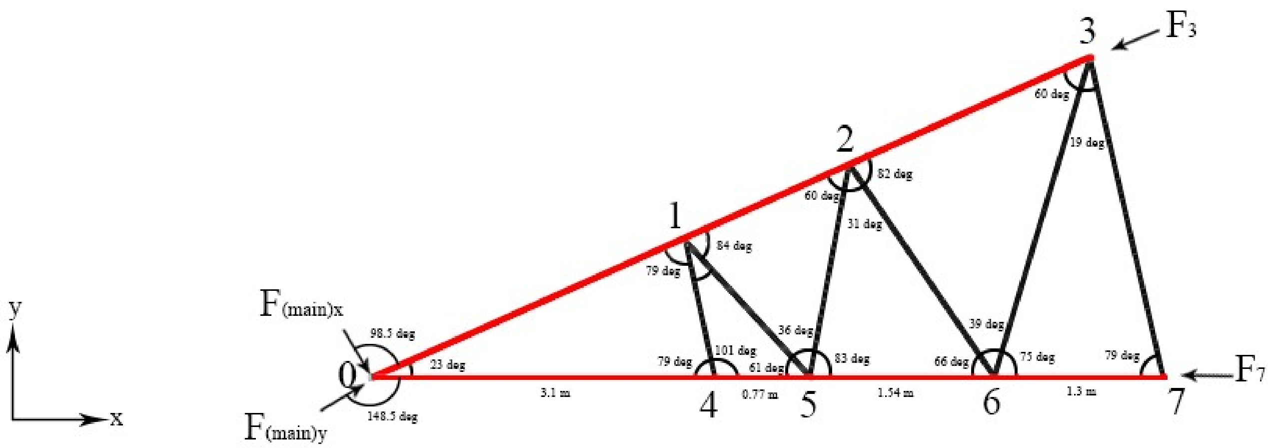

Fig. 4.12. Main Boom Butt

(Members in red are in compression)
In the butt component for the main boom, member 1-4 and 3-7 are assumed to form an isosceles triangle with members 0-3 and 0-7. With this, along with some approximate dimensions, angles can be derived from figure 4.12. Next, members from 0-3 and 0-7 are considered as two force members. In our case study, we have neglected any external forces pushing the crane from the side, apart from the counterweight, the weight of the structures themselves, and the load. The function of members crisscrossing members 0-3 and 0-7 are therefore used to redistribute stress along a different axis.
This structure analysis is further simplified by taking only one side of the structure due to the part being symmetrical along the x-y plane as shown above.
External Forces:
We have 3 external forces acting on the butt component of the main boom. Using these external forces we can calculate the forces in each member. Please refer to table 5.2.

All members are assumed to be in tension(F_x, F_y, F_z refer to sum of all forces in indicated axis)
Joint 0:
F_x: $F_{0-4}\cos(59)+F_{0-1}\cos(81.5)+5918.26/2+(6554.48/2)\cos(85.7)= 0$
F_y: $F_{0-4}\sin(59)+F_{0-1}\sin(81.5)+6284.62/2+(655.48/2)\sin(85.7)= 0$

$F_{0-4} = F_3 = -5806.74$
$F_{0-1} = F_7 = -1611.77$

Joint 1:
F_x= $-F_{1-0}\cos(23)+F_{1-4}\cos(79)+F_{1-5}\cos(61)+F_{1-2}\cos(23)$
F_y= $-F_{1-0}\sin(23)-F_{1-4}\sin(79)-F_{1-5}\sin(61)+F_{1-2}\sin(23)$
F_z= 0
Joint 2:
$F_x = -F_{2-1}\cos(23)-F_{2-5}\cos(83)+F_{2-6}\cos(60)+F_{2-3}\cos(23)$
$F_y = -F_{2-1}\sin(23)-F_{2-5}\sin(83)-F_{2-6}\sin(60)+F_{2-3}\sin(23)$
$F_z = 0$
Joint 3:
$F_x = -F_{3-2}\cos(23)-F_{3-6}(83)+F_{3-7}\sin(79)$
Fy = $-F_{3-2}\sin(23)-F_{3-6}\sin(83)-F_{3-7}\sin(79)$
Fz = 0
Joint 4:
$F_x = -F_{1-4}\cos(79)-F_{0-4}+F_{4-5}$
$F_y = F_{1-4}\sin(79)$
$F_z = 0$
Joint 5:
$F_x = -F_{1-5}\cos(61)+F_{2-5}\cos(83)-F_{4-5}+F_{5-6}$
$F_y = F_{1-5}\sin(61)+F_{2-5}\sin(83)$
$F_z = 0$
Joint 6:
$F_y = -F_{2-6}\cos(66)+F_{3-6}\cos(75)-F_{5-6}+F_{6-7}$
$F_y = F_{2-6}\sin(66)+F_{3-6}\sin(75)$
Fz = 0
Joint 7:
$F_x = -F_{6-7}-F_{3-7}\cos(79)$
$F_y = F_{3-7}\sin(79)$
$F_z = 0$

Members	Force(-ve compressive, +ve tension) MN
F_{0-4}	-5806.74
F_{4-5}	-5806.74
F_{5-6}	-5806.74
F_{7-6}	-5806.74
F_{0-1}	-1611.77
F_{1-2}	-1611.77
F_{2-3}	-1611.77

Table 4.4. Forces in Members of Main Boom Butt

4.5. Luffing Jib, Boom I, and Boom II

The Luffing Jib, Boom I and Boom II are composed of components which are structurally very similar to those of main boom. The only difference is that the forces external to the net structure is different. Hence subjecting the components to similar type but different magnitude of loading. As main Boom is the part which is subjected to biggest loadings, it is reasonable to qualitatively evaluate other similar parts. Thus by studying the main boom in detail we can understand the behaviour of the members inside the truss. The main load carriers lying along the top and bottom would suffer high degrees of compression, while the crossing members would suffer both from compressive and tensile forces. The purpose of the diagonals will also be the same as discussed for the non-simplified model of the inserts for the main boom. Which will be that of load distributor.

Conclusions

To summarize this project, we have analyzed manitowoc 31000 in five major steps with various assumptions. They are: 1) centroid location of the whole body, 2) external free body diagram, 3) components free body diagrams, 4) truss analysis of main boom, 5) qualitative analysis of main boom

1) Centroid Location: we calculate and assume centroid of the components base on their level of symmetry. Calculated centroid values are from components with high level of symmetry and assumed centroid values are determined with respect to weight distribution. Having the locations and weights of each component, we obtain centroid location of the whole body, offset due to the cabin in the x direction is put aside to simplify our study into a 2D problem.

2) External Free Body Diagram: we decide to fix the main boom and luffing jib angles to make one specific case study. In this situation the maximum load has a value of 255.20kgt and the maximum counterweight has a value of 894.84kgt. We take an allowable loading of 200kgt which enables us to study the structure with non-zero reaction forces.

3) Components Free Body Diagram: the major assumption in this step is the component connections are smooth pins (except the backhitch which is viewed as a two force member since there are only forces acting at the ends). This assumption is reasonable since each components can rotate. Without friction there are just two forces at each pin. The components FBD are calculated from the outermost to the innermost since the loading, reaction, and counterweight values are known.

4) Truss Analysis of Main Boom: divides into four subsections to study which are butt, inserts, insert 3, and top. The two major assumptions in this step are the welded insert connections are considered to be smooth pins, and the subsections are made of only two force members. These assumptions enable us to use method of joints and method of sections, however, they lead to a major change in the behaviour of the structure. Due to limited mathematical knowledge and truss analysis techniques, the main boom is studied to our best effort and corresponding tension/compression pictures are obtained.

5) Qualitative analysis of main boom: realizing the assumptions are not perfect, instead, we study the truss structures qualitatively to see the behaviours of what is really happening to the main boom.

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