How Importers Hedge Demand Uncertainty Through Dual Sourcing and Safety Inventory[1]

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Abstract

We develop a dynamic model of inventory and trade to study how an importer may hedge demand uncertainty when importing involves an order lead time. We show that when the import cost is low, the importer optimally holds safety inventory, i.e., inventory of imported goods in excess of expected sales to deal with demand surges. As the import cost rises, the firm switches from safety inventory to dual sourcing, i.e., to covering demand surges through quickly available but expensive domestic supplies while using imports for base-level demand. The endogenous adjustment of the hedging strategy implies that the volume of inventory and imports falls by more than expected sales as the import cost rises. This effect is magnified by an increase in demand uncertainty.

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1 Introduction

How optimally to hedge demand uncertainty is an issue of great practical relevance for many importers (Jain et al., 2014, and Cachon and Terwiesch, 2019). While importing goods is often cheaper than sourcing them domestically, the problem is that importing often takes considerable time so that orders have to be placed before the realization of demand is known and hence before the importing firm knows the quantity it needs.[1] How then to enjoy cost advantages from importing while ensuring that a large enough quantity of goods is available when demand turns out to be higher than expected? Two hedging strategies stand out in practice in industries ranging from apparel to electronics, toys, sporting goods, etc., namely (i) building up a safety stock of imported goods, which implies holding inventory in excess of expected sales; and (ii) relying on dual sourcing, i.e., relying on cheap but slow imports to cover regular demand while using more expensive, but quickly available supplies of the same good, often sourced domestically, to cover demand surges.

The current paper develops a dynamic model of safety inventory and dual sourcing to examine (i) how the optimal combination of hedging strategies depends on the relative cost of imported versus domestically sourced goods, and (ii) how the optimal strategy combination affects a firm's overall inventory and import response to changes in this relative cost.

Regarding our first research question, we show that it is optimal for a firm to hold safety inventory when the relative import cost is low. This is because the low relative import cost compensates for the cost of being stuck with excess inventory when demand happens to be lower than expected, specifically the cost of holding inventory for sale in future periods. As the relative import cost rises, the optimal safety inventory shrinks, and the likelihood of sourcing domestically rises. For sufficiently high relative import costs, inventory falls below expected sales and dual sourcing becomes the strategy of choice for hedging demand uncertainty; demand surges are then covered solely through fast domestic

[1] The long time delays involved in shipping goods across countries and clearing customs have been carefully documented (Hummels and Schaur, 2013). Ocean shipping times between various ports around the world can be downloaded from several webpages, such as, https://www.searates.com/de/reference/portdistance or https://www.championfreight.co.nz/times.pdf. The World Bank provides information on the time required for border and documentary compliance when goods are exported: http://www.doingbusiness.org/data/exploretopics/trading-across-borders.

sourcing, while imports are only used to cover a base-level of demand.

Regarding our second research question, namely the inventory and import response to changes in the relative import cost, we show that, due to the endogenous change in the optimal hedging strategy from safety inventory toward dual sourcing, both inventory and imports decrease more quickly than expected sales as the relative import cost rises. In this sense, changes in the relative import cost have a magnified effect on inventory and the volume of trade when a firm needs to hedge demand uncertainty. Thus if the change in relative import cost is due to trade policy, its impact on import volumes in steady state is magnified with respect to a situation without safety inventory and dual sourcing.

While dual sourcing has been used by importers in many industries for decades, it has received increased attention over the last few years, as the business environment in many industries has seemingly become more uncertain, driven not just by greater demand uncertainty, but also by supply disruptions, trade policy uncertainty, significant uncertainty regarding international shipping costs and shipping reliability, as well as increasing geopolitical tensions.[2] Demand uncertainty, in particular, is a key problem for fast fashion retailers, such as H&M, Desigual, Zara or Bershka, whose products are subject to frequent changes in fashion trends. Electronics or cellular phone manufacturers are other examples of firms facing frequent product renewals, driven in their case by rapid technological innovation. In the toy industry, sales are concentrated on a fairly short Christmas season with significant uncertainty about which products will be in high demand in any given season.

What these examples have in common is that (i) imports, often from Asia, tend to be cheap but tend to have long order lead times; (ii) holding inventory tends to be expensive since products lose their value quickly as they are replaced by new variants. Not surprisingly then, many companies in these industries engage in dual sourcing, trying to find the optimal balance of safety inventory and dual sourcing to meet demand surges.

[2]For instance, UNCTAD (2020) lists 18 broad sectors out of a total of 25 for which the average tariff is higher in 2019 than in 2010. Globaltradealert.com reports respectively 4,967 and 3,279 harmful trade policy changes worldwide in 2020 and 2021 against an average of about 2,500 per year during the period 2009-19. Of course many of the 2020-21 ones originated in the US and China. Demand uncertainty is more difficult to measure but it is difficult to deny that we are in an age of volatility. For instance, the World Uncertainty Index tends to spike when there are crises like COVID-19, the Brexit vote or wars (Ahir et al.. 2022). Finally, the Global Container Freight Index which rose from about $2,000 in August 2020 to a peak of $10,300 in September 2021 is now (May 2024) near $3,500 after having been below $2,000 one year earlier (https://terminal.freightos.com/freightos-baltic-index-global-container-pricing-index/).

Zara and Bershka, for instance, often work with two suppliers for essentially identical products: a low-cost supplier with long lead times for base orders, and a local supplier that is activated during demand surges. Nokia, a cellular phone producer, used to source products from Asia as well as a plant in its native Finland. Famosa, a toy manufacturer, sources roughly 80% of its products from China, but relies on local European suppliers when the quantity ordered from China is insufficient to meet demand (Veeraraghavan and Scheller-Wolf, 2008; Calvo and Martinez-de-Albeniz, 2016).[3]

Sporting goods companies, like Adidas and Nike, have responded to increased demand uncertainty by reshoring parts of their production, setting up what Adidas calls speedfactories, production facilities that can ramp up production on short notice and cut delivery times by up to a month. The factory in Germany was designed to make 1 million pairs of athletic shoes per year as compared with around 301 million pairs of shoes sold by Adidas each year. Nike planned to use its speedfactory in North America to produce 3 million pairs of shoes, as compared to an overall production target of 1.3 billion pairs (Boute et al., 2022).

These examples make clear that many importers are actively engaged in optimizing their sourcing and inventory strategies to hedge demand uncertainty. The value added of the present paper is to develop a tractable model of safety inventory and dual sourcing to show how the optimal combination of these two hedging strategies is linked to the relative import cost and how a change in this cost affects the volume of inventory and trade, taking into account a firm's endogenous choice of hedging strategy.

In the next section, we explain the paper's contributions to the literature in more detail. In Section 3, we propose a simple dynamic model of inventory investment and dual sourcing that we use in Section 4 to examine how, in steady state, changes in trade costs and demand uncertainty affect inventory investment, the probability of domestic sourcing, as well as the volume of imports. Section 5 concludes. In the appendix, we collect proofs of our results.

[3]Fast fashion retailers H&M and Desigual rely on single sourcing, but demand quick shipments, typically by air, in case of high demand.

2 Related Literature

Our paper is closely related to the international trade literature analyzing how firms can mitigate the impact of demand uncertainty through strategies such as sourcing and production location decisions. Evans and Harrigan (2005) explain why fast fashion retailers may want to choose production locations in or close to major consumer markets in order to be able to quickly respond to demand surges even if that entails higher labor costs. A similar tradeoff between slow but cheap versus fast but expensive sourcing modes is studied by Hummels and Schaur (2013), where the sourcing mode corresponds to the mode of transportation, either slow, cheap ocean shipping or fast, expensive air shipments. Both models, however, feature single sourcing not dual sourcing, which means that the slow, cheap source and the fast, expensive source are mutually exclusive. But, as discussed earlier, many companies seek to have the best of both worlds, using the slow, cheap source to cover base demand, and the quick, expensive source to cover demand surges, and dual sourcing is the way to do this.

Dual sourcing is featured in Hummels and Schaur (2010) and also Aizenman (2004) as a combination of ocean and air shipments in the face of random demand. However, none of these four papers studies the firm's inventory decision. Safety inventory is hence not part of the hedging strategy they consider. By contrast, we develop a dynamic model in which inventory decisions, along with dual sourcing, play a crucial role. This turns out to be quite important, since both strategies are intimately related when imports have a long order lead time and demand is uncertain: slow but cheap sourcing naturally implies that a firm holds inventory as goods are ordered before they can be used or sold. Moreover, as demand may turn out lower than expected, firms naturally accumulate excess inventory.

This interconnection between inventory and sourcing decisions has been recognized and studied extensively in the operations management literature, where dual sourcing is considered to be an essential part of a firm's inventory management strategy (see, for instance, Svoboda et al., 2021, and Xin and Van Mieghem, 2023 for recent surveys). The focus of that literature has been to come up with heuristics that firms could use to implement practical solutions to the dual sourcing problem (see, for instance, Allon and Van Mieghem, 2010, and Boute and Van Mieghem, 2015). These practical solutions typically take the form of inventory review procedures or order policies specifying critical inventory levels at which orders from one or both sources are triggered.

By contrast, we approach dual sourcing from a positive perspective. Rather than devising practical order policies we seek to understand the circumstances under which dual sourcing and safety inventory are used and how they affect inventory and trade volumes relative to expected sales as the relative import cost changes. For this purpose we deviate from the operations management approach to dual sourcing by simplifying the firm's dynamic programming problem so that we can offer closed-form solutions rather than resorting to heuristics. In particular, we build on Reagan (1982) who characterizes the (closed-form) solution to a dynamic programming problem, in which a monopolist facing demand uncertainty chooses production and inventory strategies to maximize the discounted present value of its profit stream. We extend this model to allow for dual sourcing, which allows us to study a dynamic programming problem in which the firm decides on its optimal dual sourcing and inventory strategies, and explore how the optimal mix of these strategies is affected in steady state by a change in the relative import cost.

Our model is also related to dynamic inventory models in the trade literature. These papers include Alessandria et al. (2010) and several additional papers that essentially use similar models as in Alessandria et al., such as Nadais (2017), Khan and Khederlarian (2021, 2024), and Alessandria et al. (2024). In these papers, firms hold inventories of imported goods due to the presence of fixed costs per shipment.[4] These fixed costs induce firms to order infrequently to economize on these costs, which means that they need to hold inventories to allow them to produce and sell products in between shipments. Some of these papers also feature demand or trade policy uncertainty, which adds a precautionary motive for holding inventory. By contrast, we focus on the precautionary motive for holding inventory, assuming that the fixed cost per shipment is zero. This way we can show clearly that firms, among other things, reduce the volume of inventory and imports by more than expected sales in steady state as the import cost rises. A model where inventory is based on fixed costs per shipment exhibits a force pushing in the opposite direction, as firms respond to an increase in the import cost by reducing the number of shipments.

Most of these papers focus their attention on the dynamic adjustment effects during the

[4]See also Kropf and Sauré (2014) who study fixed costs per shipment in a model with deterministic demand. Békés et al. (2017) study how firms adjust the frequency and size of shipments in response to demand volatility on their export market. They argue that the observed adjustments could, in principle, be rationalized by a stochastic inventory model.

transition to a new steady state. They show that during the transition inventory may rise relative to sales following a permanent increase in the import cost or do so in anticipation of such an increase as in Khan and Khederlarian (2021). The present paper focuses its attention solely on changes across steady states; it is thus not about transition dynamics.

Among this group of papers the ones that are most closely related to ours are Nadais (2017), and Khan and Khederlarian (2024). This is because the firms in these papers also source both imported and domestic goods, although they do not engage in dual sourcing in the sense that we—and papers in operations management—define it, namely as buying identical goods from a foreign and a domestic source for the purpose of hedging demand uncertainty.[5] In Nadais (2017), imported and domestic goods are complements, so that firms always use both sources. In Khan and Khederlarian (2024), imported and domestic varieties are imperfect substitutes, which also implies that a firm always sources both, and that an increase in the relative import cost triggers a traditional substitution effect away from the relatively more expensive imported variety to the relatively less expensive domestic variety. This impacts inventory when imports are storable and domestic goods are non-storable. By contrast, a firm in our model would never buy domestic goods in the absence of demand uncertainty, as long as imports are cheaper. The sole role of domestic goods is as an instrument to hedge demand uncertainty. An increase in the relative cost of imports hence raises the likelihood of domestic sourcing only because it induces the firm to rely more on dual sourcing and less on safety inventory to hedge demand uncertainty. In other words, we observe in our model a substitution between hedging strategies rather than a substitution effect between goods.[6]

[5] Note that dual sourcing in the sense that we use it, namely ordering the same input from an inflexible but cheap and from a flexible but expensive source to hedge demand uncertainty, is different from "multi-sourcing" analyzed, for instance, by Gervais (2018). Multi-sourcing in Gervais (2018) refers to risk-averse firms sourcing the same input from multiple suppliers to hedge idiosyncratic supply shocks, but firms are assumed not to hold any inventory.

[6] Cavallo and Krystov (2024) also make the precautionary motive for inventory a central aspect of their analysis. Their motivation, however, is very different from ours since their goal is to assess the impact of temporary and permanent stockouts on inflation during the Covid pandemic.

3 Model

In this section we build on Reagan (1982) to develop an infinite-horizon, discrete-time inventory model with imports and domestic sourcing, and with demand uncertainty. Unlike domestic sourcing which is immediate, importing involves a one-period time lag between order and delivery. The time lag implies that imports are exclusively to inventory and cannot be used to satisfy contemporaneous demand.

Inverse demand in any period t is given by the linear function $p_t = a + \epsilon_t - bq_t$, where p_t and q_t denote price and quantity sold in period t, respectively, and ϵ_t is an i.i.d. random shock uniformly distributed on $[-\Delta, \Delta]$. For each unit sold, the firm needs one unit of an homogeneous input good that can be imported or sourced domestically.[7] Importing is inflexible, as it takes one period for goods to be delivered. That is, a quantity of imports, m_t, ordered in period t is only delivered in period $t+1$. Domestic sourcing is immediate: a domestic order for quantity y_t placed in period t is delivered in period t.

Let z_t denote the *inventory* of goods at the beginning of period t.[8] In any period t, sales must come either from inventory or from domestic purchases, so that $q_t \leq z_t + y_t$. If the firm does not sell all the goods it has in inventory and has purchased domestically, i.e., $q_t < z_t + y_t$, then the unsold units become part of the available inventory in $t+1$. We refer to $z_t^0 \equiv z_t + y_t - q_t \geq 0$ as *excess inventory* in period t. Thus inventory at the beginning of period $t+1$ is equal to $z_{t+1} = m_t + z_t^0$, that is, the sum of imports purchased in period t and arriving at the beginning of period $t+1$, and the excess inventory inherited from period t.

A unit of imported goods costs v_t, which we assume includes the purchase price as well as trade costs, such as tariffs, transportation costs, and other variable transaction costs

[7] If the firm is a wholesaler or retailer, the output goods is typically the same as the input good. If the importer is a manufacturer, the input good is an intermediate input that is transformed into output one for one, where we abstract from possible substitutability between the intermediate input and other inputs, such as labor. Our model could easily accommodate labor and other inputs, especially if these are perfect complements to intermediates. For example, if producing a unit of output also requires l units of labor, so that the unit labor cost is given by $c = l\omega$, where ω denotes the wage, we can simply define the new demand intercept as $a = A - c$ where A is the original demand intercept.

[8] Notice that we implicitly assume that the firm only holds inventory of intermediate goods. A manufacturer, however, would typically also hold inventory of goods in process or of finished goods. If the purpose of holding inventory is to hedge demand uncertainty, it would not matter in which form this inventory is held. Thus the model could be extended to include a production process that allows for different forms of inventory from intermediate to finished goods.

involved in purchasing the input. The unit cost of the domestic input is given by w, which we assume to be constant over time. Storing a unit of excess inventory between periods costs γ, where $0 < \gamma < v_t$.

We make several assumptions regarding these cost parameters in order to rule out uninteresting cases. First, we assume that in any period t imports are cheaper than domestic goods even if they have to be ordered a period before they can be used, i.e., $v_t/\delta \leq w$, where $\delta < 1$ is the discount factor.[9] In the absence of demand uncertainty, the firm would thus satisfy all of its input requirements through imports. Second, we assume that $v_t > \delta\left(v_{t+1} - \gamma\right)$. This rules out the case where the firm would import so much in period t that it would voluntarily accumulate excess inventory in $t+1$. Third, we assume that $\Delta < a - v_t$, meaning that demand shocks are sufficiently small relative to the size of the market so that $m_t > 0$ in every period t.

Let us summarize the timing of decisions in any period t. The firm starts period t with inventory z_t, and by observing the realization of the demand shock ϵ_t. It then chooses sales, q_t, the quantity sourced domestically, y_t, and the quantity of imports, m_t, to be delivered in period $t+1$. The firm's objective is to maximize the discounted sum of expected future profits. We may formulate this optimization problem as a dynamic programming problem with control variables q_t, y_t, m_t, and state variable z_t. Letting $V(\cdot)$ denote the value function, we may state this problem as follows:

$$\begin{aligned} V(z_t, \epsilon_t) &= \max_{q_t, y_t, m_t} \left\{ \left(a + \epsilon_t - bq_t\right) q_t - wy_t - v_t m_t - \gamma z_t^0 + \delta EV(z_{t+1}, \epsilon_{t+1}) \right\} \\ & \text{subject to} \begin{cases} 0 \leq q_t \leq z_t + y_t, \\ 0 \leq y_t, \\ 0 < m_t \leq z_{t+1} - z_t - y_t + q_t. \end{cases} \end{aligned}$$

This dynamic problem can be reduced to two separate problems, namely the optimal decisions about q_t and y_t for a given beginning-of-period inventory z_t, and the decision about imports, m_t, and hence about z_{t+1}, the starting inventory in $t+1$. To understand why this is the case in our setting, note that our assumptions guarantee that imports occur in every period. Thus there is no period during which a firm decides to be inactive

[9]We could endogenize v_t (and w) by assuming that they are set by (or negotiated with) upstream producers with market power. Qu et al. (2018) show how this may be done in an intertemporal inventory model.

as far as importing is concerned. Moreover, excess inventory has value only next period.

We start by characterizing the optimal decisions about q_t and y_t. Notice that in period t the cost of imports ordered in $t-1$ is sunk, while the cost of ordering domestic goods in period t is avoidable. Thus, a firm does not buy domestically unless it sells all of its beginning-of-period inventory. For the same reason, the firm also never leaves any domestic goods unsold. Thus inventory z_t only ever consists of imported goods, and when we refer to inventory below, we implicitly mean inventory of imported goods.

This leaves two possibilities: (i) $q_t \leq z_t$, in which case the firm does not order domestically and may leave some inventory unsold; since unsold inventory can be taken into the next period, it has an implicit value equal to $v_t - \gamma$; and (ii) $q_t > z_t$, which implies that the firm engages in dual sourcing, i.e., it sells its entire inventory z_t and buys $y_t = q_t - z_t$ domestically at unit cost w. That is, sales, q_t, must maximize the contemporaneous profit in period t, which is equal to

$$\begin{array}{ll} (a+\epsilon_t - bq_t)q_t + (v_t - \gamma)(z_t - q_t) & \text{if} \quad q_t \leq z_t, \\ (a+\epsilon_t - bq_t)q_t - w(q_t - z_t) & \text{if} \quad q_t > z_t. \end{array} \tag{1}$$

The first-order condition with respect to q_t implies the following optimal sales

$$q_t^*(\epsilon_t) = \begin{cases} \frac{a+\epsilon_t - v_t + \gamma}{2b} & \text{if} \quad \frac{a+\epsilon_t - v_t + \gamma}{2b} \leq z_t, \\ z_t & \text{if} \quad \frac{a+\epsilon_t - w}{2b} < z_t < \frac{a+\epsilon_t - v_t + \gamma}{2b}, \\ \frac{a+\epsilon_t - w}{2b} & \text{if} \quad \frac{a+\epsilon_t - w}{2b} \geq z_t. \end{cases} \tag{2}$$

Eq. (2) shows that if demand is sufficiently small, then the firm sells only part of its inventory and does not purchase domestically. This is the case if the marginal revenue when selling the entire z_t is smaller than the value of holding on to at least one unit by storing it for next period, $v_t - \gamma$, i.e., if

$$a - 2bz_t + \epsilon_t \leq v_t - \gamma. \tag{3}$$

If the marginal revenue from selling z_t exceeds $v_t - \gamma$ but is smaller than w, then the firm sells all of its inventory, but does not make any domestic purchases. That is, we have $q_t^*(\epsilon_t) = z_t$, which means that the firm stocks out *voluntarily*, even if domestic goods are in principle available. Finally, if the marginal revenue from selling z_t exceeds w,

$$a - 2bz_t + \epsilon_t > w, \tag{4}$$

then the firm sells all of its inventory and makes domestic purchases, that is, it engages in dual sourcing. Hence, the lower is $v_t - \gamma$ and the higher is w, the wider is the interval of inventory levels for which the firm optimally decides to stock out.

From (3) and (4), we can derive the critical demand realizations for which the importer is indifferent between selling all inventory of imported goods or not, $\underline{\epsilon}(z_t)$, and for which it is indifferent between buying an additional unit domestically or not, $\overline{\epsilon}(z_t)$:

$$\underline{\epsilon}(z_t) = 2bz_t - a + v_t - \gamma, \quad \overline{\epsilon}(z_t) = 2bz_t - a + w, \tag{5}$$

with $\underline{\epsilon}(z_t) < \overline{\epsilon}(z_t)$ from our earlier assumptions. Thus, (2) can be rewritten as:

$$q_t^*(\epsilon_t) = \begin{cases} \frac{a+\epsilon_t - v_t + \gamma}{2b} & \text{if} \quad \epsilon_t \leq \underline{\epsilon}(z_t), \\ z_t & \text{if} \quad \underline{\epsilon}(z_t) < \epsilon_t < \overline{\epsilon}(z_t), \\ \frac{a+\epsilon_t - w}{2b} & \text{if} \quad \epsilon_t \geq \overline{\epsilon}(z_t). \end{cases} \tag{6}$$

Demand can be: (i) low enough so that the firm does not sell its entire inventory and therefore accumulates excess inventory, $z_t - q_t^*(\epsilon_t)$, that it may use next period; (ii) in an intermediate range such that it sells its entire inventory, $q_t^*(\epsilon_t) = z_t$, but does not order domestically and hence stocks out; or (iii) high enough that it engages in dual sourcing, i.e., sells goods from both foreign and domestic sources. Since sales are greater than inventory, the purchase of domestic goods is equal to:

$$y_t^*(\epsilon_t) = \frac{a + \epsilon_t - w}{2b} - z_t. \tag{7}$$

In order for a firm to effectively face these three options, we require the level of demand uncertainty, Δ, to be high enough so as to include the two thresholds $\underline{\epsilon}(z_t)$ and $\overline{\epsilon}(z_t)$ (i.e., $-\Delta < \underline{\epsilon}(z_t) < \overline{\epsilon}(z_t) < \Delta$). Using (5), this implies that $\overline{\epsilon}(z_t) - \underline{\epsilon}(z_t) = w - v_t + \gamma < 2\Delta$. Below, we assume that this is the case, and refer readers to Muris et al. (2023) for an analysis of the other cases. There we show, among other things, that the results derived below also hold if demand uncertainty is low in the sense that $\overline{\epsilon}(z_t) - \underline{\epsilon}(z_t) = w - v_t + \gamma >$

2Δ.

We can now proceed to the determination of the optimal imports. The optimal import quantity in t, m_t^*, is simply the difference between the desired level of inventory at the beginning of period $t+1$, z_{t+1}^*, and the excess inventory in t, i.e., $m_t^* = z_{t+1}^* - z_t^0$. Hence, to obtain m_t^*, we need to determine z_{t+1}^*.

The expected marginal revenue in period $t+1$ from a unit imported in period t is equal to:

$$E\left[MR_{t+1}\right] = \int_{-\Delta}^{\underline{\epsilon}(z_{t+1})} (v_{t+1} - \gamma)\,\frac{d\epsilon_{t+1}}{2\Delta} + \int_{\underline{\epsilon}(z_{t+1})}^{\overline{\epsilon}(z_{t+1})} (a + \epsilon_{t+1} - 2bz_{t+1})\frac{d\epsilon_{t+1}}{2\Delta} + \int_{\overline{\epsilon}(z_{t+1})}^{\Delta} w\frac{d\epsilon_{t+1}}{2\Delta}. \tag{8}$$

That is, the marginal revenue is equal to $v_{t+1} - \gamma$ for low demand realizations ($-\Delta \leq \epsilon_{t+1} \leq \underline{\epsilon}(z_{t+1})$) and thus when the firm holds on to units for the next period; it is equal to $a + \epsilon_{t+1} - 2bz_{t+1}$ when it stocks out by selling the entire inventory z_{t+1}; and it is equal to w when the demand realizations are sufficiently high ($\overline{\epsilon}(z_{t+1}) \leq \epsilon_{t+1} \leq \Delta$) that purchasing domestically is required.

Using (5) to evaluate (8), and equating the discounted expected marginal revenue in period $t+1$ to the unit import cost in period t (i.e. $\delta E\left[MR_{t+1}\right] = v_t$), the optimal inventory at the beginning of period $t+1$ is given by:

$$z_{t+1}^* = \frac{2a - (w + v_{t+1} - \gamma)}{4b} + \frac{\Delta(w + v_{t+1} - \gamma)}{2b(w - v_{t+1} + \gamma)} - \frac{\Delta v_t/\delta}{b(w - v_{t+1} + \gamma)}. \tag{9}$$

The optimal level of imports m_t^* corresponds to z_{t+1}^* if there is no excess inventory in period t. If there is excess inventory, the optimal level of imports simply corresponds to what is necessary to bring the starting inventory in $t+1$ to its optimal level.

Having characterized the solution to the dynamic problem, we are now in a position to examine its properties.

4 Comparative Statics in Steady State

In this section we examine the firm's decisions in steady state, in which $z_t^* = z^*$, and equivalently $v_t = v$ for every t. In particular, we want to know how the optimal inventory, expected imports, expected sales and expected domestic purchases change with the unit import cost v and a mean-preserving spread Δ, which represents a change in the level of demand uncertainty.

A useful reference point for this analysis is to start with the firm's expected sales, which can be computed as follows:

$$\hat{q} = \int_{-\Delta}^{\underline{\epsilon}(z)} \frac{a - v + \gamma + \epsilon}{2b} \frac{d\epsilon}{2\Delta} + \int_{\underline{\epsilon}(z)}^{\overline{\epsilon}(z)} z \frac{d\epsilon}{2\Delta} + \int_{\overline{\epsilon}(z)}^{\Delta} \frac{a - w + \epsilon}{2b} \frac{d\epsilon}{2\Delta} = \frac{a - v/\delta}{2b}. \tag{10}$$

The firm's expected sales are thus simply equal to the sales it would make with deterministic demand when importing all of its goods at an import unit cost v adjusted for the fact that imports take one period to be delivered, which yields an effective import unit cost of v/δ. Naturally, $d\hat{q}/dv = -1/2b\delta < 0$ so that expected sales decrease with a higher import unit cost.

How then do expected sales and optimal beginning-of-period inventory compare in steady state? Comparing the optimal inventory level at the beginning of a period with the sales that the firm expects to realize in that period, we have

$$z^* - \hat{q} = (2\Delta - w + v - \gamma) \frac{w - \gamma + v - 2\frac{v}{\delta}}{4b(w - v + \gamma)}, \tag{11}$$

where $z^* - \hat{q} = 0$ when $v = \frac{\delta}{2-\delta}(w - \gamma)$. This separates $z^* - \hat{q}$ into two intervals:

$$z^* - \hat{q} \begin{cases} > 0 & \text{if} \quad v < \frac{\delta}{2-\delta}(w - \gamma); \\ < 0 & \text{if} \quad v > \frac{\delta}{2-\delta}(w - \gamma). \end{cases}$$

These two intervals correspond to two separate mechanisms that a firm uses to hedge demand uncertainty:

Proposition 1. *If $v < \frac{\delta}{2-\delta}(w - \gamma)$, the firm finds it optimal to carry safety inventory ($z^* - \hat{q} > 0$); and if $v > \frac{\delta}{2-\delta}(w - \gamma)$, the firm relies on dual sourcing to cover demand surges while inventory is below expected sales ($z^* - \hat{q} < 0$).*

To better understand these two mechanisms, notice that if $v < \frac{\delta}{2-\delta}(w - \gamma)$, imported goods are sufficiently cheap so that the firm finds it optimal to build a large inventory of imported goods, one that exceeds the quantity it expects to sell, because this allows it to reduce the probability of stocking out and losing sales when demand turns out to be higher than expected. It thus carries safety inventory.

At the other extreme, if $v > \frac{\delta}{2-\delta}(w-\gamma)$, imports are sufficiently expensive that the firm accumulates a small beginning-of-period inventory, relying on domestic sourcing to cover high realizations of demand. There is no safety inventory because none is needed when domestic goods are not overly expensive relative to imports. As a result, the firm finds it optimal to hold less inventory than is required to realize expected sales.

Having shown that whether the firm finds it optimal to hedge demand uncertainty through safety inventory or through dual sourcing depends on the level of the import unit cost, we can now investigate in more detail how a change in the import unit cost affects the beginning-of-period inventory level relative to expected sales.

From (9) we obtain:

$$\frac{\partial z^*}{\partial v} = -\frac{4\Delta\left(w(1-\delta)+\gamma\right)+\delta\left(v-w-\gamma\right)^2}{4b\delta\left(w-v+\gamma\right)^2} < 0.$$

We have already established that $d\hat{q}/dv < 0$. From (11), we can further verify that:

$$\frac{\partial\left(z^*-\hat{q}\right)}{\partial v} = \frac{\left(w-v+\gamma\right)^2\left(2-\delta\right)-4\Delta\left(w-w\delta+\gamma\right)}{4b\delta\left(w-v+\gamma\right)^2} < 0.$$

Thus both expected sales and optimal inventory fall with v, and an increase in the import unit cost reduces inventory relative to expected sales. Figure (1) illustrates expected sales and optimal inventory as a function of the unit cost of imports, where we have used $d^2\hat{q}/dv^2 = 0$ and $\partial^2 z^*/\partial v^2 < 0$.[10]

How strongly a change in v affects z^* and hence $z^* - \hat{q}$ depends on the volatility of demand. In particular, we find:

$$\begin{aligned}\frac{\partial^2 z^*}{\partial v\partial\Delta} = -\frac{w(1-\delta)+\gamma}{b\delta\left(w-v+\gamma\right)^2} &< 0;\\ \frac{\partial^2\left(z^*-\hat{q}\right)}{\partial v\partial\Delta} &< 0.\end{aligned}$$

Thus a change in v has an even stronger effect on z^* and on $z^* - \hat{q}$ the more volatile is demand. There are two reasons for this. First, if imports are relatively cheap, the firm responds to greater demand uncertainty by relying on a larger safety inventory of imported

[10]The lower and upper limits for which the above results hold are not shown in the graph.

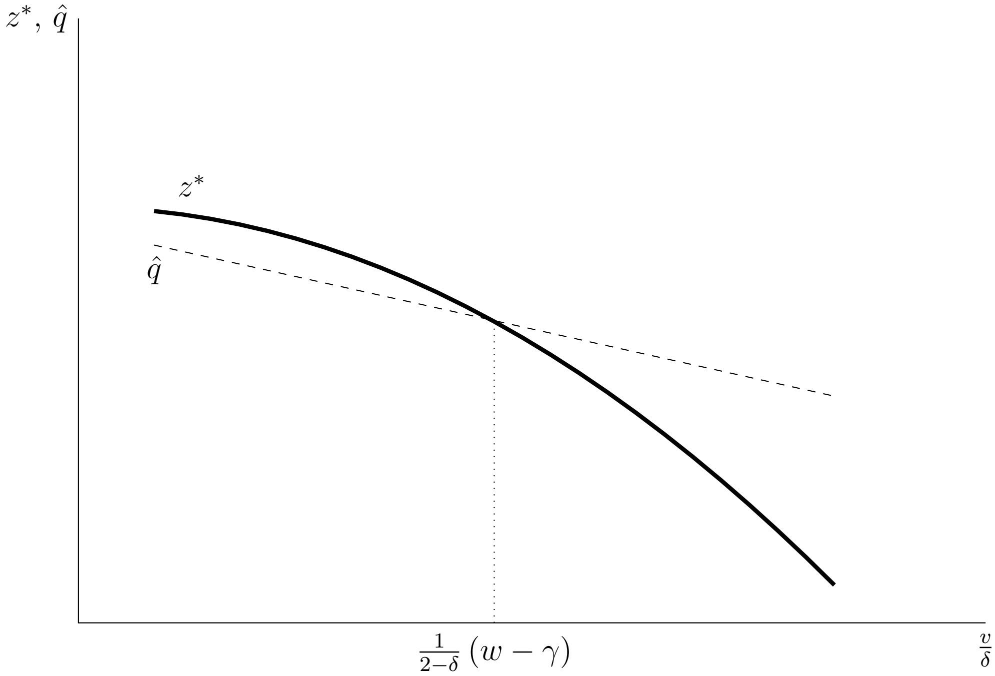

Figure 1: Optimal inventory (z^*) vs. expected sales ($\hat{q}$)

goods to hedge against stockouts. A rise in the unit import cost thus makes this stockout avoidance strategy especially costly when demand uncertainty is high, forcing the firm to more strongly cut its inventory. Second, when the firm covers high demand realizations through dual sourcing, an increase in the import cost makes the firm more willing to switch to expensive domestic sourcing the higher is the degree of demand uncertainty.

We summarize the results as follows:

Proposition 2. *An increase in the import unit cost reduces the optimal inventory z^* by more than expected sales $\hat{q}$. This effect is larger the greater is Δ.*

Proof. See Appendix A.1. □

What about imports? In steady state, expected imports are equal to the difference between expected sales and expected domestic purchases:

$$\hat{m} = \hat{q} - \hat{y} = \int_{-\Delta}^{2bz-a+v-\gamma} \frac{a-v+\gamma+\epsilon}{2b} \frac{d\epsilon}{2\Delta} + \int_{2bz-a+v-\gamma}^{\Delta} z^* \frac{d\epsilon}{2\Delta}.$$

Applying the Leibniz integral rule and the fact that $\partial z^*/\partial v < 0$, we can show directly that

$$\frac{\partial \hat{m}}{\partial v} = -\int_{-\Delta}^{2bz-a+v+\gamma} \frac{1}{2b}\frac{d\epsilon}{2\Delta} + \int_{2bz-a+v+\gamma}^{\Delta} \frac{\partial z^*}{\partial v}\frac{d\epsilon}{2\Delta} < 0.$$

This is the result of two effects, namely that expected sales are decreasing in v, and that expected domestic purchases are increasing in v, which can be confirmed directly from

$$\begin{aligned} \hat{y} &= \int_{2bz-a+w}^{\Delta} \left(\frac{a-w+\epsilon}{2b} - z^*\right)\frac{d\epsilon}{2\Delta} \\ &= \frac{1}{8b\Delta}\left(a-w+\Delta-2bz^*\right)^2. \end{aligned}$$

In other words, we observe that $\hat{m}$ decreases faster with v than $\hat{q}$, as the firm increasingly turns to dual sourcing and thus domestic purchases to cover high demand realizations. In Appendix A.2, we show that an increase in demand volatility also magnifies this effect when inventory is below expected sales, namely:

$$\frac{\partial^2 \hat{m}}{\partial v \partial \Delta} = -\frac{\partial^2 \hat{y}}{\partial v \partial \Delta} < 0 \text{ if } v > \frac{\delta}{2-\delta}\left(w-\gamma\right)$$

We may thus formulate the following proposition:

Proposition 3. *An increase in the import unit cost reduces the expected import volume by more than expected sales. This effect increases with Δ if inventory is below expected sales.*

Proof. See Appendix A.2. □

The results in this section highlight the importance of a firm's endogenous choice of safety inventory and dual sourcing for the beginning-of-period inventory, expected imports and expected sales. In particular, in our model, the fact that inventory and expected imports decrease faster than expected sales as the import unit cost rises is driven by a change in the firm's optimal hedging strategy. By focussing on steady states, we show that these results go beyond transition dynamics.

5 Conclusions

When confronted with demand shocks, importing firms facing uncertain demand typically go beyond price and quantity adjustment by implementing often a range of strategies to avoid lost sales. Although stock-outs are being observed, long lead times, delays and supply bottlenecks call for strategies on the part of importing firms to mitigate their impacts. They range from holding safety inventory, choosing locations of production closer to consumers, diversifying the sourcing of goods, to choosing additional modes of transportation. Most of these strategies are costly but they are often considered to be better responses to a demand surge than having to forego sales.

This paper considers two of these strategies, both being widely observed: dual sourcing and holding safety inventory. We do so in a simple dynamic model for which closed-form solutions exist. With it, we investigate how importers adjust safety inventory, domestic sourcing and imports to a permanent change in import prices and what role demand uncertainty plays for this adjustment.

Two central findings emerge from the analysis. First, firms use safety inventory only when the import price is low relative to the domestic price of the same good, while they use dual sourcing when it is not the case. Thus, safety inventory and dual sourcing are shown to be substitutes as hedging strategies. Second, an increase in the import price always decreases a firm's optimal inventory and import volumes by more than expected sales. Moreover this effect is larger the greater is the level of demand uncertainty. The implication is that a permanent rise in the import price decreases the 'inventory-sales' ratio irrespective of the hedging strategy (safety inventory or dual sourcing) used by the firm.

Although our analysis does not depend on a specific source for the increase in the import price, an obvious candidate is trade protection. The implication of our analysis is that the impact of a change in trade protection on the volume of trade is magnified in an environment in which firms need to hedge demand uncertainty relative to one where they do not. This is the case since, with more protection, inventory and imports both decrease more than they would otherwise. Since dual sourcing and safety inventory are important hedging strategies that firms routinely use, it is an important aspect to take into account when assessing the role of protection at the firm level. This is especially the case as demand uncertainty has been on the rise over the last few years.

Our paper has focused on how importers may hedge demand uncertainty. Another source of uncertainty is trade policy itself. Whether associated with Brexit or with the U.S. trade policy toward China, trade policy uncertainty is undoubtedly higher today than in the past. Anecdotal evidence suggest that trade policy uncertainty has significant inventory effects.[11] There are a number of studies showing that reducing trade policy uncertainty increases trade (Crowley et al., 2018, Feng et al., 2017, Handley and Limão, 2017, 2015), affects a firm's input mix and sourcing (see Handley et al., 2020), and that the ensuing trade flow dynamics are consistent with inventory adjustments (Alessandria et al., 2021). But does trade policy uncertainty have the same impacts as demand uncertainty on a firm's safety inventory and dual sourcing? There is more to be done to understand firms' behavior in a dynamic model of international trade.

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[11] For instance, Hasbro, a U.S.-based toymaker which outsources a large fraction of its production to Asia, not only has to deal with the demand uncertainty associated with its toys during the critical Christmas shopping season but also with trade policy uncertainty. Since it has essentially no domestic sources in the United States able to supply close substitutes, it has to rely on imports. This has consequences not only for the level of inventory it wants to hold but also for the timing of its orders (New York Times, Aug. 15, 2019, 'Trump delays a holiday tax, but toymakers are still worried').

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Appendix

A.1 Proof of Proposition 2

We have

$$\frac{\partial\left(z^{*}-\hat{q}\right)}{\partial v}=\frac{\left(w-v+\gamma\right)^{2}\left(2-\delta\right)-4\Delta\left(w-w\delta+\gamma\right)}{4b\delta\left(w-v+\gamma\right)^{2}}<0,$$

since

$$
\begin{aligned}
& 4\Delta\left(w-w\delta+\gamma\right)-\left(w-v+\gamma\right)^{2}\left(2-\delta\right) \\
> \ & 4\Delta\left(w-v+\gamma\right)-\left(w-v+\gamma\right)^{2}\left(2-\delta\right) \\
= \ & \left(w-v+\gamma\right)\left[4\Delta-\left(w-v+\gamma\right)\left(2-\delta\right)\right] \\
> \ & \left(w-v+\gamma\right)\left[4\Delta-2\Delta\left(2-\delta\right)\right] \\
= \ & \left(w-v+\gamma\right)2\Delta\delta>0.
\end{aligned}
$$

where the first inequality follows from $v/\delta < w$, and the second inequality follows from $w - v + \gamma < 2\Delta$.

A.2 Proof of Proposition 3

Notice that $\frac{\partial^2 \hat{m}}{\partial v \partial \Delta} = -\frac{\partial^2 \hat{y}}{\partial v \partial \Delta}$, since $\frac{\partial^2 \hat{q}}{\partial v \partial \Delta} = 0$. Thus, one needs to prove first that $\hat{y}$ is increasing in v; i.e. that

$$
\frac{\partial \hat{y}}{\partial v} = -\frac{(a+\Delta-w-2bz)}{4b\Delta}\left(1+2b\frac{\partial z}{\partial v}\right) > 0.
$$

We observe first that $(a+\Delta-w-2bz) > 0$ since $\overline{\epsilon}(z_t) = 2bz_t - a + w < \Delta$. Hence we need to show that $1 + 2b\frac{\partial z}{\partial v} < 0$ or

$$
\begin{aligned}
& 1-\frac{1}{2\delta\left(w-v+\gamma\right)^{2}}\left(4\Delta\left(w-w\delta+\gamma\right)+\delta\left(v-w-\gamma\right)^{2}\right)<0 \\
\iff \ & \frac{1}{2\delta\left(w-v+\gamma\right)^{2}}\left(4\Delta\left(w-w\delta+\gamma\right)+\delta\left(v-w-\gamma\right)^{2}\right)>1 \\
\iff \ & 4\Delta\left(w-w\delta+\gamma\right)+\delta\left(v-w-\gamma\right)^{2}>2\delta\left(w-v+\gamma\right)^{2} \\
\iff \ & 4\Delta\left(w-w\delta+\gamma\right)>\delta\left(w-v+\gamma\right)^{2}.
\end{aligned}
$$

This is true since $4\Delta(w - w\delta + \gamma) > 4\Delta(v - v\delta + \gamma\delta) > \delta(w - v + \gamma)^2$, where the first inequality comes from $w > v$ and $\delta < 1$, and the second inequality comes from $\overline{\epsilon}(z_t) = 2bz_t - a + w < \Delta$.

Next we want to show that

$$
\frac{\partial^2 \hat{y}}{\partial v \partial \Delta} = -\frac{\partial\left(\frac{(a+\Delta-w-2bz)}{4b\Delta}\left(1+2b\frac{\partial z}{\partial v}\right)\right)}{\partial \Delta} > 0,
$$

which can be written as

$$-\frac{4b\Delta(1-2b\frac{\partial z}{\partial \Delta})-4b\left(a+\Delta-w-2bz\right)}{\left(4b\Delta\right)^2}\left(1+2b\frac{\partial z}{\partial v}\right)-\frac{\left(a+\Delta-w-2bz\right)}{4b\Delta}\left(2b\frac{\partial^2 z}{\partial v\partial \Delta}\right)>0. \tag{A.1}$$

From Section 4 we know that $\frac{\partial^2 z}{\partial v \partial \Delta} < 0$ and $a+\Delta-w-2bz>0$ because $\bar{\epsilon}(z_t) = 2bz_t - a + w < \Delta$. Hence the last term in (A.1) is positive. It can also be easily shown that $1+2b\frac{\partial z}{\partial v}<0$ which means that the first term is positive provided that $4b\Delta(1-2b\frac{\partial z}{\partial \Delta}) - 4b\left(a+\Delta-w-2bz\right)>0$. This expression can be rewritten as

$$\Delta(1-2b\frac{\partial z}{\partial \Delta})-\left(a+\Delta-w-2bz\right)>0$$
$$\frac{\delta^2\left(v-w-\gamma\right)^2+2\Delta\left(1-\delta\right)\left(v(2-\delta)-\delta(w-\gamma)\right)}{2\delta^2\left(w+\gamma-v\right)}>0$$

Hence, a sufficient condition is $v(2-\delta)-\delta(w-\gamma)>0$; equivalently, $v>\frac{\delta}{2-\delta}\left(w-\gamma\right)$, which holds when inventory is lower than expected sales.