Behaviour And Games On Networks

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Thanks to Matthew Jackson for access to his teaching resources.

23 March 2015

Games on Networks

So far we have studied diffusion and learning on networks, but what if:

- There are decisions to be made that involve:
  - Complementarities
  - Strategic interplay
  - Interdependencies
- Various reasons for interactive effects:
  - Friends, neighbours, society's choice can influence one's own choice.
  - There are external effects, like higher payoff if one is well connected.
  - Sense of identity: act consistent with some stereotype

Graphical Games

Individual decisions often depend on the relative proportions of neighbours taking actions, e.g.:

- Whether to buy a product, or learn a new language.
- This results in multiple equilibria, some people may be willing to adopt a new tech only if others do:
  - No one adopts,
  - Or some non-trivial portion of population adopts it.
- One way of introducing such strategic behaviour is to model the interactions as a game.
- A useful class of such interactions and games are called Graphical Games.

General Definition

- There are n players who are connected by a network g.
- Each player takes an action in $\{0, 1\}$.
- The payoff of player i is given by:

$$u_i(x_i, x_{N_i(g)})$$

- Where $x_{N_i(g)}$ is the profile of actions taken by neighbours of i in network g.
- Therefore, the payoff depends on:
  - How many neighbours choose each action,
  - How many neighbours a player has.

General Definition

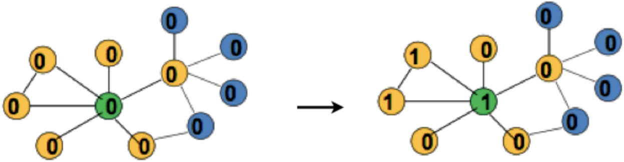
- Note that a players action is related to his indirect neighbours actions, since a player's neighbours actions is influenced by their neighbours and so forth:
- The equilibrium conditions tie together all the behaviours in the network.
- The network could be directed or undirected.

Example 1: Simple Complement

- Agent i is willing to choose 1 iff at least t of his neighbours do:

$$u_i(0, m_{N_i(g)}) = 0$$
$$u_i(1, m_{N_i(g)}) = -t + m_{N_i(g)}$$

- Where $m_{N_i(g)}$ is the number of neighbours choosing action 1.
- Example: An agent is willing to take action 1 if and only if (iff) at least two neighbours do ($t = 2$):



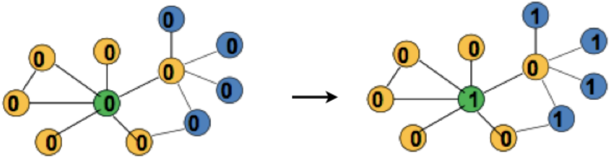
Example 2: 'Best Shot' Public Good Game

- An agent is willing to take action 1 if and only if no neighbours do:

$$u_i(0, m_{N_i(g)}) = \begin{cases} 1 & \text{if } m_{N_i(g)} > 0 \\ 0 & \text{if } m_{N_i(g)} = 0 \end{cases}$$

$$u_i(1, m_{N_i(g)}) = 1 - c, \qquad 0 < c < 1$$

- Taking action 1 is costly (c): a player prefers that a neighbour take the action rather than doing it himself,
- But taking the action is better than having nobody take the action.



Example 3: Match Majority

- Agent i prefers to do what majority of neighbours do:

$$u_i(0, m_{N_i(g)}) = 1 - \frac{m_{N_i(g)}}{d_i}$$
$$u_i(1, m_{N_i(g)}) = \frac{m_{N_i(g)}}{d_i}$$

- As an example: consider a game where there are two types of agents, "conformists" and "rebels".
- Conformists like to take an action that matches the majority of their neighbours, while rebels refer to take an action that matches the minority of their neighbours,

Equilibria

Given the graphical game structure, we can use game theory to make predictions about players' behaviour and how it depends on the network structure.

- In a graphical game, a pure strategy Nash equilibrium is a profile of strategies $x = (x_1, ..., x_n)$ such that:

$$u_i(1, x_{N_i(g)}) \geq u_i(0, x_{N_i(g)}) \qquad \text{if } x_i = 1, \tag{1}$$

$$u_i(0, x_{N_i(g)}) \geq u_i(0, x_{N_i(g)}) \qquad \text{if } x_i = 0 \tag{2}$$

- Equilibrium condition requires that each player chooses the action that gives them the highest payoff given his neighbours actions.
- No player should regret the choice that he made given the action taken by others.

Strategic Complements & Subsititutes

- Strategic Complements: Choice to take an action by my friends **increases** my relative payoff to taking that action (e.g., friend learns to play a video game). Formally, $\forall i, m > m'$:

$$u_i(1, m) - u_i(0, m) \geq u_i(1, m') - u_i(0, m')$$

- Strategic Substitutes: Choice to take an action by my friends **decreases** my relative payoff to taking that action (e.g., roommate buys a stereo/fridge). Formally, $\forall i, m > m'$:

$$u_i(1, m) - u_i(0, m) \leq u_i(1, m') - u_i(0, m')$$

Threshold and Externalities

Useful Observation

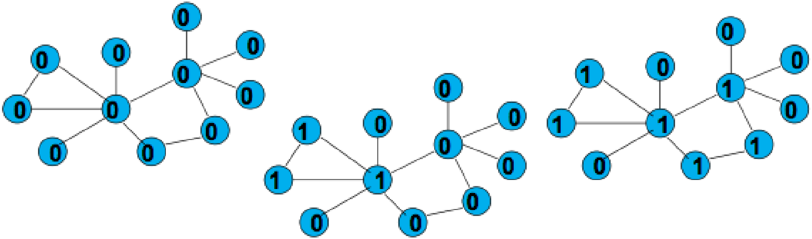
- Complements: there is a threshold $t(d)$, such that i prefers 1 if $m_{N_i(g)} > t(d)$ and 0 if $m_{N_i(g)} < t(d)$.
- Substitutes: there is a threshold t(d), such hat i prefers 1 if $m_{N_i(g)} < t(d)$ and 0 if $m_{N_i(g)} > t(d)$.

- We have Externality when:
  - Others' behaviours affect my **utility/welfare**.
  - Others' behaviours affect my **decisions, actions, consumptions, opinions**.
  - Others' actions affect the **relative payoffs to my behaviours**.

Equilibria in Simple Complement Game

Equilibrium structure is a Complete Lattice when:

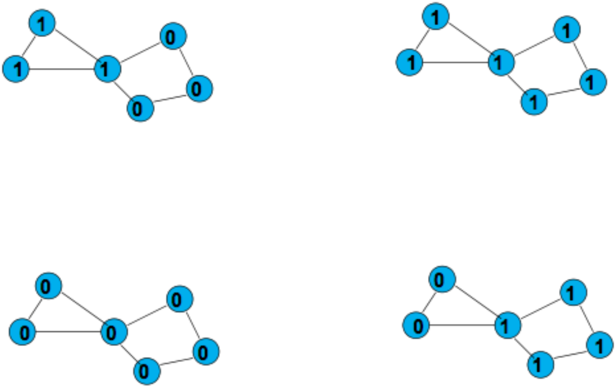
- There exist a maximum equilibrium such that each players action is at least as high as in every other equilibrium,
- Similarly, there is a minimum equilibrium where actions take their lowest values out of all other equilibria.



Equilibria in Simple Complement Game

Proposition

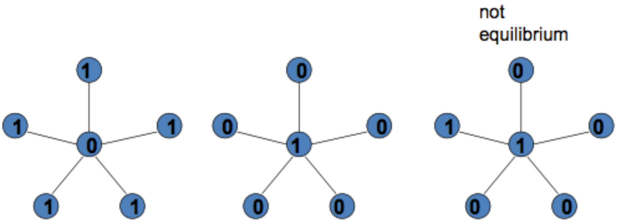
In a graphical game of strategic complements the set of pure strategy equilibria is a (nonempty) complete lattice.



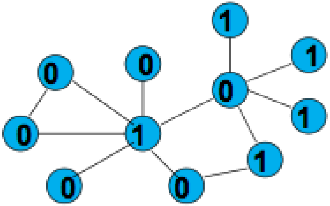
Equilibria in Best Shot Public Good Game

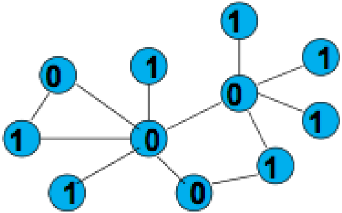
Maximal Independent Set:

- **Independent Set**: A set S of nodes such that no two nodes in S are linked.
- **Maximal**: Every node in the network is either in S or linked to a node in S.
- In the **best shot** game, the maximal independent set is the set of all agents who choose 1. By definition, none of these agents are connected to each other:



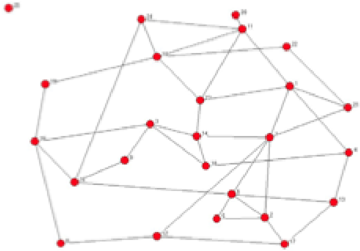
Equilibria in Best Shot Public Good Game

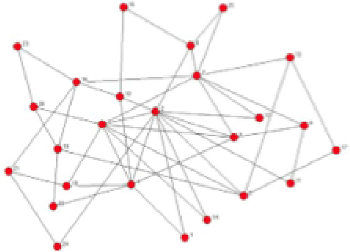




How Do Equilibria Vary With Networks?

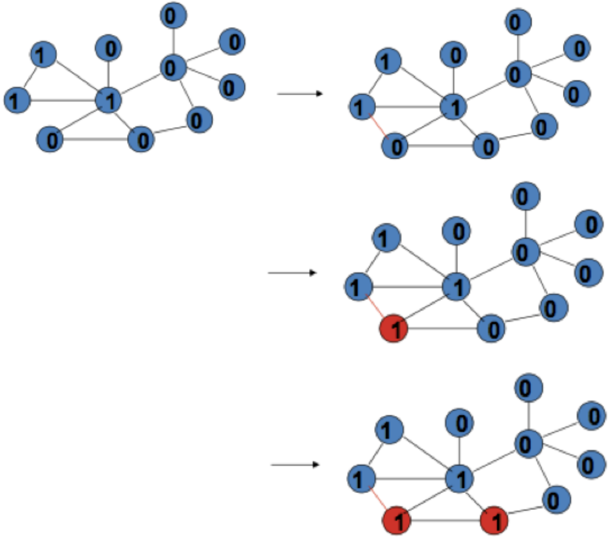
- What Happens as Network Becomes More Connected?
- What Happens as Link Structure is Rearranged?





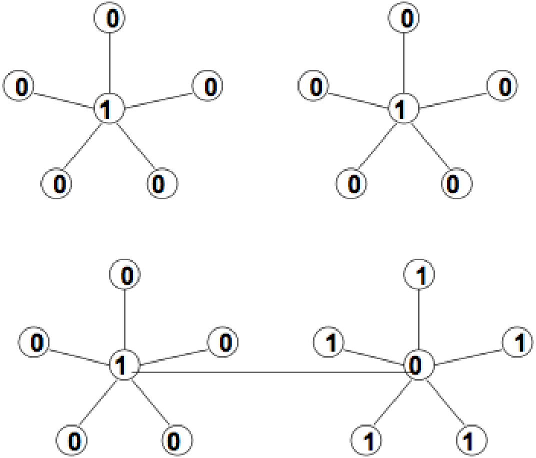
Adding Links: Strategic Complements

New equilibrium where all players take weakly higher actions ($t(d) = 2$):

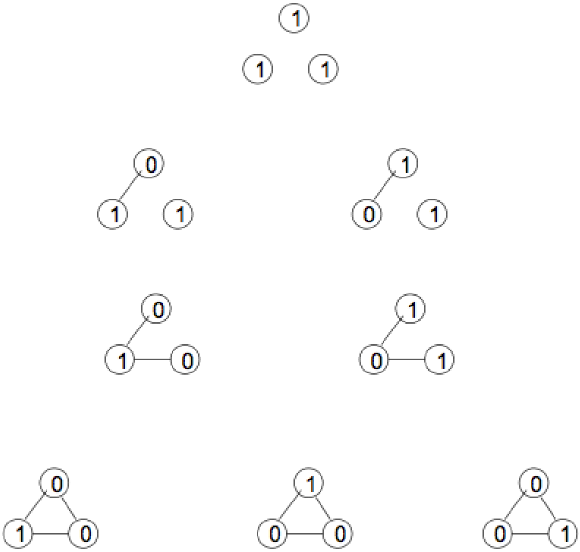


Adding Links: Strategic Substitutes

Best shot game: care only about maximum action in neighbourhood.



Adding Links: Strategic Substitutes



Conclusion

- Individuals' positions in the network matters:
  - Higher actions in complements
  - Lower actions in substitutes
- Network structure matters, adding links:
  - "increases" provision in complements,
  - "decreases" in substitutes.
- Welfare is ambiguous.