Diffusion And Learning In Networks

Rogayeh Dastranj Tabrizi
email: rda18@sfu.ca
Office: WMC 3607
Office Hours: Thursdays 12pm-2pm

Department of Economics
Simon Fraser University

Thanks to Matthew Jackson for access to his teaching resources.

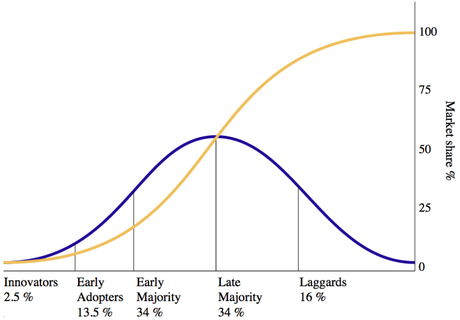
16 March 2015

Diffusion and Learning

- How does network structure impact behaviour?
  - Diffusion
    - Spread of disease
    - Ideas, basic information
    - Buy a product or not?
  - Opinion and Information: Learning
  - Choices and Decisions: Games on Networks
- Questions:
  - Extent of diffusion?
  - How does it depend on the process as well as the network?

Diffusion of Innovations: Rogers(1962)

Diffusion of innovations is a theory that seeks to explain how, why, and at what rate new ideas and technology spread through cultures[1].



With successive groups of consumers adopting the new technology (blue), its market share (yellow) will eventually reach the saturation level.

[1] http://en.wikipedia.org/wiki/Diffusion_of_innovations

Bass Model

Bass Model is a benchmark model with no explicit social or network structure:

- There are two actions or states or behaviours: 0 and 1
- $F(t)$ is the fraction of people who have adopted action 1 at time t.
- p is the rate of spontaneous innovation or adoption
- q is the rate of imitation of adoption

$$\frac{dF(t)}{dt} = (p + qF(t))(1 - F(t))$$

$$\Rightarrow \quad F(t) = \frac{1 - e^{-(p+q)t}}{1 + \frac{q}{p}e^{-(p+q)t}}$$

Getting the S-Shape Graph

Let us look at these equations again:

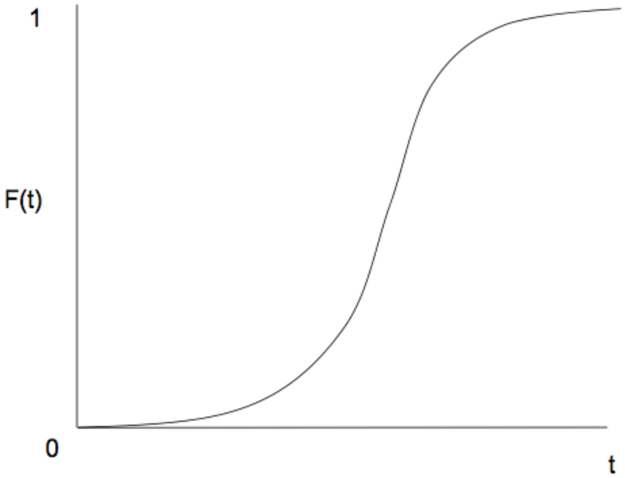
$$\frac{dF(t)}{dt} = (p + qF(t))(1 - F(t))$$

$$\Rightarrow \quad F(t) = \frac{1 - e^{-(p+q)t}}{1 + \frac{q}{p}e^{-(p+q)t}}$$

- When $F(t) = 0 \quad \Rightarrow \quad \frac{dF(t)}{dt} = p$.
- When $F(t) \to 1 \quad \Rightarrow \quad \frac{dF(t)}{dt} = 0$
- When $F(t) = \epsilon \quad \Rightarrow \quad \frac{dF(t)}{dt} = (p + q\epsilon)(1 - \epsilon)$.
- To get the initial convexity, we need $(p + q\epsilon)(1 - \epsilon) > p$:

$$q(1 - \epsilon) > p \quad \Rightarrow \quad q > p$$

S-Shape Graph



Beyond Bass Model

Reach of contagion is bounded by the component structure:

- Some players or nodes are never reached: they are immune
- Some links fail to transmit information or innovation
- What is the extent of diffusion? Does it always happen?

Bring in the network and interaction structure:

- SIR model – Susceptible, Infected, Removed
- SIS model – Susceptible, Infected, Susceptible

SIR Model

This is a good and simple model for many infectious diseases:

- Infect for some random time,
- Some links randomly (and independently) fail.

- Consider a random network on n nodes.
- Delete fraction p of the nodes
- Delete fraction $1 - f$ of the links
- If we starts at a node in giant component of the remaining network, then the giant component of that network is the extent of the infection.

SIS Model

The SIS model is extensively studied in epidemiology, and it allows nodes to **change** behaviour back and forth over time.

- Nodes are infected (I) or susceptible (S).
- Probability of infection is proportional to number of neighbours that are infected at rate ν.
- Nodes could also get well at random in any period at some rate δ.
- If $\delta > \nu$, then nodes recover faster than getting sick, so no infection stays long.
- Otherwise, the infection stays at some level and low recovery rates (low δ) can lead to large infections.

Summary

- Network structure affects diffusion:
  - Threshold for infection and contagion
  - Extent of infection
  - Who becomes infected
- High degree nodes are more prone to infection and serve as conduits.
- Tractable simulations can go a long way to offering predictions.

Learning

What if we care not just about who is infected or not, and we want to keep track of information?

- Learn from actions of others
- Learn from talking to others
- Infer indirect information

Some Questions:

- When does a society reach consensus?
- How does network structure affect consensus?
- Who is influential?
- What affects the speed of learning?

Empirical Studies

- Lazarsfeld et al (1944): Interviews in Ohio town 1940 election:
  - Identified opinion leaders who became informed via media, and others who became informed via opinion leaders
- Katz and Lazarsfeld (1955): Interviews of women in Decatur Illinois 1950's:
  - Repeated interviews over months,
  - Changes in opinions linked to individuals who influenced multiple others,
  - Opinion leaders were of same social status as influenced individuals,
  - Opinion leaders had large families and tended to be older.

Empirical Studies

Conley and Udry (2005): Pineapples in Ghana

- 132 pineapple farmers in Ghana
- Build network based on the question: "*Have you ever gone to ... for advice about your farm?*"
- There are links if either person says yes: median degree is 2 and max degree is 8.
- Observe past own fertilizer use and productivity and that of neighbours.
- Examine the probability that individuals change their fertilizer use.
- Regress on indices of good and bad news about neighbours productivity who used same/different amounts of fertilizer.

Bayesian Learning – Bala, Goyal (1998)

How to model learning of the type seen in Conlet and Udry?

- There are N players in an undirected network g.
- They can choose action A or B each period.
- Action A pays 1 for sure, Action B pays 2 with probability p and 0 with probability $1 - p$.
- Each period get a payoff based on choice and also observe neighbours' choices.
- Maximize discounted stream of payoffs:

$$E[\sum_t \delta^t \pi_{it}]$$

- Note that p is unknown and subject to learning.

Updating Beliefs

- Individuals learn from neighbours' choices, not just the actions,
- But this can be very complicated,
- So assume bounded rationality: *Agents only update on observed outcomes and not on choices*
- using this assumption, agents do not require information about larger network structure or other beliefs, or beliefs on beliefs!!

Updating Beliefs

Proposition:

If p is not exactly 1/2, then with probability 1 there is a time such that all agents in a given component play just one action (and all play the same action) from that time onward.

Sketch of Proof:

- Suppose some player plays B infinitely often. Then it should be that $p > 1/2$, or that agent would stop playing b.
- With probability 1, all agents who see B played infinitely often converge to a belief that B pays 2 with prob p>1/2.
- Neighbours of agent must play B, after some time, and so forth.
- All agents must play B from some time on

Conclusion

There are many more complicated and sophisticated models that study network structure and learning:

- These models study who has influence?
- what is the convergence speed?
- When is the consensus accurate?
- What is the impact of the network structure on learning and convergence?