Network Formation II - Strategic Networks

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Game Theoretic Models of Network Formation

In the game theoretic models of network formation, we should consider:

- Costs and benefits for each agent associated with each network
- Agents may choose links
- Contrast incentives and social efficiency

Some important questions:

- Which networks are likely to form?
- Are the networks that form stable or efficient?
- Can interventions improve efficiency?
- Can such models provide insight into observed characteristics of networks?

Modelling Choices

How should we model incentives for form and sever links?

- Is consensus needed? (directed or undirected links)
- Can agents coordinate changes in the network?
- Is the process dynamic or static?
- What do agents know when making a decision?
- Can they compensate each other for the friendship?
- How sophisticated are agents? Do they make errors?
- What happens on the network?

An Economic Analysis - Jackson Wolinsky (1996)

An Undirected Network Formation model:

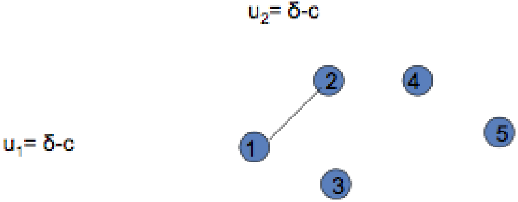
- $u_i(g)$: payoff to i if the network is g
- $0 \leq \delta \leq 1$: benefit parameter for i from connection between i and j
- $c_{ij} \geq 0$: cost to i of link to j
- $l(i,j)$: shortest path length between i, j

Therefore the payoff function is:

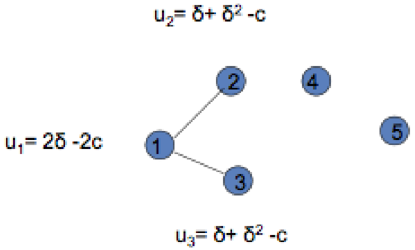
$$
\begin{aligned}
u_i(g) &= \sum_{j}^{N} \left(\delta^{l(i,j)} - g_{ij} c_{ij}\right) \qquad (1) \\
&= \sum_{j}^{N} \delta^{l(i,j)} - \sum_{j \in N_i(g)} c_{ij}
\end{aligned}
$$

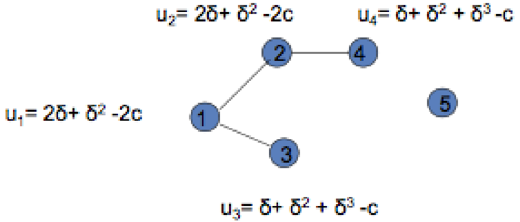
An Example - Symmetric Version

- Benefit from a friend is $\delta < 1$
- Benefit from a friend of a friend is δ^2
- cost of a link is $c > 0$



An Example - Symmetric Version

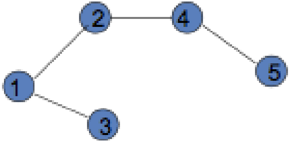




An Example - Symmetric Version

$u_2 = 2\delta + 2\delta^2 - 2c$ $u_4 = 2\delta + \delta^2 + \delta^3 - 2c$

$u_1 = 2\delta + \delta^2 + \delta^3 - 2c$

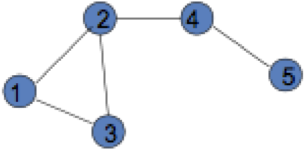


$u_5 = \delta + \delta^2 + \delta^3 + \delta^4 - c$

$u_3 = \delta + \delta^2 + \delta^3 + \delta^4 - c$

$u_2 = 3\delta + \delta^2 - 3c$ $u_4 = 2\delta + 2\delta^2 - 2c$

$u_1 = 2\delta + \delta^2 + \delta^3 - 2c$



$u_5 = \delta + \delta^2 + 2\delta^3 - c$

$u_3 = 2\delta + \delta^2 + \delta^3 - 2c$

Modelling Incentives and Equilibirum

Some Questions:

- Which networks are best for society?
- Which networks are formed by agents?

How to Model:

- Model this as a game where each agent announce who they wish to be linked to simultaneously.
- Links are formed if and only if both agents name each other.
- Since both agents need to confirm to form a link, the equilibrium concept is Nash equilibrium.
- In equilibrium, no agent can benefit from changing his actions, i.e. forming new links or severing other links.

Equilibrium Example With Two Agents

	"Yes" to link	"No" to link
"Yes" to link	1,1	0,0
"No" to link	0,0	0,0

Both announce each other is an equilibrium (yes,yes). Neither announces the other is an equilibrium (no, no)

Pairwise Stability

We have pairwise stability when:

- No agent benefits from severing a link: relationships must be beneficial to be maintained.

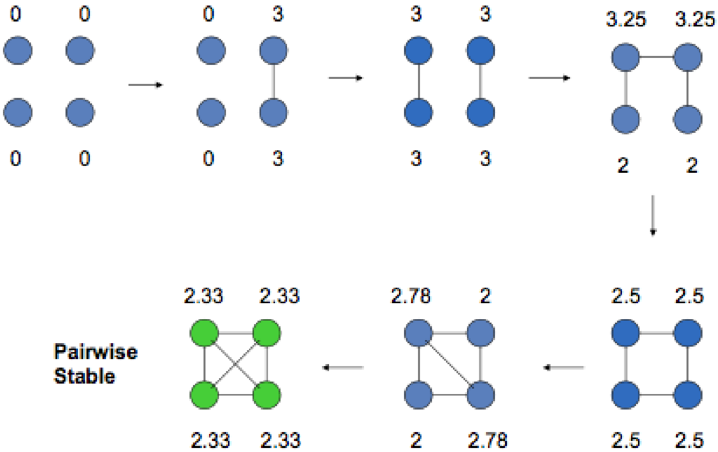
$$u_i(g) \geq u_i(g - ij) \quad \forall i, ij \in g$$

- No two agents both benefit from adding a link (at least one strictly): beneficial relationships are pursued when available.

$$u_i(g + ij) \geq u_i(g) \quad \text{implies} \quad u_j(g + ij) < u_j(g) \quad \text{for} \quad ij \notin g$$

- A weak concept, but often narrows things down.

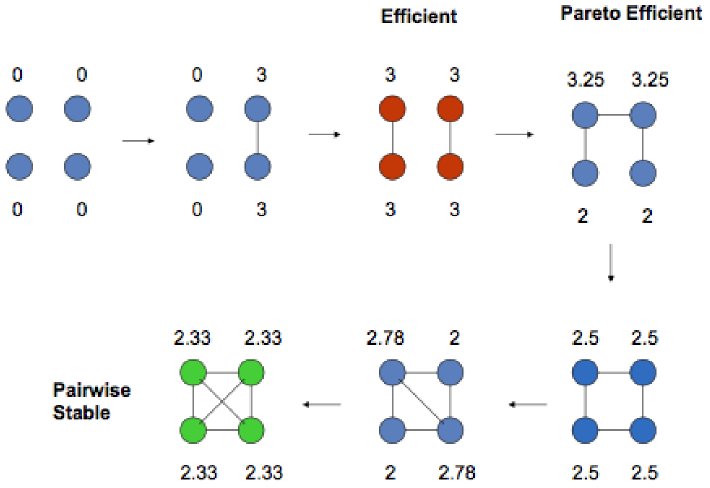
Pairwise Stability



Efficiency

- Pareto Efficient Network g: There does not exist g' such that $u_i(g') \geq u_i(g)$ for all i with strict inequality for some.
- Efficient Network g (or strong efficiency): A network g is called efficient if $\sum_i u_i(g) \geq \sum_i u_i(g')$ for all $g' \in G$.
- There always exist at least one efficient network, given that there are only finitely many networks.

Efficiency


Efficient
Pareto Efficient
0 0
0 0
0 3
0 3
3 3
3 3
3.25 3.25
2 2
2.5 2.5
2.5 2.5
2.78 2
2 2.78
Pairwise Stable
2.33 2.33
2.33 2.33

Externalities

- Positive Externality:

$$u_k(g+ij) \geq u_k(g) \quad \text{if} \;\; ij \notin g, \;\; \forall k \neq i, j$$

- Negative Externality:

$$u_k(g+ij) \leq u_k(g) \quad \text{if} \;\; ij \notin g, \;\; \forall k \neq i, j$$

- There are inefficiencies due to positive or negative externalities.

Back to Jackson Wolinsky (1996)

An Undirected Network Formation model:

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Therefore the payoff function is:

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&= \sum_{j}^{N} \delta^{l(i,j)} - \sum_{j \in N_i(g)} c_{ij}
\end{aligned}
$$

Bloch-Jackson 2005 Model

Distance Based Utility Model:

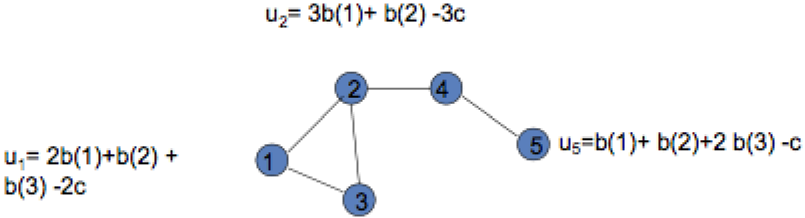
- Payoff to i if the network is g:

$$u_i(g) = \sum_{j}^{N} \left(bl(i,j)) - d_i(g)c\right)$$

- $b(l)$: decreasing benefit function for i from connection between i and j
- $c \geq 0$: cost to i of link to j
- $l(i,j)$: shortest path length between i, j

An Example - Symmetric Version

- Benefit from a friend is $b(1)$
- Benefit from a friend of a friend is $b(2) < b(1)$
- cost of a link is $c > 0$



Efficient Network: Symmetric Connection Model

The **unique efficient** network structure in the distance-based model is:

- Complete network is $b(2) < b(1) - c$
- Star encompassing all nodes if $b(1) - b(2) < c < b(1) + \frac{(n-2)}{2} b(2)$
- Empty network if $b(1) + \frac{(n-2)}{2} b(2) < c$

Inefficiencies in Connection Model is due to positive externalities.

Pairwise Stable Networks

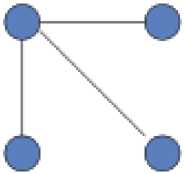
In the distance-based connection model is:

- At low cost $c < b(1) - b(2)$: **Complete network** is pairwise stable.
- At medium low cost $b(1) - b(2) < c < b(1)$: **Star network** is pairwise stable, but there are other pairwise stable networks as well.
- At medium high cost $b(1) < c < b(1) + \frac{(n-2)}{2} b(2)$: Star network **is not** pairwise stable.
- At high cost $b(1) + \frac{(n-2)}{2} b(2) < c$: **Empty network** is the only pairwise stable network.

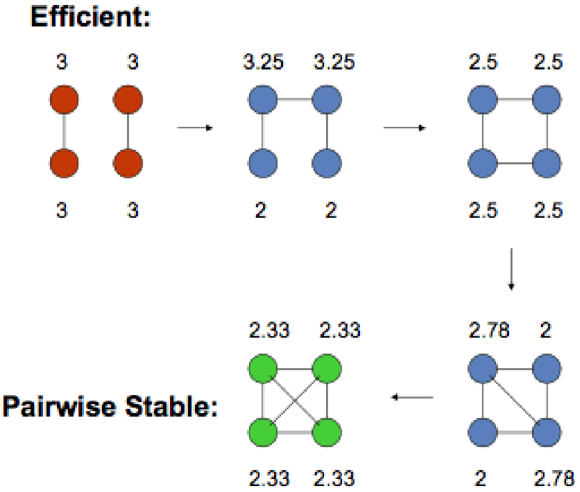
Externality

Look at the following example to see why for example at medium high cost star is not pairwise stable:

- Payoff to the center agent: $3b(1) - 3c$.
- Not pairwise stable if $b(1) < c$.
- But overall payoff is $6b(1) + 6b(2) - 6c$.
- Central agent does not account for the fact that peripheral players gain indirect benefits.



Tension Between Stability & Efficiency



Summary

Strength of economics approach:

- Payoffs allow for welfare analysis (incentives vs efficiency)
- Tie the nature of externalities to network formation

Challenges to economic approach:

- Overly regular network structures emerge from models: Need for some heterogeneity
- How to identify payoff structure in applications?