# NOTES ON POINT ELASTICITY

Elasticity is a mathematical concept that provides a measurement for the change in an output when an input is changed. Both the input and output need to be positive and the functional relationship is differentiable.

In mathematics, elasticity of a function of $x$ ($f\,[x]$)is typically estimated at a fixed point ($a$) as:

$$\frac{a}{f[a]}\,\frac{d\,f[a]}{dx} \quad \rightarrow \quad \approx \frac{\%\Delta f[a]}{\%\Delta x} = \frac{\Delta f[a]}{f} \Big/ \frac{\Delta x}{x}$$

where ($d\,f[a]$ / $dx$) is the derivative ($df[x]$ / $dx$) evaluated at the point $a$. For example, if $a = 2$ and $f[x] = 2x^2$ then ($df[x]$ / $dx$) = $4x$ and ($d\,f[a]$ / $dx$) = 4(2) = 8.

Dividing the change by the level of the variable has two implications: i) units cancel out leaving a (dimension-less) number; ii) units of measurement no longer matter, e.g., if $x$ is price then it does not matter if measured in dollars or cents.

An ‘elastic’ function is one where % changes in output for a given % change in input > 1
An ‘inelastic’ function is one where % changes in output for a given % change in input < 1
Unitary elasticity functions = 1 across the domain of the function.

Elasticity concepts were adopted in Economics to provide a number for ‘price elasticity of demand’, ‘price elasticity of supply’, ‘income elasticity of demand’ and the like. In Economics 100 courses this concept is usually introduced as a discrete variable (without using calculus) as the Arc elasticity to distinguish this concept from the Point elasticity that is defined using calculus. The Arc elasticity recognizes the need to evaluate the elasticity at the midpoint. For example, the price ($P$) elasticity of quantity demanded ($Q_d$) as:

$$\frac{\dfrac{P_1 + P_2}{2}}{\dfrac{Q_{d1} + Q_{d2}}{2}} * \frac{\Delta Q_d}{\Delta P}$$

where the $\Delta$ is evaluated for the two points given. This discrete approach recognizes that a straight-line demand function will have an elasticity that varies along the function.