Numerical Simulation of Stochastic Differential Equations

Handout for the Short Course:
The Third Montreal Scientific Computing Days, 2006
`http://www.crm.umontreal.ca/Comp06/indexen.html`

The lectures are designed to give an accessible introduction to the numerical solution of stochastic differential equations (SDEs). They are based on the opening chapters of a book that is currently in preparation: *An Introduction to the Numerical Simulation of Stochastic Differential Equations*, by Desmond J. Higham and Peter E. Kloeden.

The treatment here is designed to give postgraduate students a feel for the basic concepts. Hence, it is not intended to be mathematically rigorous. To learn more about the numerical solution of stochastic differential equations (SDEs), we recommend the following sources:

- [4] and [7] are comprehensive modern references
- [2] is a more elementary text that includes MAPLE computations

Below are some resources that cover SDE theory, with an asterisk indicating that numerical methods are also discussed.

- [1] is perhaps the most accessible of the rigorous texts; it starts from a very basic level and works up to a definition of SDEs
- [6]* is a fairly gentle non-rigorous text
- [3] and [5]* are at the more accessible end of the rigorous category
- the Berkeley lecture notes *An Introduction to Stochastic Differential Equations, Version 1.2** by Lawrence C. Evans will appeal to those with a differential equations background; these are currently available at `math.berkeley.edu/~evans/SDE.course.pdf`

Some MATLAB codes that accompany the lectures can be found at the *LMS Short Course on Computational Differential Equations* section of the webpage `http://www.maths.strath.ac.uk/~aas96106/algfiles.html`

Lecture 1, Part 1: Background Material

- Random variables
- Monte Carlo simulation
- Brownian motion

Lecture 1, Part 2: Stochastic Differential Equations

- Ito stochastic integrals
- Ito SDEs
- Examples of SDEs

Lecture 2, Part 1: Euler–Maruyama method

- Definition
- Weak convergence
- Strong convergence
- Linear Stability

Lecture 2, Part 2: Application: Mean Exit Times

- Statement of problem and background theory
- Monte Carlo simulations

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References

[1] Z. Brzeźniak and T. Zastawniak, *Basic Stochastic Processes*, Springer, Berlin, 1999.

[2] S. Cyganowski, P. Kloeden, and J. Ombach, *From Elementary Probability to Stochastic Differential Equations with MAPLE*, Springer, Berlin, 2002.

[3] F. C. Klebaner, *Introduction to Stochastic Calculus with Applications*, Imperial College Press, London, 1998.

[4] P. E. Kloeden and E. Platen, *Numerical Solution of Stochastic Differential Equations*, Springer-Verlag, Berlin, 1992.

[5] X. Mao, *Stochastic Differential Equations and Applications*, Horwood, Chichester, 1997.

[6] T. Mikosch, *Elementary Stochastic Calculus (with Finance in View)*, World Scintific, Singapore, 1998.

[7] G. N. Milstein and M. V. Tretyakov, *Stochastic Numerics for Mathematical Physics*, Springer-Verlag, Berlin, 2004.