

A Comparison of the Experimental Properties of a Compound Pendulum with Simple Theory

A. Curzon
Department of Physics
Simon Fraser University, Burnaby BC
Canada V5A 1S6

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Abstract

We measured the period of oscillation, P , of a rigid, 1-metre long bar as a function of distance, l , of the point of suspension from the centre of gravity. The observed curve of P versus l agrees within experimental error with a curve calculated from simple theory, although in the region of the minimum values of P , the theoretical curve presents values that are systematically about 1% too low. This deviation is not consistent with viscous effects proportional to the angular velocity of the bar and probably arises because of a more complex experimental damping mechanism. As expected from theory, the graph of P^2l versus l^2 is linear and indicates a value for the radius of gyration of the bar which agrees within 1% with the value of 28.9 cm calculated from the geometry of the bar. We also determined the acceleration due to gravity to be

$$g = 971 \pm 8 \text{ cm/s}^2.$$

The various disagreements between the theoretical and experimental results do not exceed $\pm 1\%$ and are attributed to damping mechanisms not considered in the simple theory.

Introduction[†]

A compound pendulum is a rigid body swinging in a vertical plane about any horizontal axis passing through the body. Pendulums have many practical applications including timekeeping and measuring gravitational field strength. The simple pendulum is treated in many elementary physics texts but is an idealization which does not include the mass of the arm which supports the swinging bob. Newman and Searle (1951, 22-23) have developed a theoretical treatment of the compound pendulum based on Newton's laws. In order to see if this simple theory will allow accurate analysis of real pendulums, we carried out an experimental investigation of a compound pendulum, focusing on the dependence of P on l , where P is the period for small oscillations of the pendulum when the distance between the centre of gravity and the axis of rotation is l . In this laboratory report, we present the results of this experiment and compare our experimental results with the theory of Newman and Searle

Theory

According to Newman and Searle (1951, 22-23), the period, P , of a compound pendulum is

$$P = 2\pi \left(\frac{k^2 + l^2}{lg} \right)^{1/2}, \quad (1)$$

where g is the acceleration of gravity and k is the radius of gyration, which is defined by

$$k^2 = \frac{\int \rho r^2 dV}{\int r dV}, \quad (2)$$

where ρ is the density of the material of the pendulum at a distance, r , from the centre of gravity.

Equation 1 applies if the damping is negligible and if α , the angular amplitude of oscillation, is infinitely small. When α is finite, the period is given by

$$P' = P \left(1 + \frac{1}{4} \sin^2 \frac{\alpha}{2} \right). \quad (3)$$

[†] This sample lab report was originally written by A. Curzon (SFU Department of Physics) in 1993. It was updated for style and format in 2001 by S. Stevenson and S. Whitmore (SFU School of Engineering Science) and M. Chen (SFU Department of Physics).

If we assume that the damping force is $MR(k^2+l^2)\omega$ where ω is the angular velocity of the pendulum, M is its mass, and R is a constant, then according to Stephens and Bate (1950, 358), the period of the damped motions is

$$P_D = 2\pi \left[\frac{lg}{(k^2 + l^2)} - \frac{R^2}{4} \right]^{1/4} . \quad (4)$$

When the damping is small, $R^2/4 < lg/(k^2+l^2)$, and in this case, we can use equations 4 and 1 to obtain the relation

$$P_D - P = P_D^3 R^2 / 32\pi^2 . \quad (5)$$

Experimental Procedure

The compound pendulum used in the present experiments consists of a rectangular iron bar 100 cm long x 3.80 cm wide x 0.95 cm thick in which a number of holes each 0.47 cm in diameter have been drilled with 5 cm between the centres of adjacent holes (see Figure 1). The axes of the holes are perpendicular to the face of the largest area of the bar, and the axis of one of the holes (A) passes through the centre of gravity of the bar. In the current experiments, the bar was suspended by means of an axle which passed through one of the holes and which was supported on a ball bearing mount so that the rod could oscillate in a vertical plane with a minimum of friction at the bearing.

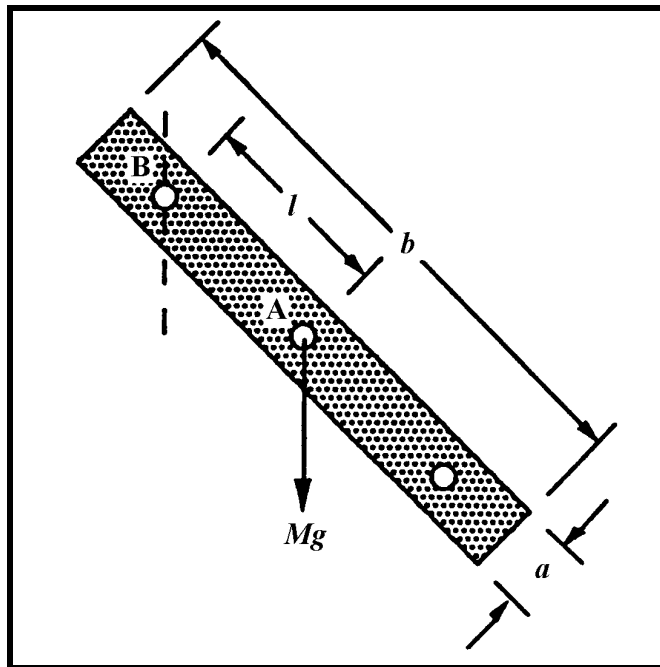


Figure 1: The Compound Pendulum

The period of oscillation was obtained by timing twenty swings with a stop watch. To obtain information about errors, several such sets of readings were obtained, and the period, P , was calculated by averaging. The amplitude of oscillation was kept below 10° to ensure that the period of oscillation was within 0.2% of the period for infinitely small oscillations (see equation 3). The distance, l , between the axes of holes A and B was measured with a metre ruler, enabling us to determine l within 1% accuracy.

Results and Discussion

Figure 2 presents a graph of P versus l .

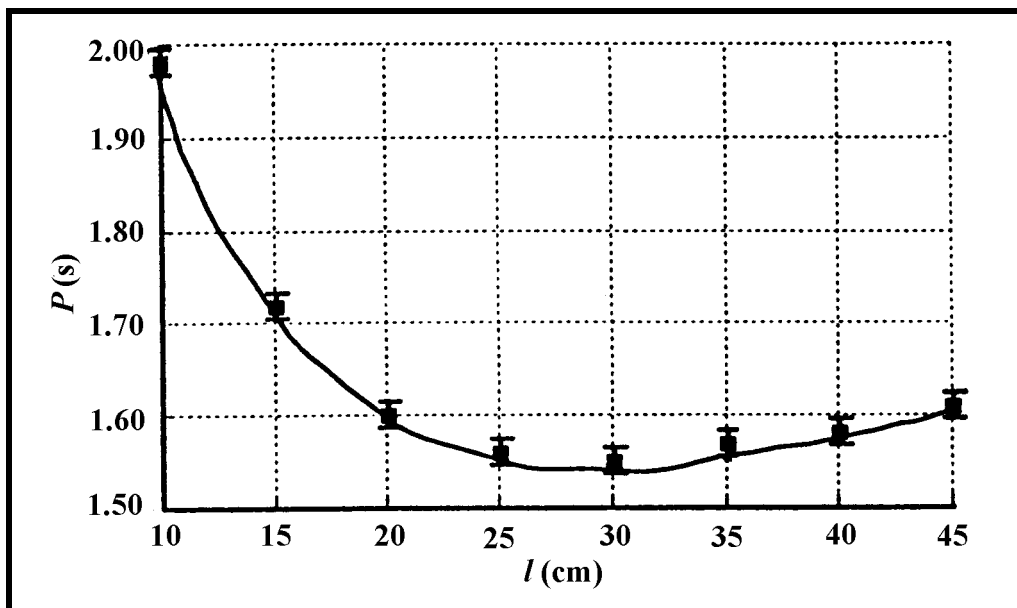


Figure 2: Graph of P versus l for a compound pendulum where P is the period in seconds and l is the distance in cm between the centre of gravity of the pendulum and the point of suspension. (A solid line indicates calculated values; points with error bars indicate experimental results.)

By differentiation of equation 1, it is easily shown that the minimum value of P is

$$P_{\min} = 2\pi\sqrt{2k/g} . \quad (6)$$

From Figure 2, we obtain

$$P_{\min} = 1.54 \pm 0.01 \text{ s; hence,}$$

$$k = 29.5 \pm 0.5 \text{ cm,} \quad (7)$$

where we have used the value of

$$g = 981 \text{ cm/s}^2 \quad (8)$$

for the acceleration of gravity.

Equation 1 has two solutions, l_1 and l_2 , for a fixed value of P . Algebraic manipulations of the equation demonstrate that

$$\sqrt{l_1 l_2} = k. \quad (9)$$

When $P = 1.58$ s, the observed values for l were $l_1 = 42.5 \pm 0.01$ cm and $l_2 = 20.0 \pm 0.1$ cm. These values provide the result, $k = 29.2 \pm 0.1$ cm, which is in reasonable agreement with the value of k obtained from P_{\min} (equation 6).

When equation 2 is applied to a uniform bar of width, a , and length, b (see Figure 1), the result is

$$k^2 = a^2/12 + b^2/12. \quad (10)$$

In the present experiments, the values of a and b were 3.80 ± 0.01 cm and 100.0 ± 0.1 cm, respectively. Hence, the calculated value of k is

$$k = 28.9 \pm 0.01 \text{ cm}. \quad (11)$$

For simplicity, this calculation neglects the presence of holes. When the holes are considered, the calculated value of k is reduced but remains within 1% of that given in equation 11.

The curve shown in Figure 2 was calculated using equations 1, 7, and 8. The calculated and measured values of P agree within 1% or less, with the biggest discrepancy being observed in the region of P_{\min} , where the calculated values are consistently low. This discrepancy cannot be explained in terms of viscous damping proportional to the angular velocity of the bar because as equation 5 demonstrates, the discrepancy between the observed damped period, P_D , and the undamped period, P , should decrease as P_D decreases. The difference in the calculated and observed results may be due to damping in the roller bearing.[‡]

Equation 1 may be rewritten in the form

[‡] Note that in a scientific paper, you must produce evidence for this sort of conjecture. For example, the experiment should be repeated after the bearing has been lubricated. However, in an elementary laboratory, you lack the time to follow up all possible explanations. Despite this limitation, you must nevertheless make an effort to determine plausible explanations for discrepancies.

$$P^2l = \frac{4\pi^2}{g}(k^2 + l^2). \quad (12)$$

This equation indicates that the graph of P^2l versus l^2 should be a straight line of slope $4\pi^2/g$ and intercept $4\pi^2k^2/g$ on the P^2l axis as illustrated in Figure 3.

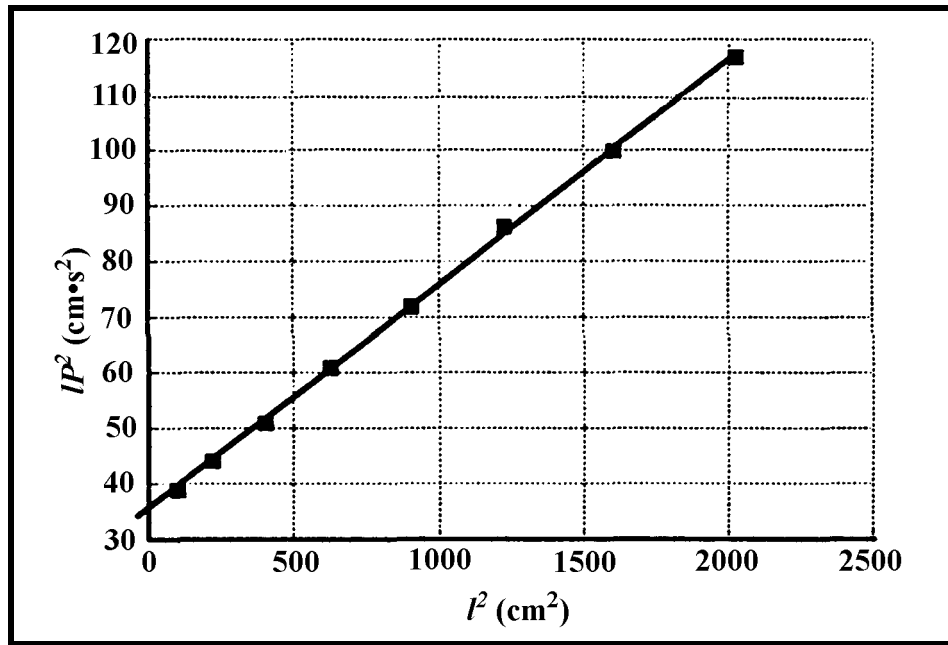


Figure 3: Graph of P^2l versus l^2 for a compound pendulum.
Slope = $0.0404 \pm 0.0004 \text{ s}^2/\text{cm}$.
Intercept on the IP^2 axis = $35.4 \pm 0.4 \text{ cm}^2/\text{s}$.

Figure 3 shows that the graph of P^2l versus l^2 is a straight line whose slope and intercept on the P^2l axis lead to the values

$$k = 28.9 \pm 0.04 \text{ cm} \quad (13)$$

$$g = 971 \pm 8 \text{ cm/s}^2. \quad (14)$$

The value of k agrees well with the calculated result (see equation 11), and g is close to the accepted value of 981 cm/s^2 . In Figure 3, the majority of the length of the straight line has points where P is not close to P_{min} (because P^2l is plotted). Hence, k is largely determined by points having P in excess of P_{min} . Figure 2 shows that the calculated values of P agree best with the experimental values for P in excess of P_{min} . This fact explains why the straight line graph gives a value of k that agrees well with the calculated value (see equations 11 and 13). Similarly, the difference between the k values given by equations 7 and 11 occurs because the value given in equation 7 depends on the observed P_{min} , which is not the same as the result calculated from simple theory.

The random errors in the times and distances measured in the present experiments were about $\pm 1\%$. We used the range of possible straight lines that could be drawn in Figure 3 to fit the data in order to obtain the errors given in equations 13 and 14. In other cases, we used the standard rules for combining errors.

As noted previously, the measured values of P are systematically about 1% too high when P has values near P_{\min} . This result is probably due to friction at the bearing and not due to any fundamental limitation of the theory. Therefore, we can reasonably assert that this experiment demonstrates that the behaviour of a compound pendulum is in accord with Newton's laws of rotation.

References

- Newman, F.H. and V.H.L. Searle. 1951. *The General Properties of Matter*. Edward Arnold: London.
- Stephens, R.W.B. and A.E. Bate. 1950. *Wave Motion and Sound*. Edward Arnold: London.