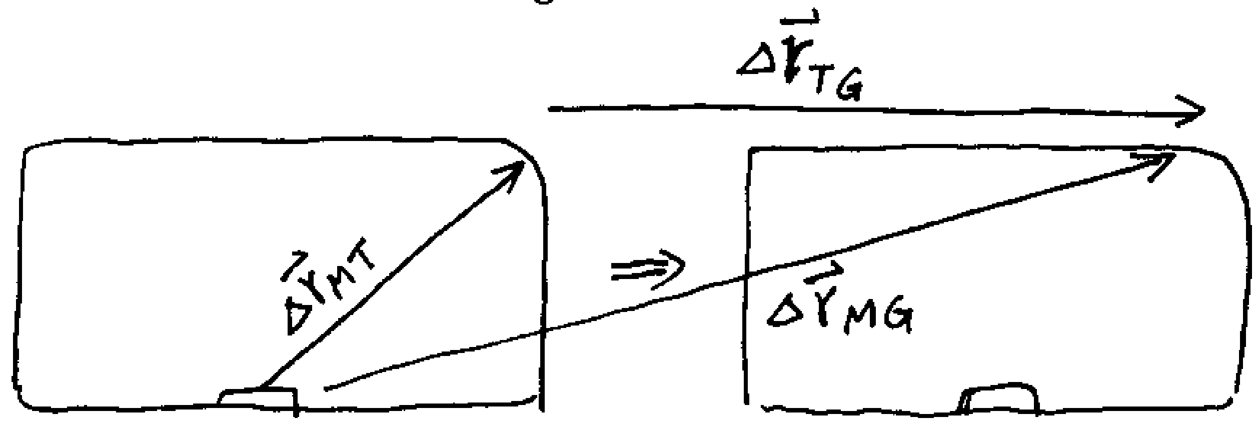
Phys101 Lecture 4
Relative Motion

Key point:

Vector relationship among the *absolute velocity*, *relative velocity* and *the velocity of the moving reference frame*.

Reference: 3-9

e.g. A man walking in a moving train



In time interval Δt

The man moves from door to corner : displacement = $\Delta\vec{r}_{MT}$

MT — man relative to Train

The train moves to the right by a displacement $\Delta\vec{r}_{TG}$.

TG — Train relative to Ground.

Question : How do we find the displacement of the man with repect to the ground ? $\Delta\vec{r}_{MG}$

From the diagram, and the Head-To-Tail Rule:

$$\Delta \vec{r}_{MG} = \Delta \vec{r}_{MT} + \Delta \vec{r}_{TG} . \qquad (1)$$

↑ absolute displacement — ↑ relative displacement — ↑ displacement of the moving reference.

Displacement of the object measured using a coordinate system fixed on the ground.

Displacement of the object measured in a coordinate system attached to a moving reference frame.

Displacement of the moving reference frame measured using a coordinate system fixed on the ground.

For velocity relationship,

$$\frac{eq(1)}{\Delta t}, \text{ and let } \Delta t \rightarrow 0. \quad \text{We get}$$

$$\vec{v}_{MG} = \vec{v}_{MT} + \vec{v}_{TG}$$

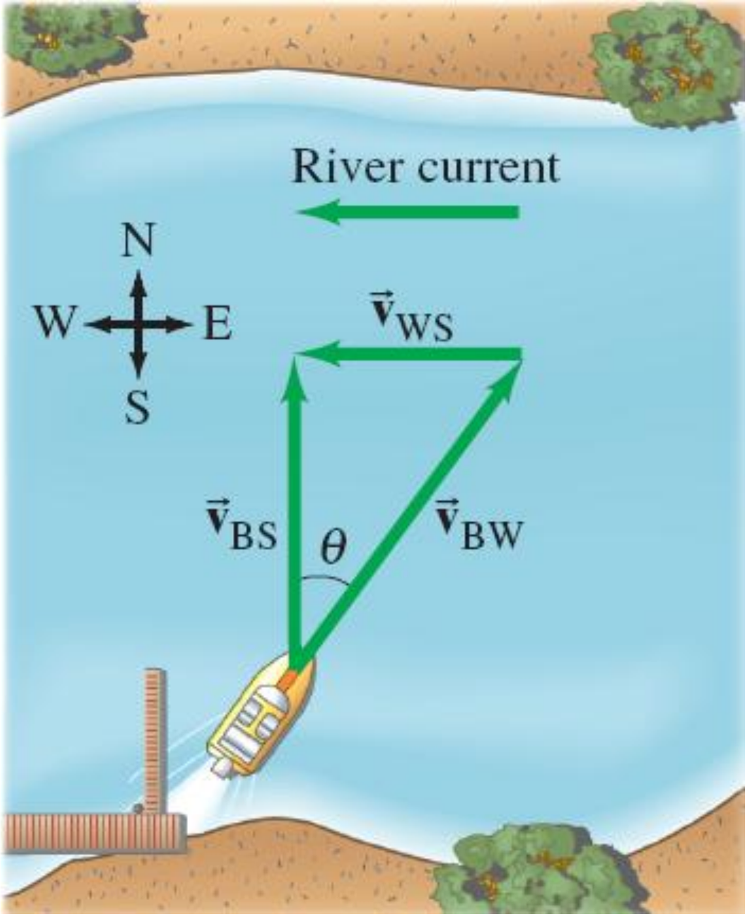
i.e.,

$$\text{absolute velocity} = \text{relative velocity} + \text{velocity of moving reference}$$

Vector sum!

Similarly, for acceleration,

$$\vec{a}_{MG} = \vec{a}_{MT} + \vec{a}_{TG}$$



A Boat in a River

$$\vec{\mathbf{v}}_{BS} = \vec{\mathbf{v}}_{BW} + \vec{\mathbf{v}}_{WS}.$$

If a man is walking on the boat,

$$\vec{v}_{MS} = \vec{v}_{MB} + \vec{v}_{BS}$$

$$\vec{v}_{MS} = \vec{v}_{MB} + \vec{v}_{BW} + \vec{v}_{WS}$$

$\vec{v}_{BW}$ is the velocity of the boat relative to water,

$\vec{v}_{BS}$ is the velocity of the boat relative to the shore,

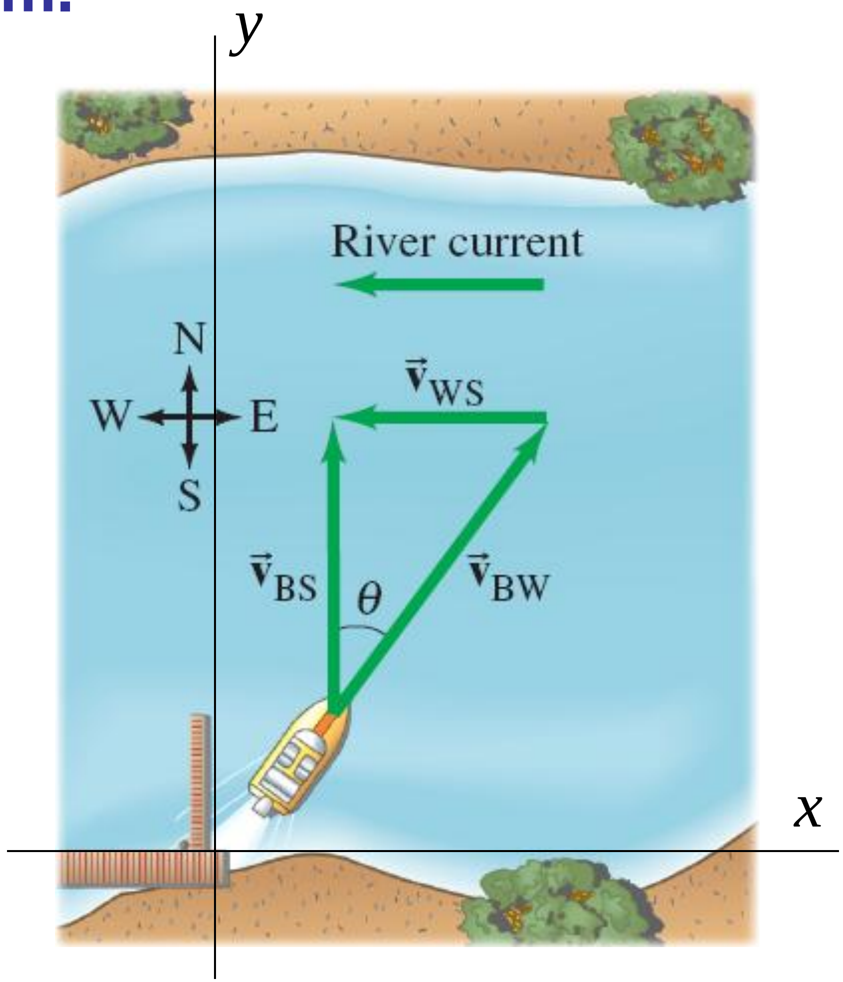
$\vec{v}_{WS}$ is the velocity of water relative to the shore,

$\vec{v}_{MB}$ is the velocity of the man relative to the boat,

$\vec{v}_{MS}$ is the velocity of the man relative to the shore.

Example 3-14: Heading upstream.

A boat's speed in still water is v_{BW} = 1.85 m/s. If the boat is to travel directly across a river whose current has speed v_{WS} = 1.20 m/s, at what upstream angle must the boat head?



$$\vec{\mathbf{v}}_{BS} = \vec{\mathbf{v}}_{BW} + \vec{\mathbf{v}}_{WS}.$$

x-component:

$$0 = v_{BW} \sin\theta - v_{WS}$$

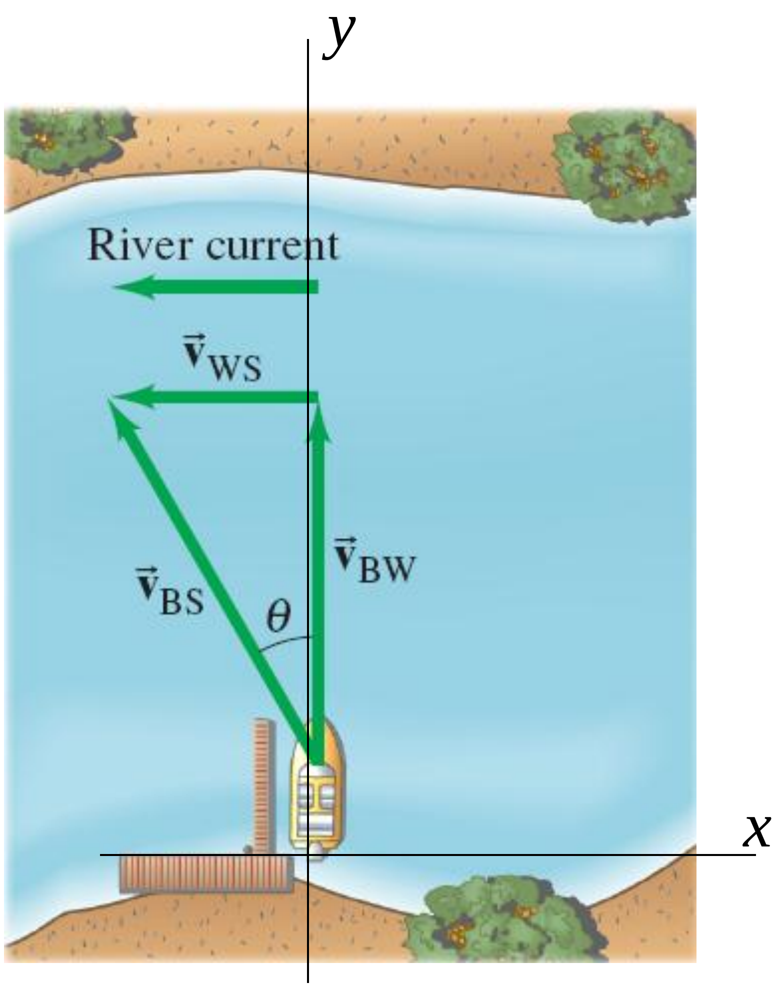
$$\sin\theta = \frac{v_{WS}}{v_{BW}}$$

$$\theta = \sin^{-1}\frac{v_{WS}}{v_{BW}} = \sin^{-1}\frac{1.20}{1.85} = 40.4°$$

The x-component of v_{BW} cancels v_{WS}.

Example 3-15: Heading across the river.

The same boat (v_{BW} = 1.85 m/s) now heads directly across the river whose current is still 1.20 m/s. What is the velocity (magnitude and direction) of the boat relative to the shore?



$$\vec{v}_{BS} = \vec{v}_{BW} + \vec{v}_{WS}.$$

It's a right triangle,

$$v_{BS} = \sqrt{v_{BW}^2 + v_{WS}^2} = \sqrt{1.85^2 + 1.20^2} = 2.21 m/s$$

$$\theta = \tan^{-1}\frac{v_{WS}}{v_{BW}} = \tan^{-1}\frac{1.20}{1.85} = 33.0°$$

Next: Dynamics

Key concept/method:

Free body diagram

Ref: Chapter 4 – the whole chapter.