Chapter 6

Gravitation and Newton's Synthesis

Units of Chapter 6

- **Newton's Law of Universal Gravitation**

- **Vector Form of Newton's Law of Universal Gravitation**

- **Gravity Near the Earth's Surface; Geophysical Applications**

- **Satellites and "Weightlessness"**

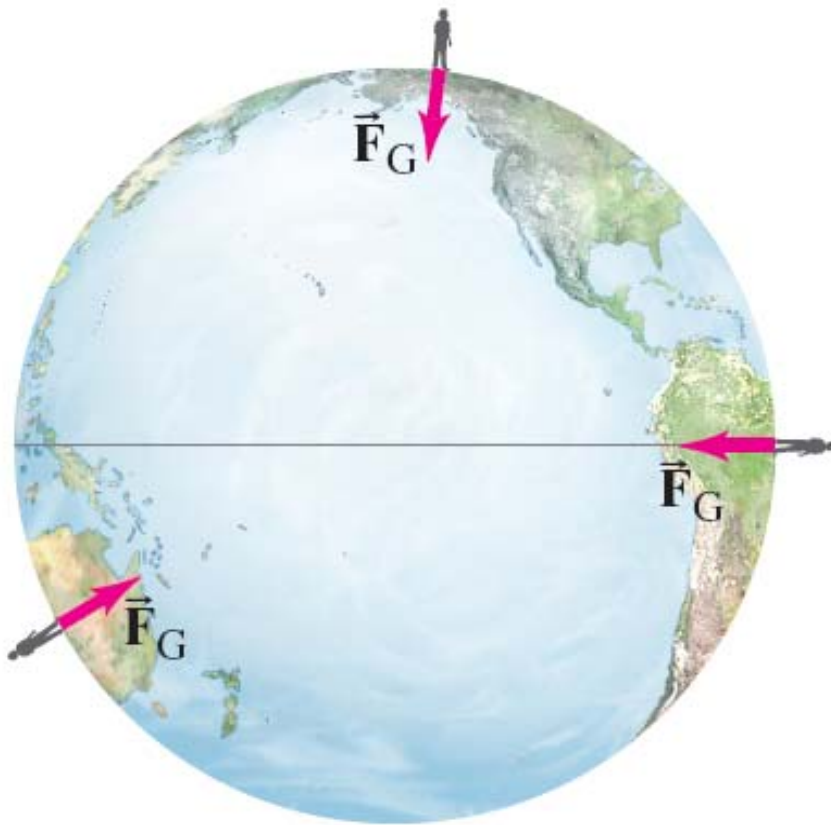- **Kepler's Laws and Newton's Synthesis**

- **Gravitational Field**

Units of Chapter 6

- **Types of Forces in Nature**

- **Principle of Equivalence; Curvature of Space; Black Holes**

6-1 Newton's Law of Universal Gravitation

If the force of gravity is being exerted on objects on Earth, what is the **origin** of that force?

Newton's realization was that the force must come from the **Earth.**

He further realized that this force must be what keeps the **Moon** in its orbit.

6-1 Newton's Law of Universal Gravitation

The gravitational force on you is one-half of a third law pair: the **Earth** exerts a **downward** force on you, and you exert an **upward** force on the Earth.

When there is such a **disparity** in masses, the reaction force is undetectable, but for bodies more equal in mass it can be **significant**.

Moon

Gravitational force exerted on Moon by Earth

Earth

Gravitational force exerted on Earth by the Moon
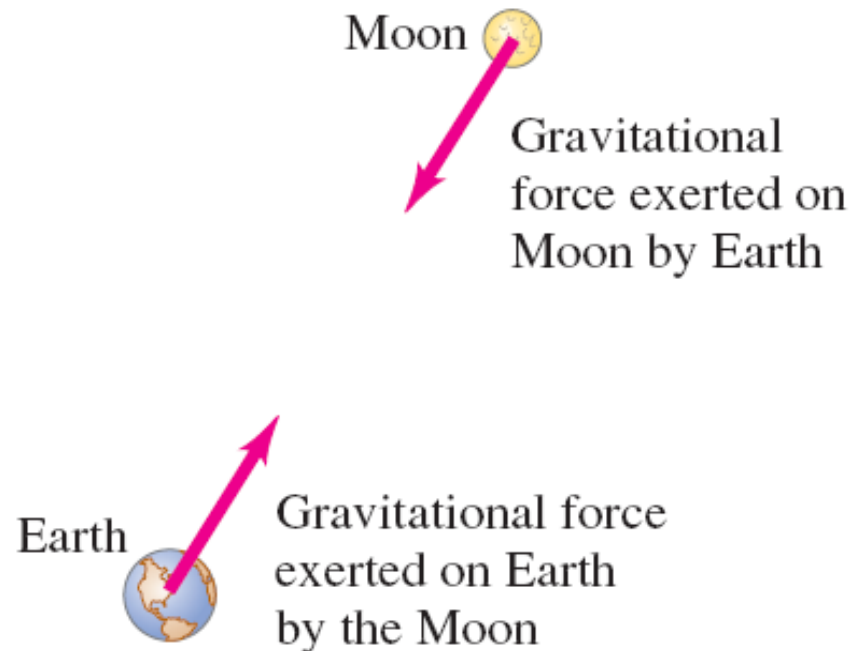
6-1 Newton's Law of Universal Gravitation

Therefore, the gravitational force must be proportional to **both** masses.

By observing planetary orbits, Newton also concluded that the gravitational force must decrease as the **inverse** of the **square** of the distance between the masses.
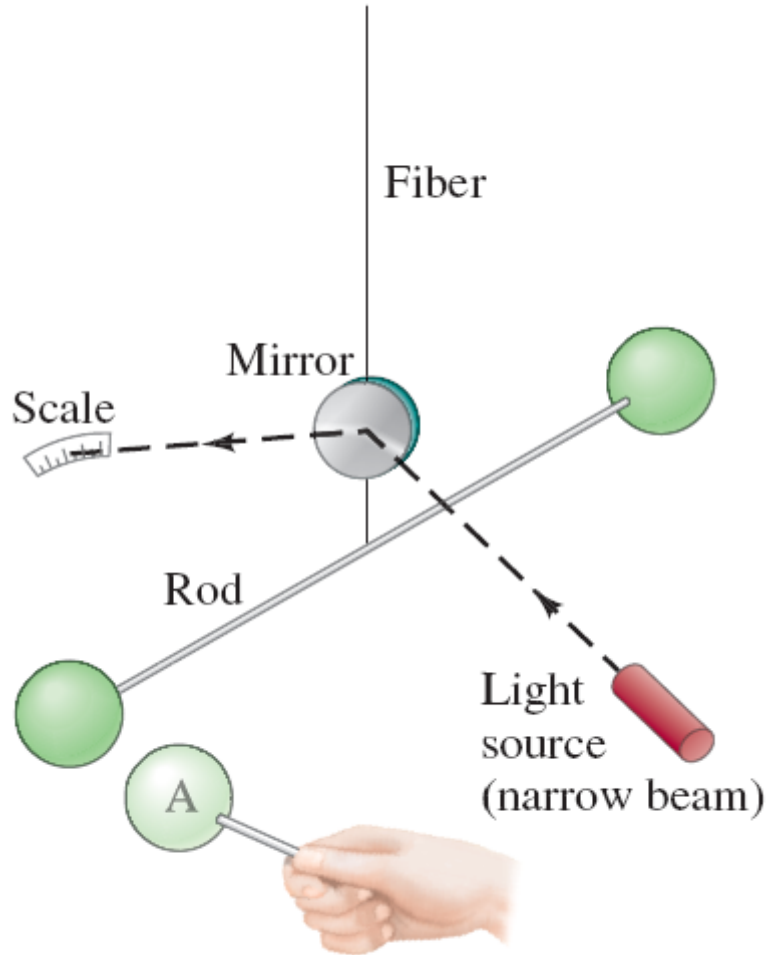
In its final form, the law of universal gravitation reads:

$$F = G \frac{m_1 m_2}{r^2},$$

where

$$G = 6.67 \times 10^{-11} \, \text{N} \cdot \text{m}^2/\text{kg}^2.$$

The **magnitude** of the gravitational constant G can be measured in the laboratory.

This is the **Cavendish** experiment.

6-1 Newton's Law of Universal Gravitation

Example 6-1: Can you attract another person gravitationally?

A 50-kg person and a 70-kg person are sitting on a bench close to each other. Estimate the magnitude of the gravitational force each exerts on the other.
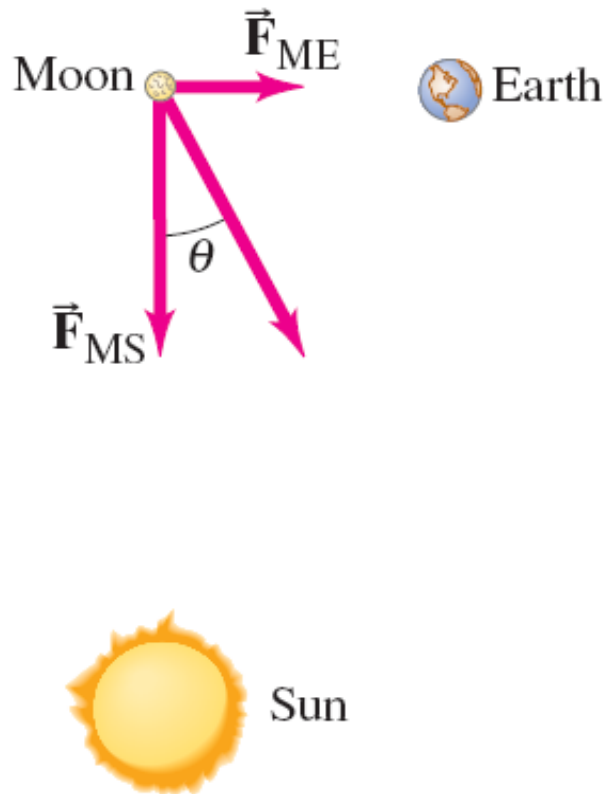
6-1 Newton's Law of Universal Gravitation

Example 6-2: Spacecraft at $2r_E$.

What is the force of gravity acting on a 2000-kg spacecraft when it orbits two Earth radii from the Earth's center (that is, a distance r_E = 6380 km above the Earth's surface)? The mass of the Earth is m_E = 5.98 x 10^{24} kg.

Motion

$2r_E$

r_E
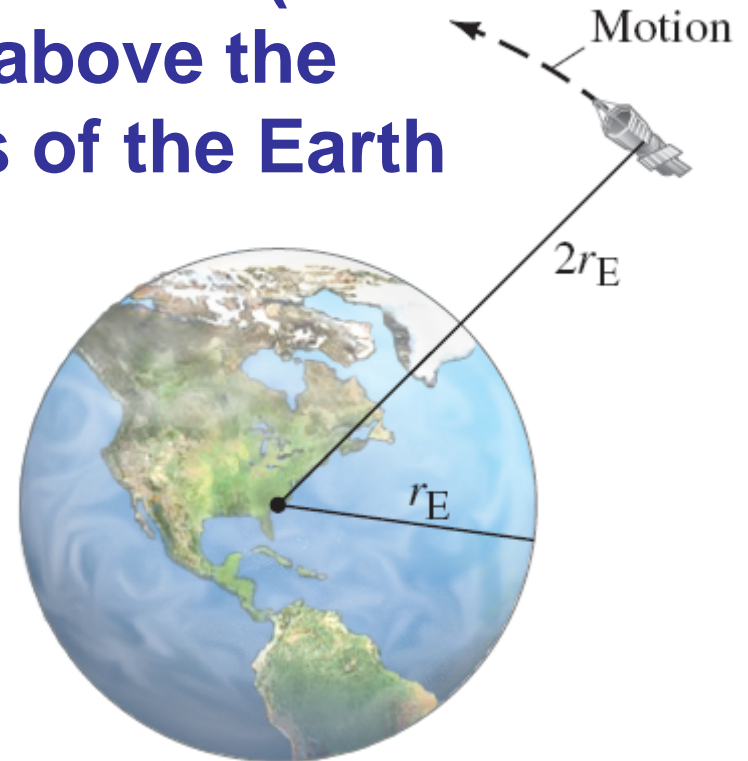
6-1 Newton's Law of Universal Gravitation

Example 6-3: Force on the Moon.

Find the net force on the Moon (m_M = 7.35 x 10^{22} kg) due to the gravitational attraction of both the Earth (m_E = 5.98 x 10^{24} kg) and the Sun (m_S = 1.99 x 10^{30} kg), assuming they are at right angles to each other.

6-1 Newton's Law of Universal Gravitation

Using calculus, you can show:

Particle **outside** a thin spherical shell: gravitational force is the **same** as if all mass were at center of shell

Particle **inside** a thin spherical shell: gravitational force is **zero**

Can model a **sphere** as a series of thin shells; **outside** any spherically symmetric mass, gravitational force acts as though all mass is at center of sphere

6-2 Vector Form of Newton's Universal Gravitation

In vector form,

$$\vec{\mathbf{F}}_{12} = -G \frac{m_1 m_2}{r_{21}^2} \hat{\mathbf{r}}_{21}.$$

This figure gives the directions of the displacement and force vectors.
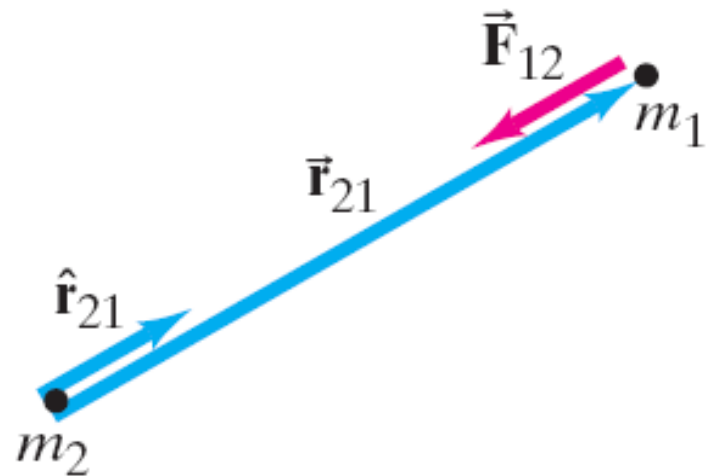
6-2 Vector Form of Newton's Universal Gravitation

If there are many particles, the total force is the vector sum of the individual forces:

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \cdots + \vec{F}_{1n} = \sum_{i=2}^{n} \vec{F}_{1i}.$$

6-3 Gravity Near the Earth's Surface; Geophysical Applications

Now we can relate the gravitational constant to the local acceleration of gravity. We know that, on the surface of the Earth:

$$mg = G \frac{m m_E}{r_E^2}.$$

Solving for g gives:

$$g = G \frac{m_E}{r_E^2}.$$

Now, knowing g and the radius of the Earth, the mass of the Earth can be calculated:

$$m_E = \frac{g r_E^2}{G} = 5.98 \times 10^{24} \, \text{kg}.$$

6-3 Gravity Near the Earth's Surface; Geophysical Applications

Example 6-4: Gravity on Everest.

Estimate the effective value of g on the top of Mt. Everest, 8850 m (29,035 ft) above sea level. That is, what is the acceleration due to gravity of objects allowed to fall freely at this altitude?

6-3 Gravity Near the Earth's Surface; Geophysical Applications

**TABLE 6–1**
Acceleration Due to Gravity at Various Locations on Earth

Location	Elevation (m)	g (m/s^2)
New York	0	9.803
San Francisco	0	9.800
Denver	1650	9.796
Pikes Peak	4300	9.789
Sydney, Australia	0	9.798
Equator	0	9.780
North Pole (calculated)	0	9.832

The acceleration due to gravity varies over the Earth's surface due to altitude, local geology, and the shape of the Earth, which is not quite spherical.

# 6-3 Gravity Near the Earth's Surface; Geophysical Applications

Example 6-5: Effect of Earth's rotation on g.

Assuming the Earth is a perfect sphere, determine how the Earth's rotation affects the value of g at the equator compared to its value at the poles.
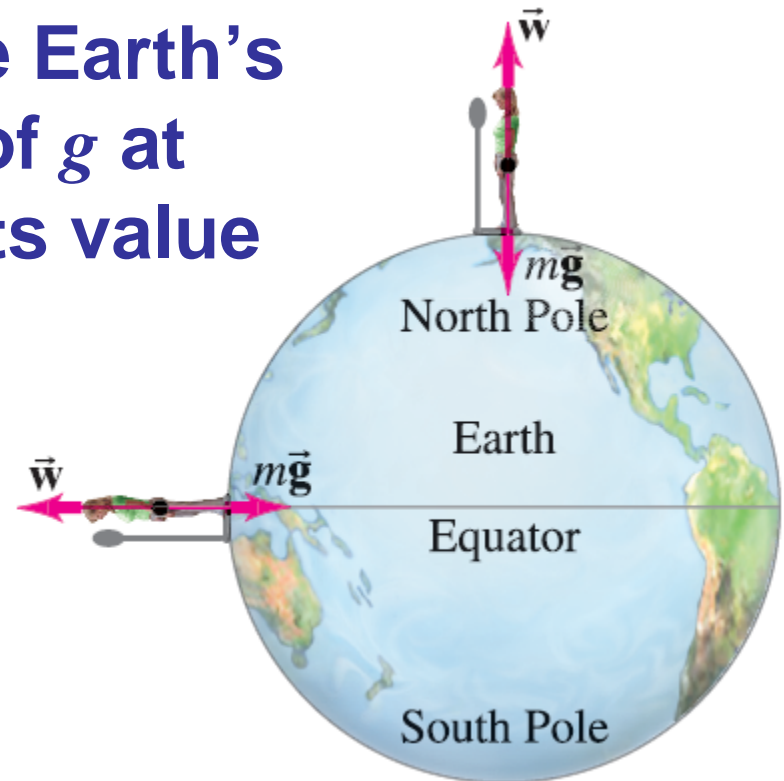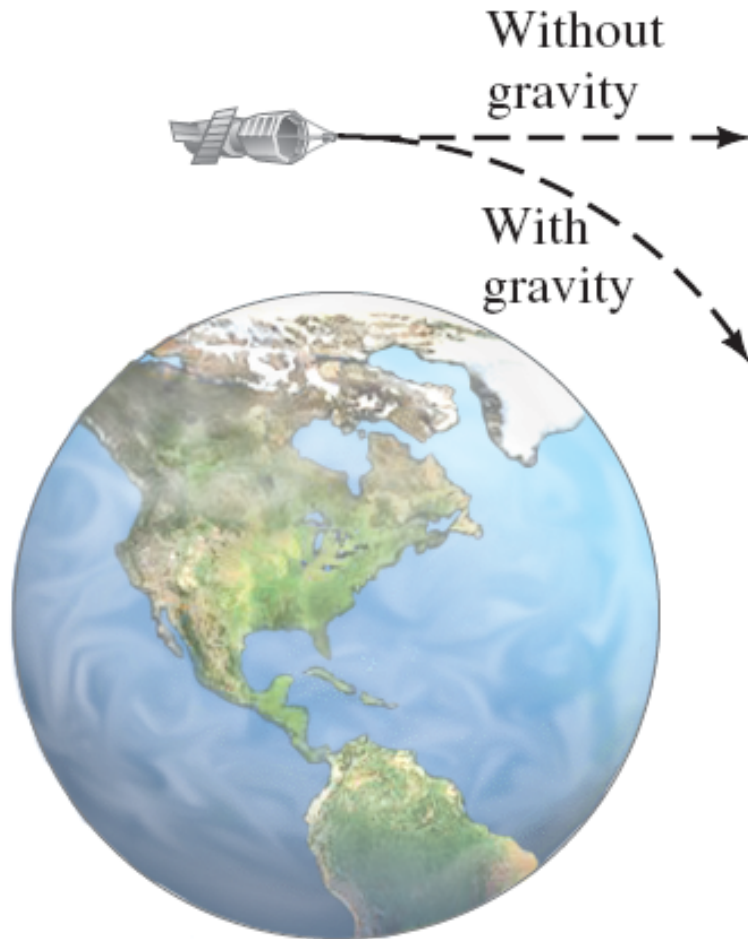
6-4 Satellites and "Weightlessness"

Satellites are routinely put into orbit around the Earth. The tangential speed must be high enough so that the satellite does not return to Earth, but not so high that it escapes Earth's gravity altogether.
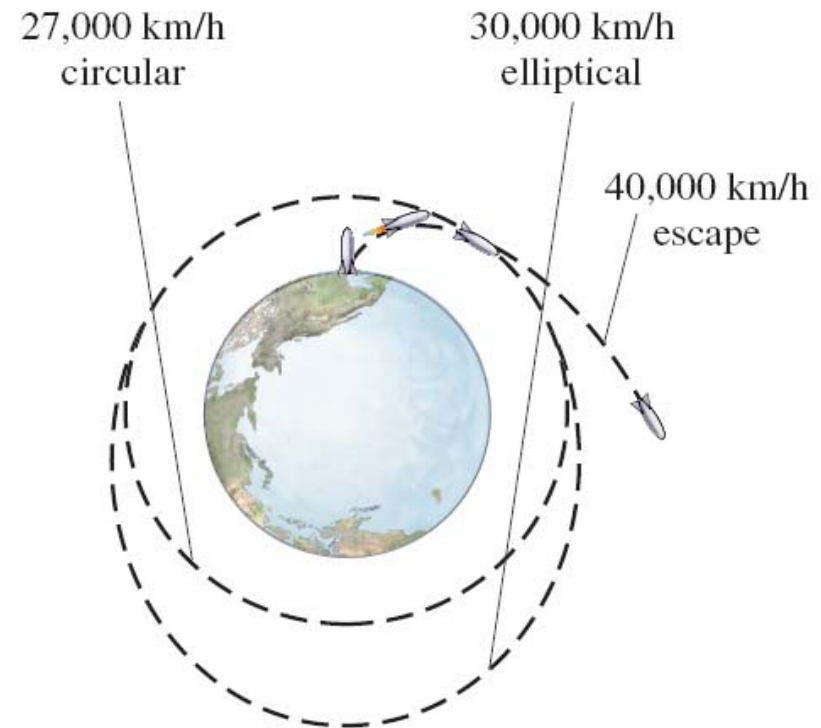
6-4 Satellites and "Weightlessness"

The satellite is kept in orbit by its speed—it is continually falling, but the Earth curves from underneath it.

Without gravity

With gravity

6-4 Satellites and "Weightlessness"

Example 6-6: Geosynchronous satellite.

A geosynchronous satellite is one that stays above the same point on the Earth, which is possible only if it is above a point on the equator. Such satellites are used for TV and radio transmission, for weather forecasting, and as communication relays. Determine (a) the height above the Earth's surface such a satellite must orbit, and (b) such a satellite's speed. (c) Compare to the speed of a satellite orbiting 200 km above Earth's surface.
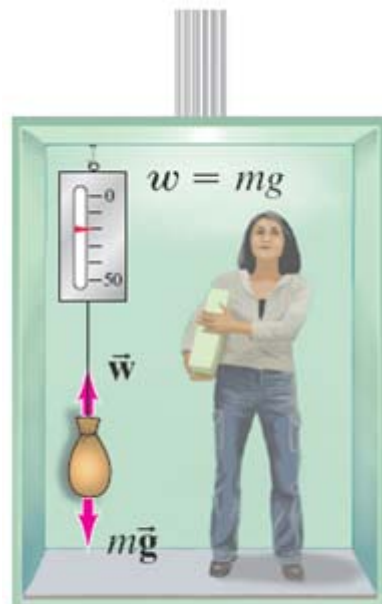
6-4 Satellites and "Weightlessness"

Conceptual Example 6-7: Catching a satellite.

You are an astronaut in the space shuttle pursuing a satellite in need of repair. You find yourself in a circular orbit of the same radius as the satellite, but 30 km behind it. How will you catch up with it?
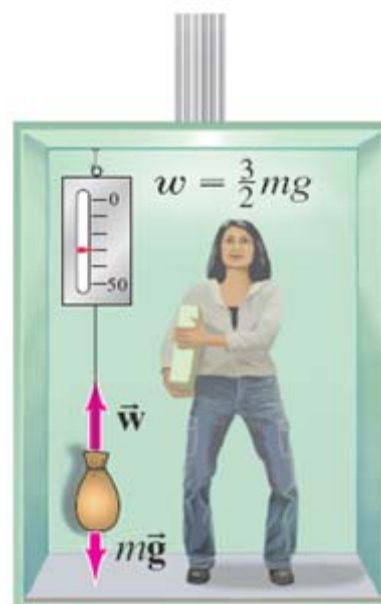
6-4 Satellites and "Weightlessness"

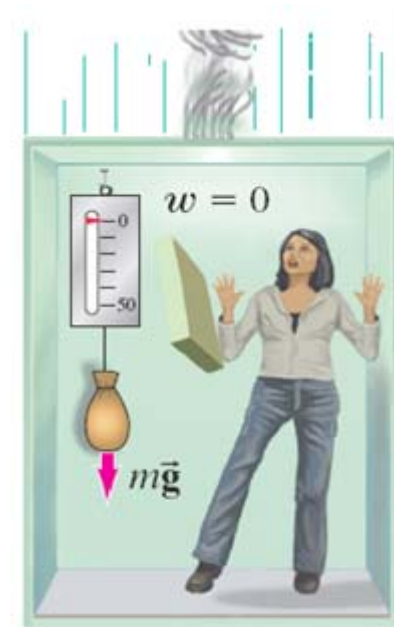**Objects in orbit are said to experience weightlessness. They do have a gravitational force acting on them, though!
The satellite and all its contents are in free fall, so there is no normal force. This is what leads to the experience of weightlessness.**

$w = mg$

$\vec{\mathbf{w}}$

$m\vec{\mathbf{g}}$

$\vec{\mathbf{a}} = 0$

$w = \frac{3}{2}mg$

$\vec{\mathbf{w}}$

$m\vec{\mathbf{g}}$

$\vec{\mathbf{a}} = \frac{1}{2}\vec{\mathbf{g}} \text{ (up)}$

$w = 0$

$m\vec{\mathbf{g}}$

$\vec{\mathbf{a}} = \vec{\mathbf{g}} \text{ (down)}$

6-4 Satellites and "Weightlessness"

More properly, this effect is called apparent weightlessness, because the gravitational force still exists. It can be experienced on Earth as well, but only briefly:
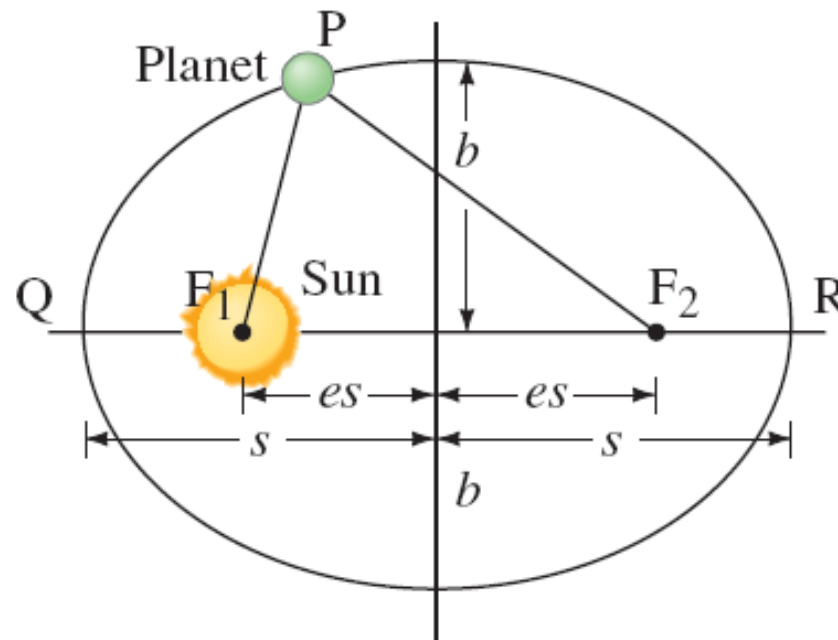
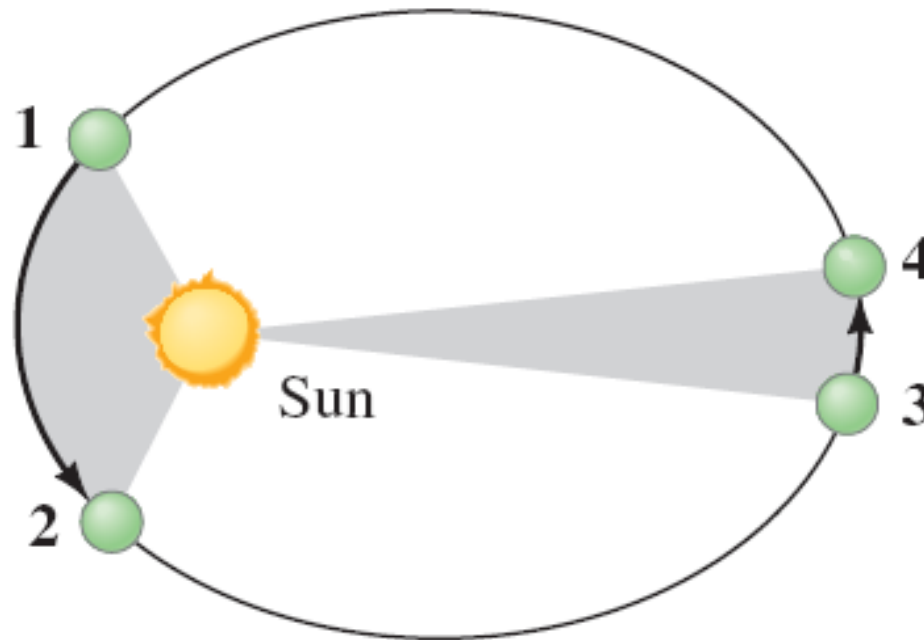# 6-5 Kepler's Laws and Newton's Synthesis

Kepler's laws describe planetary motion.

1. **The orbit of each planet is an ellipse, with the Sun at one focus.**

2. An imaginary line drawn from each planet to the Sun sweeps out equal areas in equal times.

6-5 Kepler's Laws and Newton's Synthesis

3. The square of a planet's orbital period is proportional to the cube of its mean distance from the Sun.

TABLE 6–2 Planetary Data Applied to Kepler's Third Law

Planet	Mean Distance from Sun, s (10^6 km)	Period, T (Earth yr)	s^3/T^2 $\left(10^{24} \frac{\text{km}^3}{\text{yr}^2}\right)$
Mercury	57.9	0.241	3.34
Venus	108.2	0.615	3.35
Earth	149.6	1.0	3.35
Mars	227.9	1.88	3.35
Jupiter	778.3	11.86	3.35
Saturn	1427	29.5	3.34
Uranus	2870	84.0	3.35
Neptune	4497	165	3.34
Pluto	5900	248	3.34

6-5 Kepler's Laws and Newton's Synthesis

Kepler's laws can be derived from **Newton's laws.** In particular, Kepler's third law follows directly from the law of universal gravitation—equating the gravitational force with the centripetal force shows that, for any two planets (assuming circular orbits, and that the only gravitational influence is the Sun):

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{s_1}{s_2}\right)^3.$$

6-5 Kepler's Laws and Newton's Synthesis

Example 6-8: Where is Mars?

Mars' period (its "year") was first noted by Kepler to be about 687 days (Earth-days), which is (687 d/365 d) = 1.88 yr (Earth years). Determine the mean distance of Mars from the Sun using the Earth as a reference.
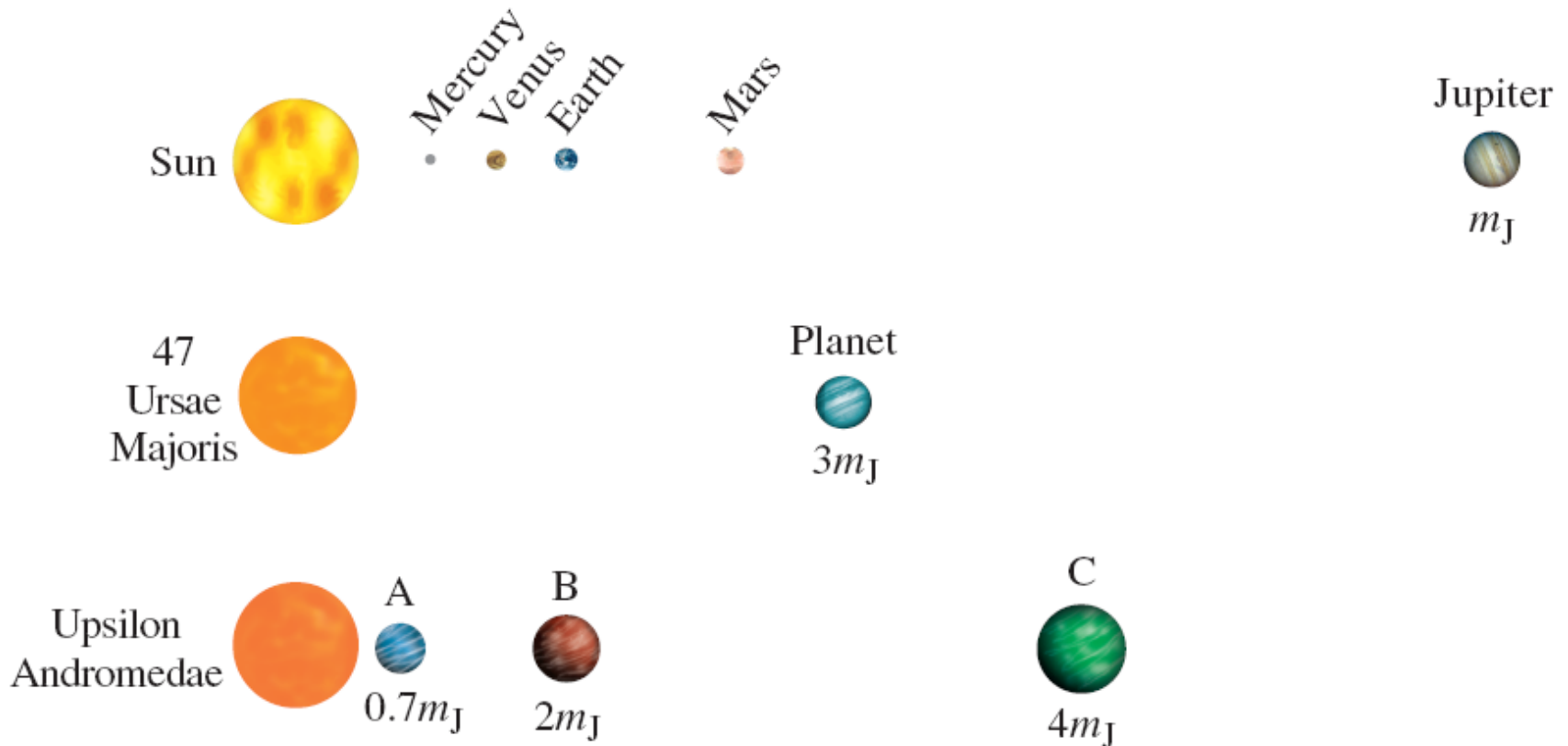
6-5 Kepler's Laws and Newton's Synthesis

Example 6-9: The Sun's mass determined.

Determine the mass of the Sun given the Earth's distance from the Sun as r_{ES} = 1.5 x 10^{11} m.

6-5 Kepler's Laws and Newton's Synthesis

Irregularities in planetary motion led to the discovery of Neptune, and irregularities in stellar motion have led to the discovery of many planets outside our solar system.

# 6-5 Kepler's Laws and Newton's Synthesis

Example 6-10: Lagrange point.

The mathematician Joseph-Louis Lagrange discovered five special points in the vicinity of the Earth's orbit about the Sun where a small satellite (mass m) can orbit the Sun with the same period T as Earth's (= 1 year).
One of these "Lagrange Points," called L1, lies between the Earth (mass m_E) and Sun (mass m_E), on the line connecting them. That is, the Earth and the satellite are always separated by a distance d. If the Earth's orbital radius is R_{ES}, then the satellite's orbital radius is ($R_{ES} - d$). Determine d.
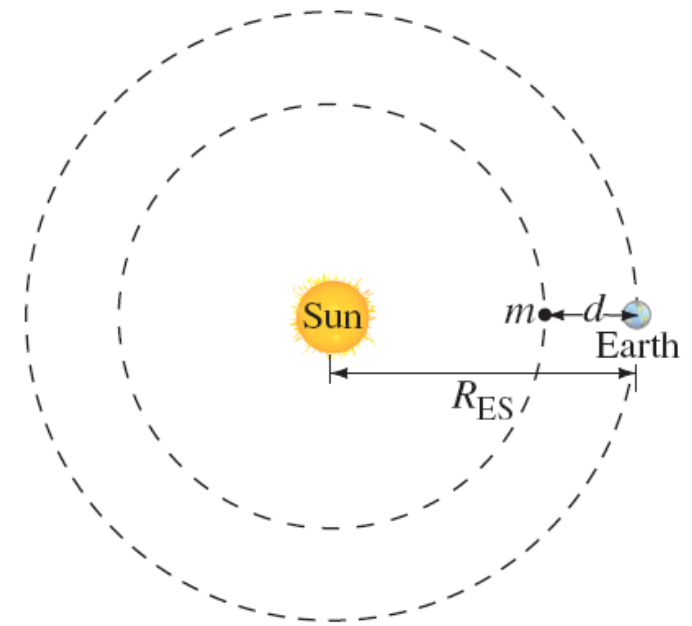
6-6 Gravitational Field

The gravitational field is the gravitational force per unit mass:

$$\vec{\mathbf{g}} = \frac{\vec{\mathbf{F}}}{m}.$$

The gravitational field due to a single mass M is given by:

$$\vec{\mathbf{g}} = -\frac{GM}{r^2}\hat{\mathbf{r}}.$$

6-7 Types of Forces in Nature

Modern physics now recognizes four fundamental forces:

1. **Gravity**

2. **Electromagnetism**

3. **Weak nuclear force** (responsible for some types of radioactive decay)

4. **Strong nuclear force** (binds protons and neutrons together in the nucleus)

6-7 Types of Forces in Nature

So, what about **friction**, the **normal** force, **tension**, and so on?

Except for gravity, the forces we experience every day are due to **electromagnetic** forces acting at the **atomic** level.

6-8 Principle of Equivalence; Curvature of Space; Black Holes

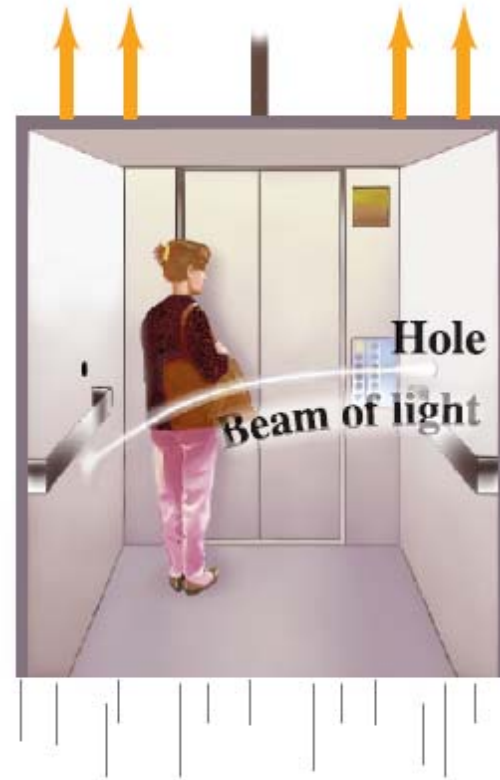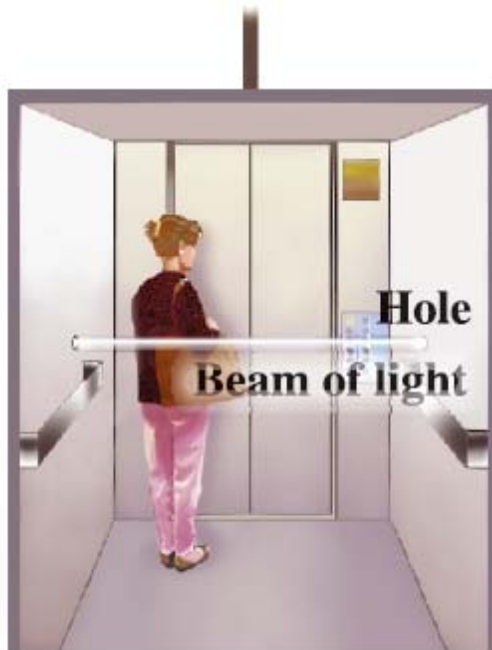**Inertial mass: the mass that appears in Newton's second law**

Gravitational mass: the mass that appears in the universal law of gravitation

Principle of equivalence: inertial mass and gravitational mass are the same
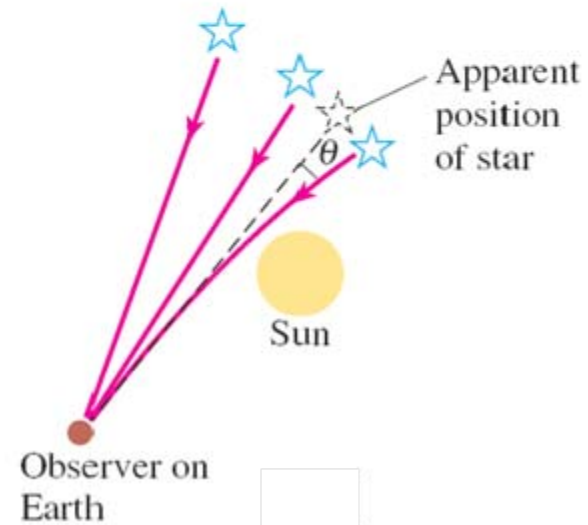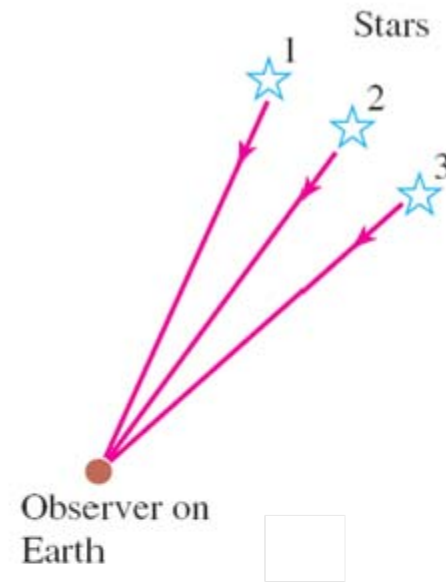
Therefore, light should be deflected by a massive object:

6-8 Principle of Equivalence; Curvature of Space; Black Holes

This bending has been measured during total solar eclipses:

Stars
1
2
3

Observer on Earth

Apparent position of star

θ

Sun

Observer on Earth

6-8 Principle of Equivalence; Curvature of Space; Black Holes

One way to **visualize** the curvature of space (a **two-dimensional** analogy):

Weight
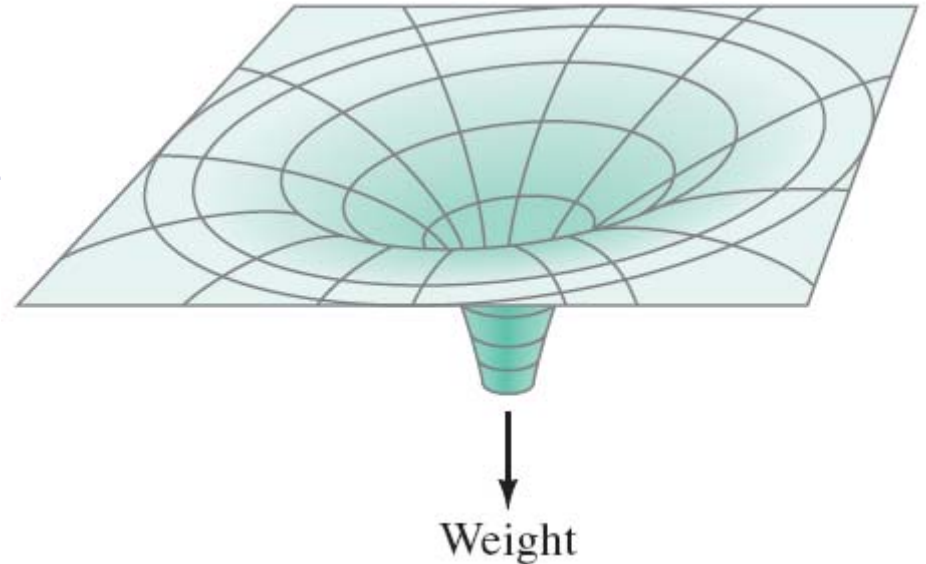
If the gravitational field is strong enough, even light cannot escape, and we have a **black hole.**

Summary of Chapter 6

- **Newton's law of universal gravitation:**

$$F = G\frac{m_1 m_2}{r^2}$$

- **Total force is the vector sum of individual forces.**

- **Satellites are able to stay in Earth orbit because of their large tangential speed.**

- **Newton's laws provide a theoretical base for Kepler's laws.**

Summary of Chapter 6

- **Gravitational field is force per unit mass.**

- **Fundamental forces of nature: gravity, weak nuclear force, electromagnetism, strong nuclear force**