

Phys101 Lectures 11, 12

Momentum and Collisions

Key points:

- **Momentum and impulse**
- **Condition for conservation of momentum and why**
- **How to solve collision problems**
- **Centre of mass**

Ref: 7-1,2,3,4,5,6,7,8,9,10.

Momentum is a vector:

$$\vec{p} = m\vec{v}.$$

It's a quantity that represents the amount and direction of motion. It was once called "the quantity of motion". Now we know that kinetic energy is another quantity of motion.

Newton's 2nd Law

$$\vec{F} = m\vec{a} = m \frac{d\vec{v}}{dt} = \frac{d\vec{p}}{dt}$$

$$\Delta\vec{p} = \vec{F} \Delta t = \vec{J}$$

where $\vec{J} = \vec{F} \Delta t$ is called the impulse of force \vec{F} .

Impulse-momentum principle : The net impulse on an object is equal to the change in momentum of the object.

$$\vec{J} = \Delta\vec{p}$$

$$\vec{F} \Delta t = \vec{p}_2 - \vec{p}_1$$

Example: Force of a tennis serve.

For a top player, a tennis ball may leave the racket on the serve with a speed of 55 m/s (about 120 mi/h). If the ball has a mass of 0.060 kg and is in contact with the racket for about 4 ms (4×10^{-3} s), estimate the average force on the ball.

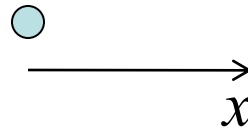
[Solution]

$$\vec{J} = \vec{F}_{ave} \Delta t = \Delta \vec{p}$$

$$F_{ave} = \frac{\Delta p}{\Delta t} = \frac{mv - 0}{\Delta t}$$

$$= \frac{0.06 \times 55}{0.004}$$

$$= 800 \text{ (N)}$$



i-clicker question 11-1

A 0.140-kg baseball is dropped from rest. It has a speed of 1.20 m/s just before it hits the ground. It rebounds with a speed of 1.00 m/s. The ball is in contact with the ground for 0.00140 s. What is the average force exerted by the ground on the ball during that time? (ignore gravity).

- A) 20 N
- B) 100 N
- C) 220 N
- D) 240 N
- E) 120 N

$$\vec{F} \Delta t = \Delta \vec{p}$$

$$F_x \Delta t = \Delta p_x$$

$$F_x = \frac{\Delta p_x}{\Delta t} = \frac{m(v_{fx} - v_{ix})}{\Delta t}$$

$$= \frac{0.140[1.00 - (-1.20)]}{0.0014}$$

$$= 220 \text{ (N)}$$



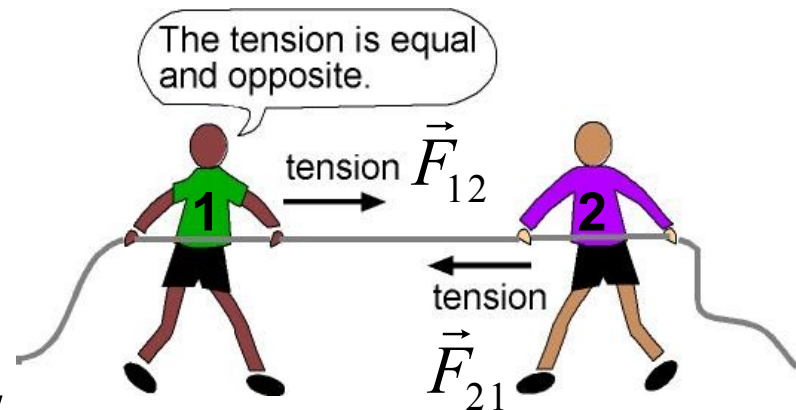
Gravitational force: $mg = 0.14 \times 9.8 = 1.37 \text{ N}$.

Many-body System

Two students pulling each other on ice.

Internal forces: \vec{F}_{12} and \vec{F}_{21} .

External forces: $m_1\vec{g}$, $m_2\vec{g}$, \vec{F}_{N1} and \vec{F}_{N2}



Newton's law: $m_1\vec{g} + \vec{F}_{N1} + \vec{F}_{12} = \frac{d\vec{p}_1}{dt}$, $m_2\vec{g} + \vec{F}_{N2} + \vec{F}_{21} = \frac{d\vec{p}_2}{dt}$

Adding the two equations

$$m_1\vec{g} + \vec{F}_{N1} + m_2\vec{g} + \vec{F}_{N2} + \vec{F}_{12} + \vec{F}_{21} = \frac{d\vec{p}_1}{dt} + \frac{d\vec{p}_2}{dt}$$

$$\sum \vec{F}_{ext} + \sum \vec{F}_{int} = \frac{d\vec{P}}{dt}. \quad (\vec{P} \text{ is the total momentum of the system})$$

Since the internal forces always cancel, $\vec{F}_{21} = -\vec{F}_{12}$, $\sum \vec{F}_{int} = 0$

$$\sum \vec{F}_{ext} = \frac{d\vec{P}}{dt} \quad (\text{only external forces can change the total momentum})$$

Demo

Conservation of Momentum

$$\text{When } \sum \vec{F}_{ext} = 0, \quad \frac{d\vec{P}}{dt} = 0, \quad \text{i.e., } \vec{P} = \text{constant}$$

This is the law of conservation of linear momentum:

When the net external force on a system of objects is zero, the total momentum of the system remains constant.

Note 1: If one of the components of the net external force is zero, the corresponding component of the total momentum of the system is conserved (even though the total momentum vector may or may not be conserved).

Note 2: For a one-object system, the condition for momentum conservation is that the net force acting on the object is zero.

i-clicker question 11-2

The condition for momentum to be conserved is

- (A) It's a closed system.**
- (B) The net external force is zero.**
- (C) No nonconservative work.**
- (D) The momentum is never conserved.**
- (E) The momentum is always conserved.**

Example: Rifle recoil.

Calculate the recoil velocity of a 5.0-kg rifle that shoots a 0.020-kg bullet at a speed of 620 m/s.

[Solution]

X-component of external Force is zero;

Thus the x-component of the total momentum is conserved:

$$P_{xf} = P_{xi} = 0$$

$$m_B v'_{Bx} + m_R v'_{Rx} = 0$$

$$v'_{Rx} = -\frac{m_B v'_{Bx}}{m_R} = -\frac{0.02 \times 620}{5} = -2.5 \text{ (m / s)}$$



Collisions



Approach



Collision



If elastic



If inelastic

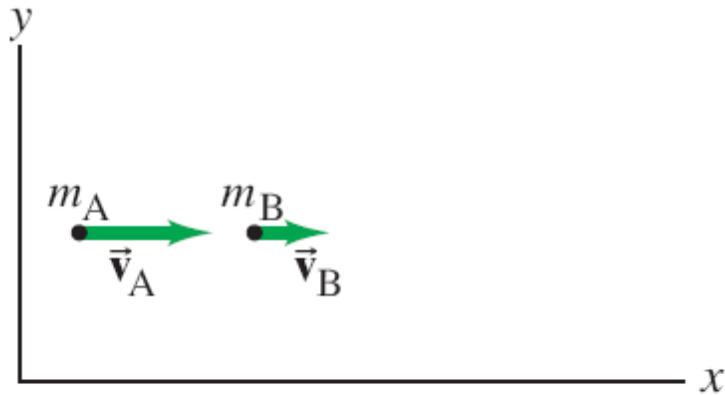
Momentum is conserved in all collisions.

Why?

Because the impulse of external forces can be ignored (much smaller than internal impulse).

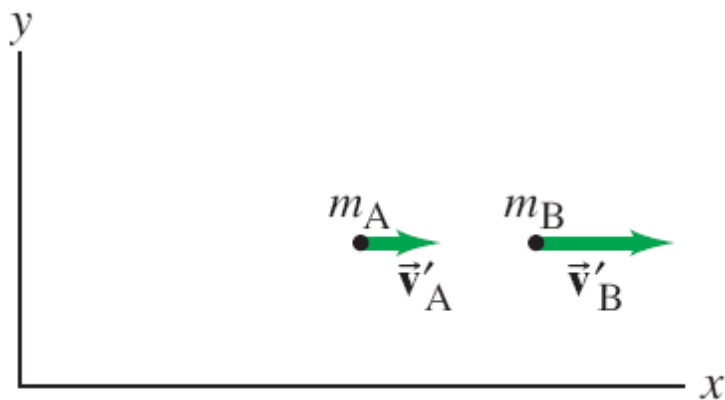
Collisions in which kinetic energy is conserved as well are called elastic collisions, and those in which it is not are called inelastic.

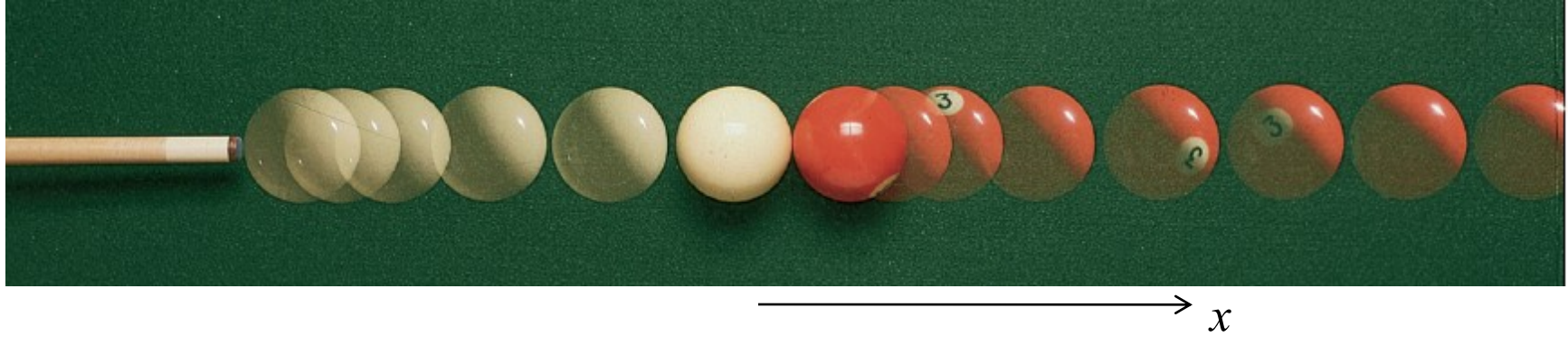
Elastic Collisions in One Dimension



Here we have two objects colliding **elastically**. We know the masses and the initial speeds.

Since both momentum and kinetic energy are conserved, we can write **two** equations. This allows us to solve for the **two** unknown final speeds.





Example: Billiard ball A of mass m moving with speed v_A collides head-on with ball B of equal mass. What are the speeds of the two balls after the collision, assuming it is elastic? Assume ball B is initially at rest ($v_B = 0$).

Conservation of momentum:

$$mv_A = mv'_A + mv'_B \quad (\text{x - comp})$$

Conservation of kinetic energy:

$$\frac{1}{2}mv_A^2 = \frac{1}{2}mv'^2_A + \frac{1}{2}mv'^2_B$$

$$v_A = v'_A + v'_B \quad (1)$$

$$v_A^2 = v'^2_A + v'^2_B \quad (2)$$

$$(1)^2 - (2): \quad 0 = 2v'_A v'_B$$

$$\text{Solution 1:} \quad v'_A = 0$$

$$v'_B = v_A$$

$$\text{Solution 2:} \quad v'_B = 0$$

$$v'_A = v_A$$

Solution 2 should be rejected because it means no collision.

Inelastic Collisions

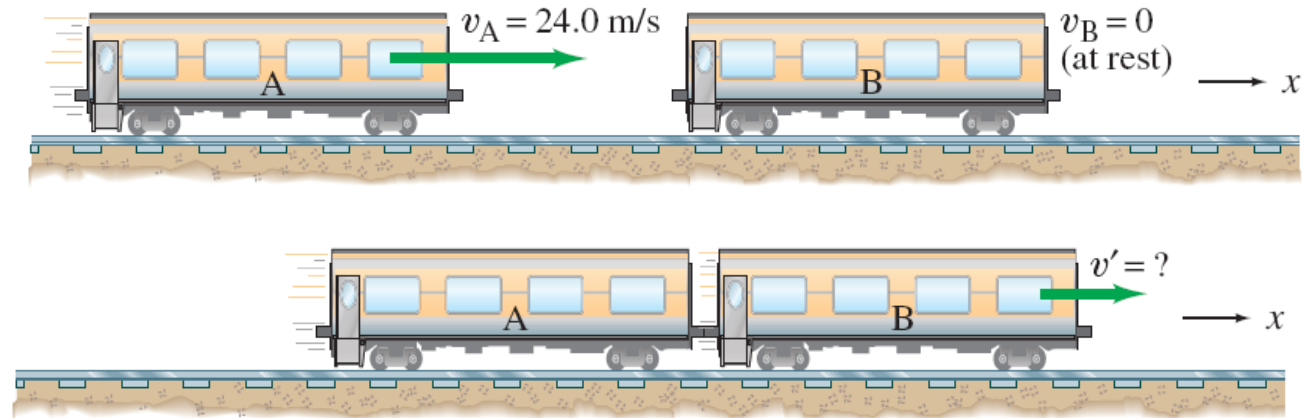
With inelastic collisions, some of the initial kinetic energy is lost to thermal or potential energy. Kinetic energy may also be gained during explosions, as there is the addition of chemical or nuclear energy.

A completely inelastic collision is one in which the objects stick together afterward, so there is only one final velocity.

$$\vec{v}'_A = \vec{v}'_B$$

Example: Railroad cars collide

A 10,000-kg railroad car, A, traveling at a speed of 24.0 m/s strikes an identical car, B, at rest. If the cars lock together as a result of the collision, what is their common speed immediately after the collision?



Conservation of momentum (x-comp.):

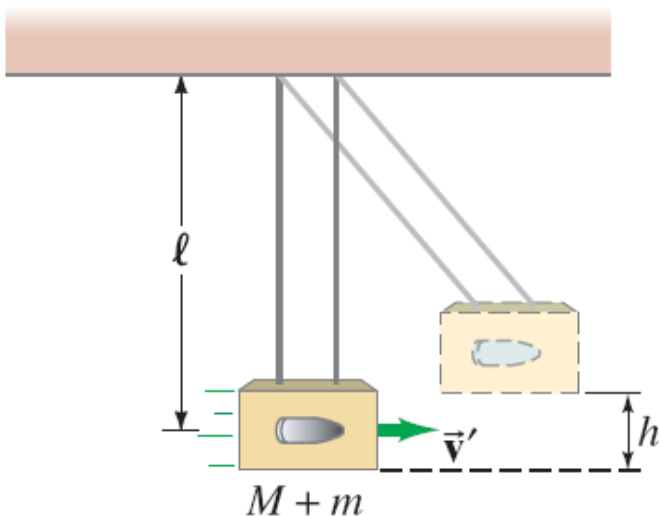
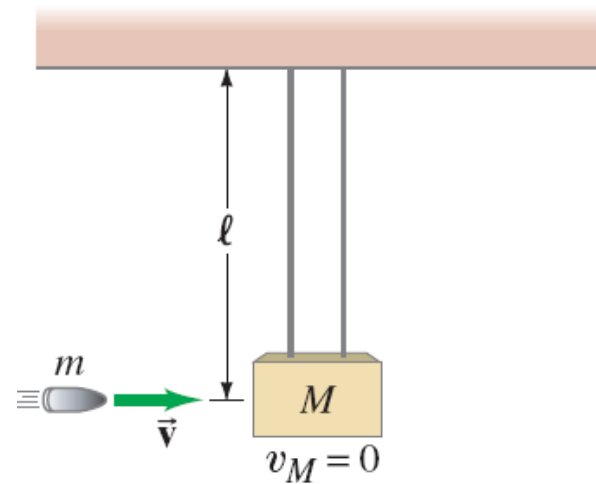
$$mv_A = mv'_A + mv'_B$$

Hit and stick (perfectly inelastic):

$$v'_A = v'_B$$

Solve:

$$v'_A = v'_B = \frac{v_A}{2}$$



Example: Ballistic pendulum.

The ballistic pendulum is a device used to measure the speed of a projectile, such as a bullet. The projectile, of mass m , is fired into a large block of mass M , which is suspended like a pendulum. As a result of the collision, the pendulum and projectile together swing up to a maximum height h . Determine the relationship between the initial horizontal speed of the projectile, v , and the maximum height h .

[Solution] Two events:

1. Hit and stick; $mv = (m + M)v'$
2. Swing. $\frac{1}{2}(m + M)v'^2 = (m + M)gh$

Eliminate v' : $\frac{1}{2} \frac{m^2}{m + M} v^2 = (m + M)gh$, $v = \frac{m + M}{m} \sqrt{2gh}$

Collisions in Two or Three Dimensions

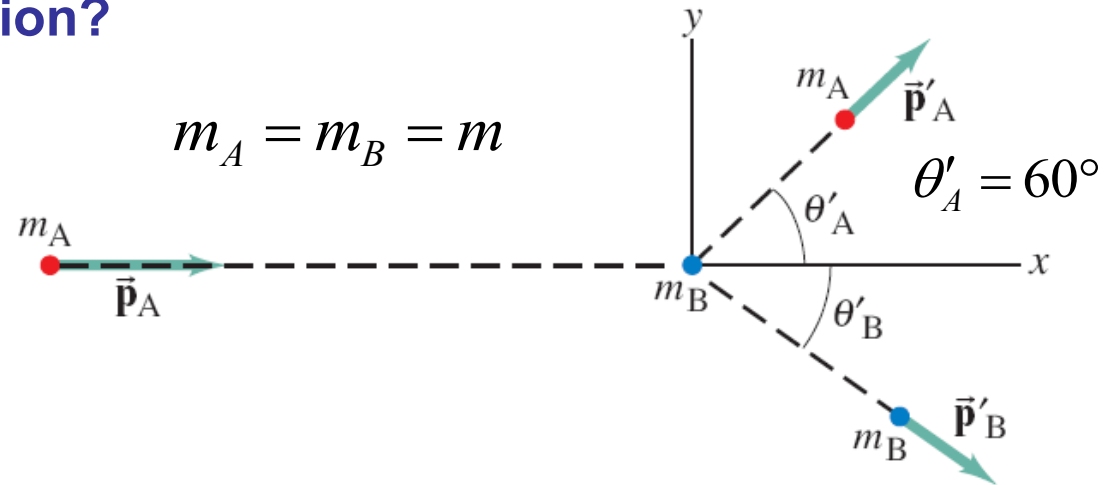
Example: Proton-proton collision.

A proton traveling with speed 8.2×10^5 m/s collides elastically with a stationary proton in a hydrogen target. One of the protons is observed to be scattered at a 60° angle. At what angle will the second proton be observed, and what will be the velocities of the two protons after the collision?

Elastic collision:

$$\frac{1}{2} m v_A^2 = \frac{1}{2} m v_A'^2 + \frac{1}{2} m v_B'^2$$

i.e., $v_A^2 = v_A'^2 + v_B'^2$ (1)



Conservation of momentum:

$$m \vec{v}_A = m \vec{v}'_A + m \vec{v}'_B, \quad \text{i.e., } \vec{v}_A = \vec{v}'_A + \vec{v}'_B,$$

$$x: \quad v_A = v'_A \cos \theta'_A + v'_B \cos \theta'_B \quad (2)$$

$$y: \quad 0 = v'_A \sin \theta'_A + v'_B \sin \theta'_B \quad (3)$$

3 unknowns: v'_A, v'_B, θ'_B .

3 equations: (1), (2), (3).

$$v'_A = 4.1 \times 10^5 \text{ m/s}$$

$$v'_B = 7.1 \times 10^5 \text{ m/s}$$

$$\theta'_B = -30^\circ.$$

i-clicker question 11-3

A car with a mass of 1200 kg and a speed of 12 m/s heading north approaches an intersection. At the same time, a minivan with a mass of 1300 kg and speed of 24 m/s heading east is also approaching the intersection. The car and the minivan collide and stick together. Consider the total momentum and the total kinetic energy of the two vehicles before and after the collision.

What is the velocity of the wrecked vehicles just after the collision?

Hit and stick:

Conservation of momentum:

$$\vec{p}_C = \vec{p}_V = \vec{p}_R$$

B. The total momentum is conserved but the total kinetic energy is not conserved.

$$\vec{p}_C + \vec{p}_V = \vec{p}_R$$

C. Neither the total momentum nor the total kinetic energy is conserved.

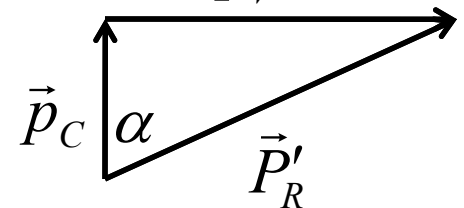
$$P'_R = \sqrt{(m_V v_V)^2 + (m_C v_C)^2} = 34400 \text{ (kg} \cdot \text{m/s)}$$

D. The total kinetic energy is conserved but the total momentum is not conserved.

$$v_R = \frac{P'_R}{m_V + m_C} = \frac{34400}{1300 + 1200} = 13.7 \text{ (m/s)}$$

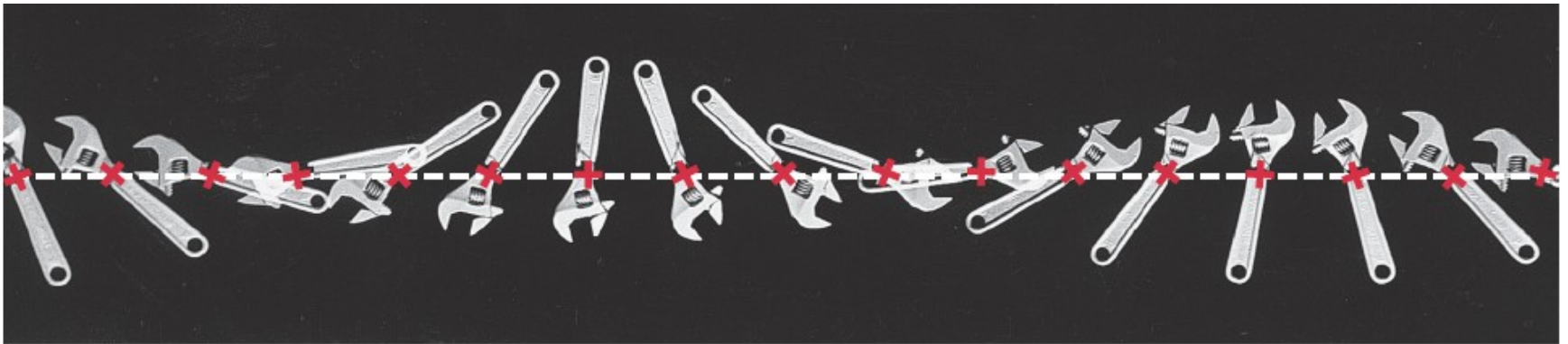
E. The change in total momentum equals the change in total kinetic energy.

$$\alpha = \tan^{-1} \frac{p_V}{p_C} = \tan^{-1} \frac{m_V v_V}{m_C v_C} = 65^\circ$$



Center of Mass (CM)

The general motion of an object can be considered as the sum of the translational motion of the CM, plus rotational, vibrational, or other forms of motion about the CM.



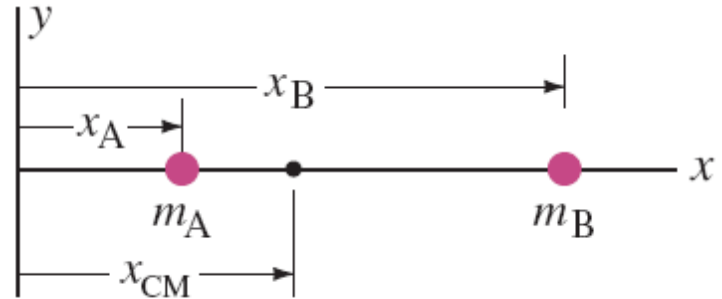
The centre of mass is practically the same as the **centre of gravity** because the gravitational force goes through the CM. Also, the gravitational potential energy should be measured at the CM.

Center of Mass (CM)

For two particles, the center of mass lies closer to the one with the most mass:

$$x_{\text{CM}} = \frac{m_A x_A + m_B x_B}{m_A + m_B} = \frac{m_A x_A + m_B x_B}{M},$$

where M is the total mass.



For example, if $x_A=1\text{m}$, $x_B=2\text{m}$, $m_A=10\text{kg}$, $m_B=10\text{kg}$; then $x_{\text{CM}}=1.5\text{m}$.

For example, if $x_A=1\text{m}$, $x_B=2\text{m}$, $m_A=10\text{kg}$, $m_B=40\text{kg}$; then $x_{\text{CM}}=1.8\text{m}$.

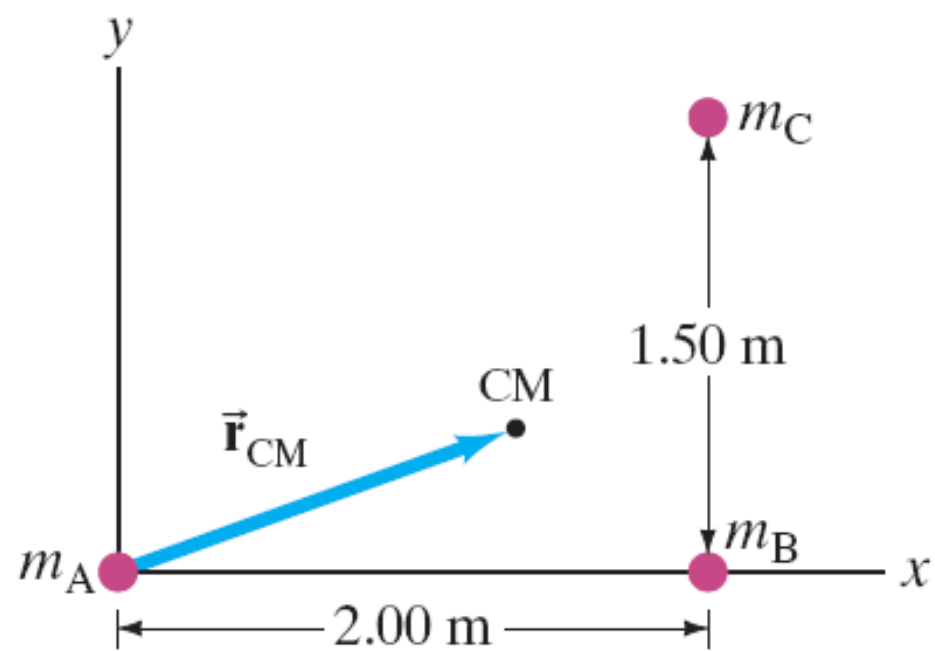
For example, if $x_A=1\text{m}$, $x_B=2\text{m}$, $m_A=40\text{kg}$, $m_B=10\text{kg}$; then $x_{\text{CM}}=1.2\text{m}$.

In general,

$$x_{\text{CM}} = \frac{m_A x_A + m_B x_B}{m_A + m_B}, \quad y_{\text{CM}} = \frac{m_A y_A + m_B y_B}{m_A + m_B},$$

Exercise: Three particles in 2-D.

Three particles, each of mass 2.50 kg, are located at the corners of a right triangle whose sides are 2.00 m and 1.50 m long, as shown. Locate the center of mass.



[Solution]

$$x_{CM} = \frac{m_A x_A + m_B x_B + m_C x_C}{m_A + m_B + m_C}$$

$$= \frac{1}{3} (x_A + x_B + x_C) \quad (\because m_A = m_B = m_C)$$

$$= \frac{1}{3} (0 + 2.0 + 2.0)$$

$$= 1.33 \text{ (m)}$$

$$y_{CM} = \frac{m_A y_A + m_B y_B + m_C y_C}{m_A + m_B + m_C}$$

$$= \frac{1}{3} (y_A + y_B + y_C)$$

$$= \frac{1}{3} (0 + 0 + 1.5)$$

$$= 0.50 \text{ (m)}$$

Example: CM of L-shaped flat object.

Determine the CM of the uniform thin L-shaped object shown.

[Solution] The object consists of two rectangular parts: A and B, whose centres of mass are (x_A, y_A) and (x_B, y_B) .

$$x_A = 1.03m, \quad y_A = 0.10m$$

$$x_B = 1.96m, \quad y_B = -0.74m$$

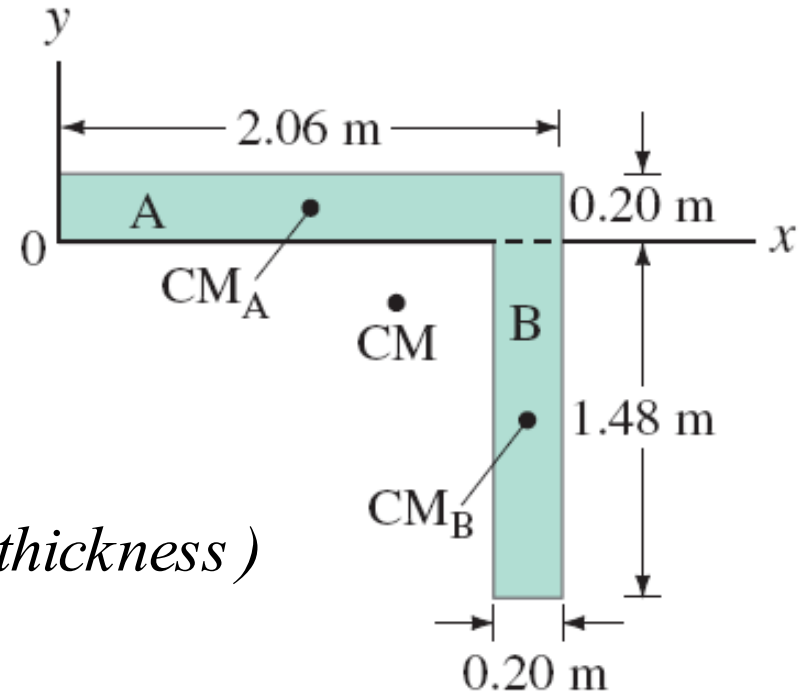
$$m_A = \rho t(2.06 \times 0.20)$$

$$= 0.412\rho t \quad (\rho = \text{density}, \quad t = \text{thickness})$$

$$m_B = \rho t(1.48 \times 0.20) = 0.296\rho t$$

$$x_{CM} = \frac{m_A x_A + m_B x_B}{m_A + m_B} = \frac{0.412 \times 1.03 + 0.296 \times 1.96}{0.412 + 0.296} = 1.42 \text{ (m)}$$

$$y_{CM} = \frac{m_A y_A + m_B y_B}{m_A + m_B} = \frac{0.412 \times 0.10 + 0.296 \times (-0.74)}{0.412 + 0.296} = -0.25 \text{ (m)}$$



Center of Mass and Translational Motion

The total momentum of a system of particles is equal to the product of the total mass and the velocity of the center of mass.

$$\vec{P}_{Total} = M\vec{v}_C$$

The sum of all the forces acting on a system is equal to the total mass of the system multiplied by the acceleration of the center of mass:

$$M\vec{a}_{CM} = \Sigma \vec{F}_{ext.}$$

Therefore, the center of mass of a system of particles (or objects) with total mass M moves like a single particle of mass M acted upon by the same net external force.

Conceptual Example: A two-stage rocket.

A rocket is shot into the air as shown. At the moment it reaches its highest point, a prearranged explosion separates it into two parts of equal mass. Part I is stopped in midair by the explosion and falls vertically to Earth. Where does part II land? Assume $g = \text{constant}$.

