Phys101 Lecture 3
Vectors and Projectile Motion

Key points:

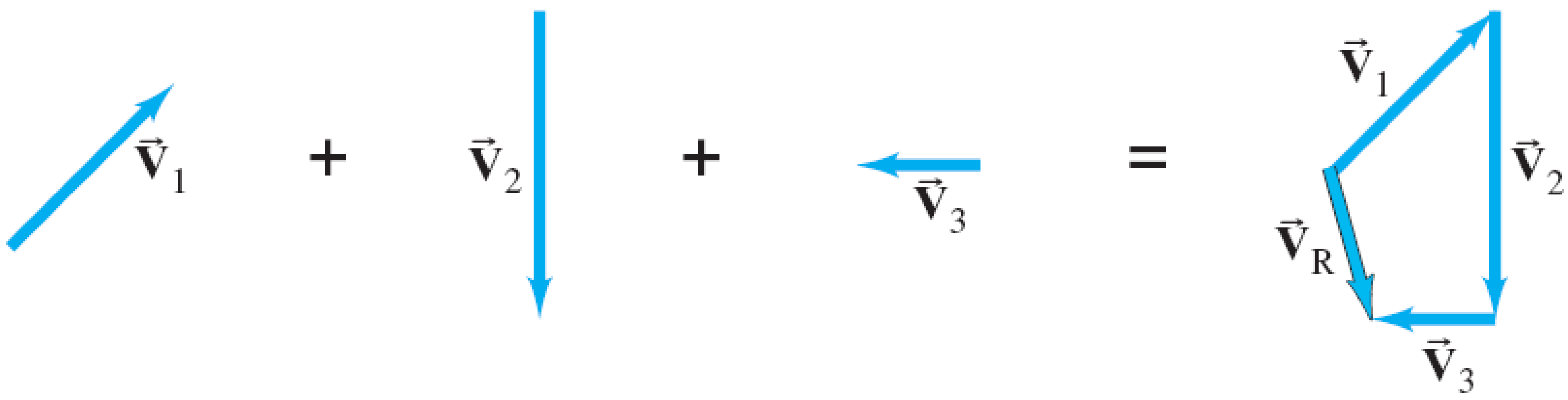
- **How to use vector components (and why?)**
- **Equations of projectile motion depend on how you choose the coordinate system.**

Sections covered:

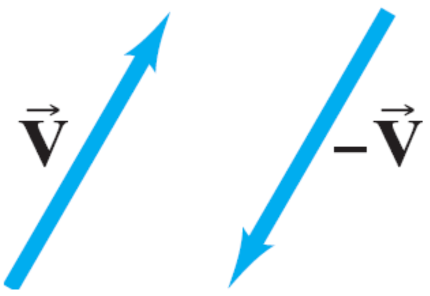
3-1,2,3,4,5,6.

Addition of Vectors—Graphical Methods

Vectors can be added **graphically** by using the **tail-to-tip** method.

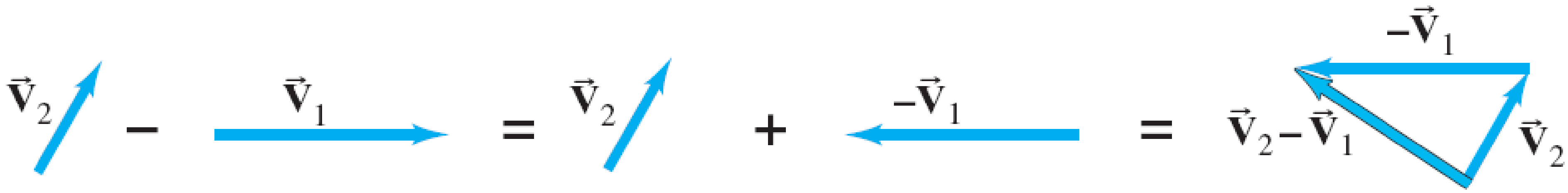


Subtraction of Vectors



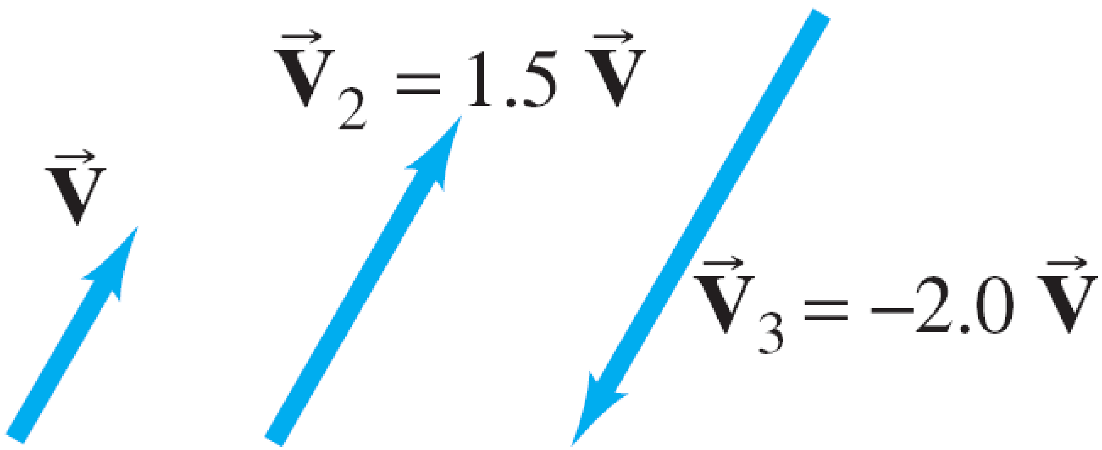
In order to **subtract** vectors, we define the **negative** of a vector, which has the same magnitude but points in the **opposite** direction.

Then we add the negative vector.



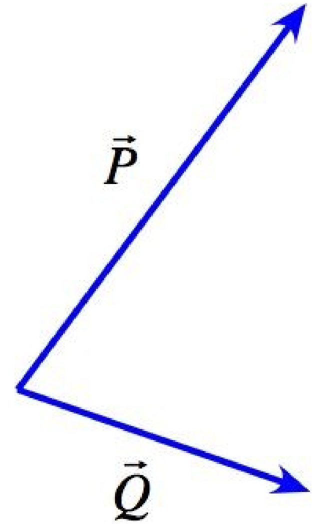
Multiplication of a Vector by a Scalar

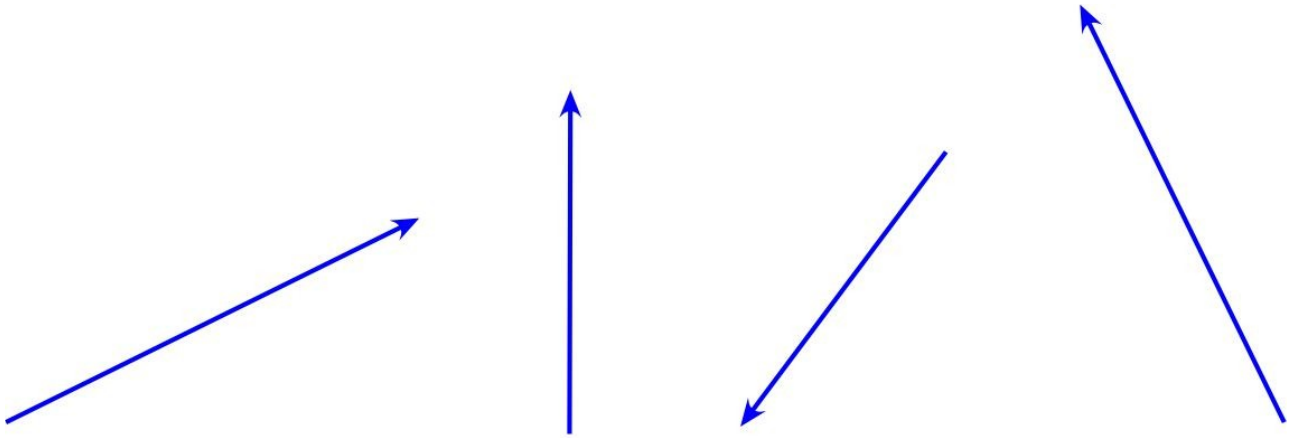
A vector $\vec{\mathbf{V}}$ can be multiplied by a **scalar** c; the result is a vector $c\vec{\mathbf{V}}$ that has the same **direction** but a **magnitude** cV. If c is **negative**, the resultant vector points in the opposite direction.



i-Clicker Question 3-2

Which of the vectors below best represents the vector sum $\vec{P} + \vec{Q}$?

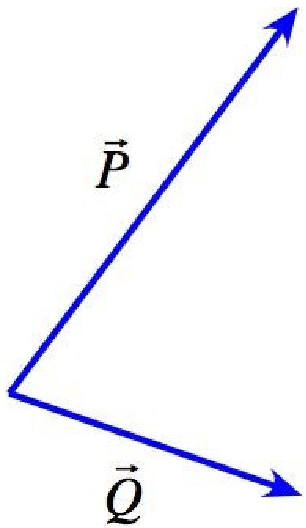


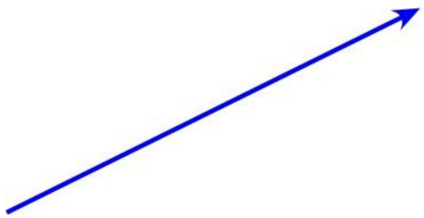


A. B. C. D.

Answer

Which of the vectors below best represents the vector sum $\vec{P}+\vec{Q}$?

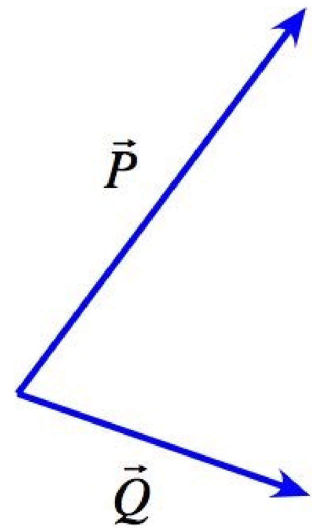


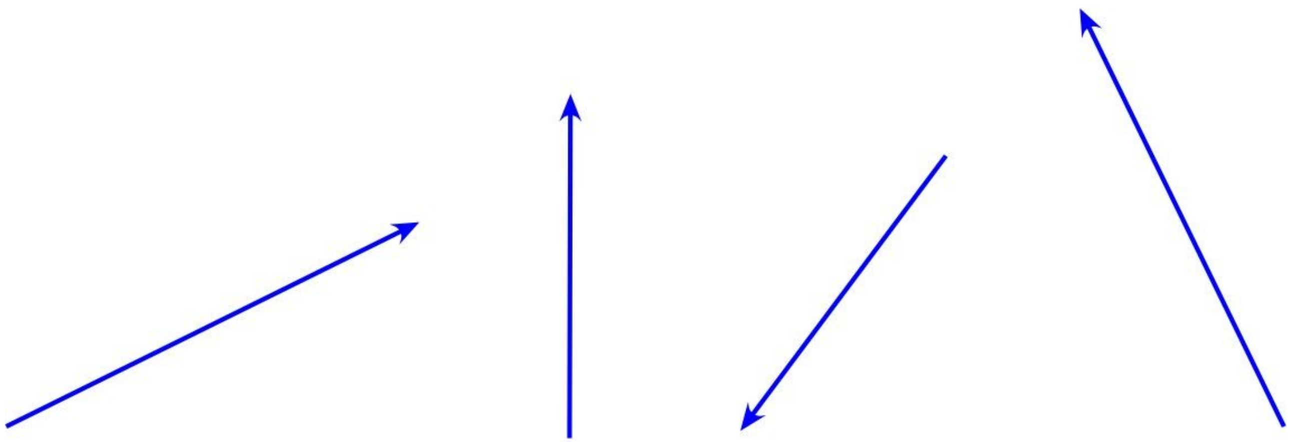


A.

i-Clicker Question 3-3

Which of the vectors below best represents the difference $\vec{P} - \vec{Q}$?

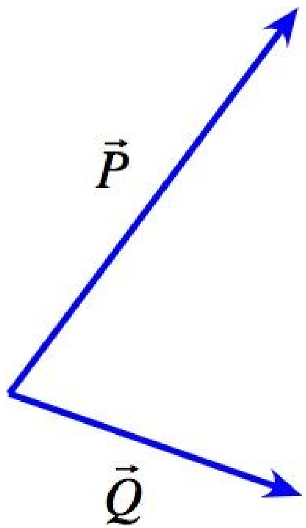




A. B. C. D.

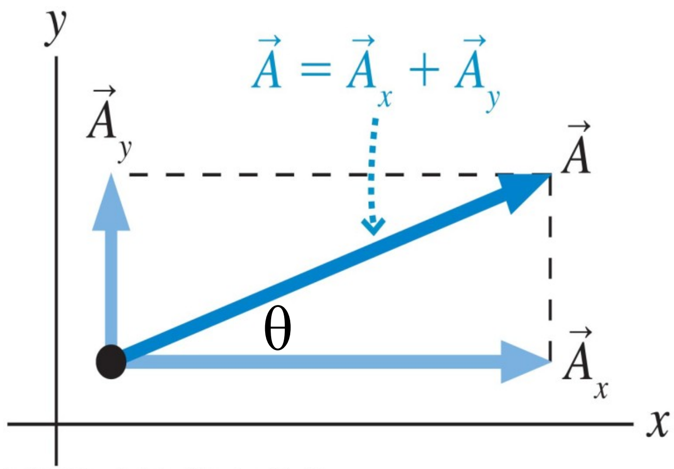
Answer

Which of the vectors below best represents the difference $\vec{P} - \vec{Q}$?



B.

Vector Components and Unit Vectors



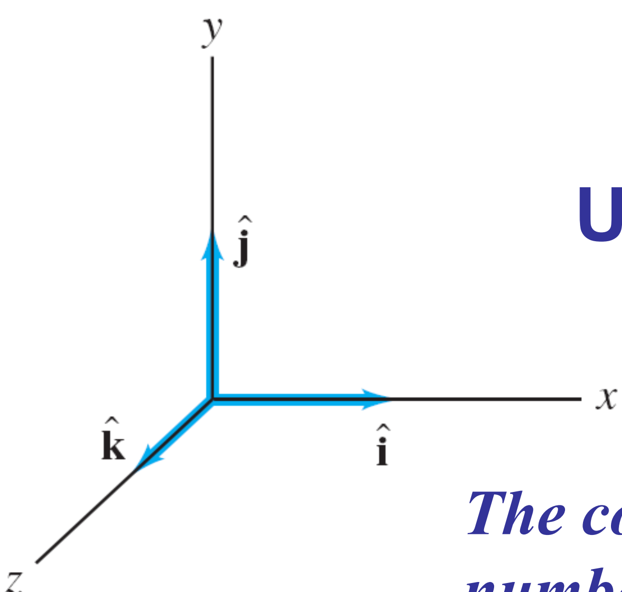
$$\vec{A} = \vec{A}_x + \vec{A}_y = A_x\hat{i} + A_y\hat{j}$$

$$A_x = A\cos\theta$$

$$A_y = A\sin\theta$$

$$A = \sqrt{A_x^2 + A_y^2}$$

$$\theta = \tan^{-1}\frac{A_y}{A_x}$$



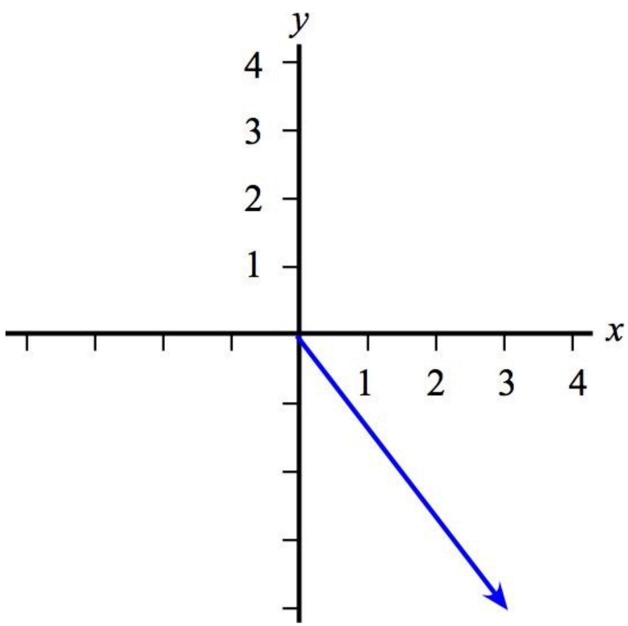
Unit vectors have magnitude 1.

$$\vec{\mathbf{V}} = V_x\hat{\mathbf{i}} + V_y\hat{\mathbf{j}} + V_z\hat{\mathbf{k}}.$$

The components Ax, Ay, Vx, Vy, Vz are just signed numbers! They are not vectors any more!

I-clicker question 3-4:

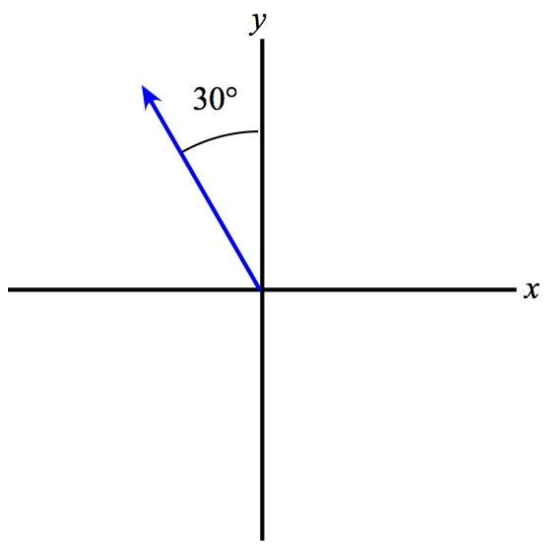
What are the *x*- and *y*-components of this vector?



A. 3, -4
B. 4, 3
C. -3, 4
D. 4, -3
E. -3, -4

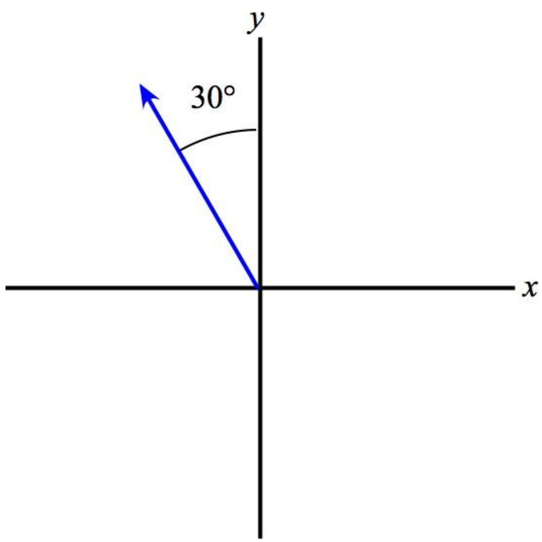
I-clicker question 3-5:

The length of the following vector is 4.0 units. What are the *x*- and *y*-components of this vector?



A. 3.5, 2.0
B. -2.0, 3.5
C. -3.5, 2.0
D. 2.0, -3.5
E. -3.5, -2.0

The length of the following vector is 4.0 units. What are the *x*- and *y*-components of this vector?



B. -2.0, 3.5

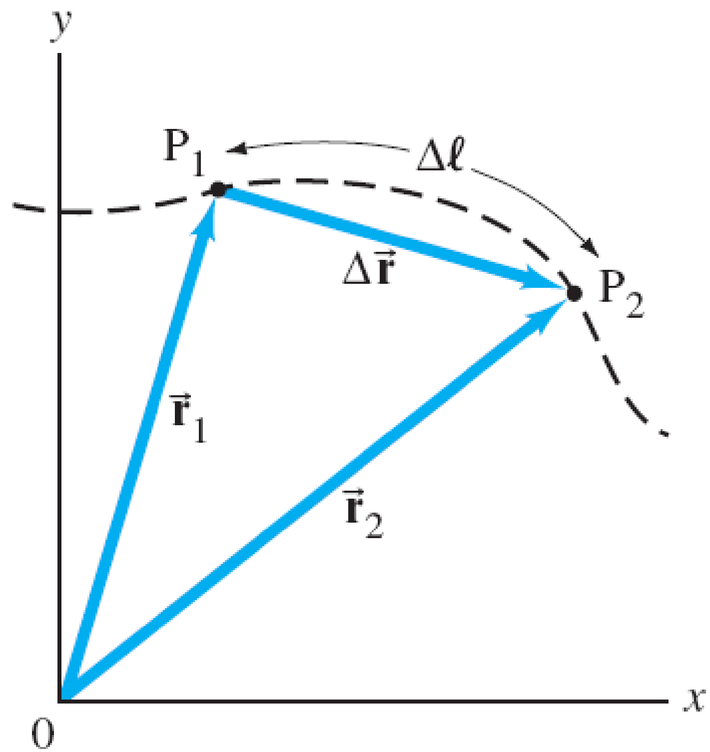
2-D Kinematics

In two or three dimensions, the displacement is a vector:

$$\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_2 - \vec{\mathbf{r}}_1 .$$

$\vec{r}_1$ – initial position vector

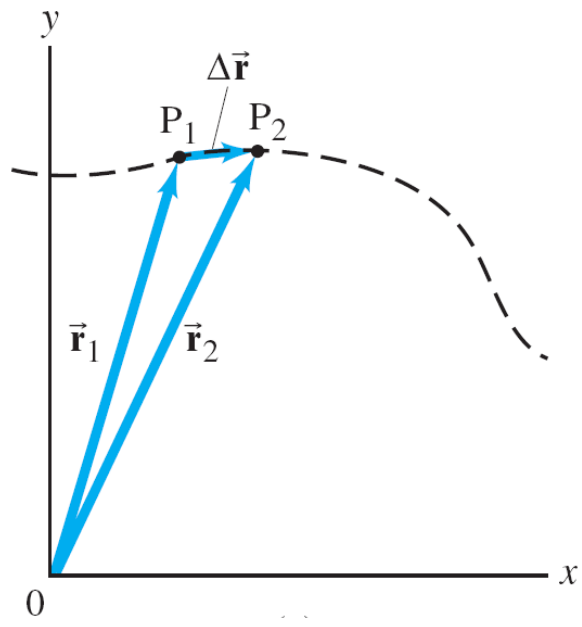
$\vec{r}_2$ – final position vector

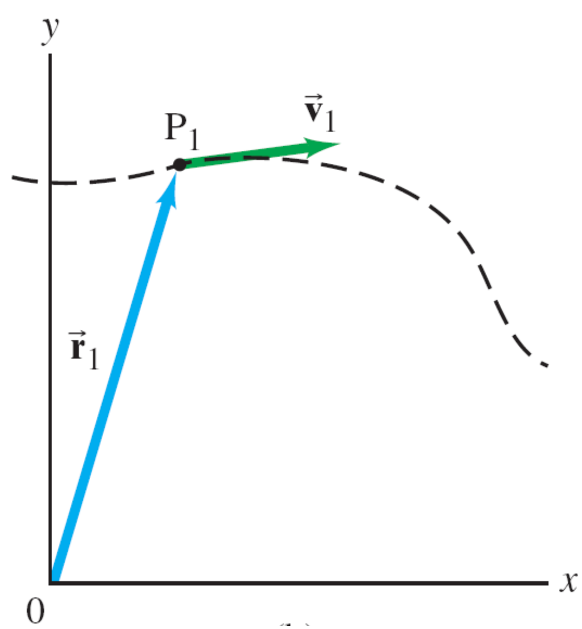


It's vector subtraction!
Don't just subtract the magnitude.

Instantaneous velocity

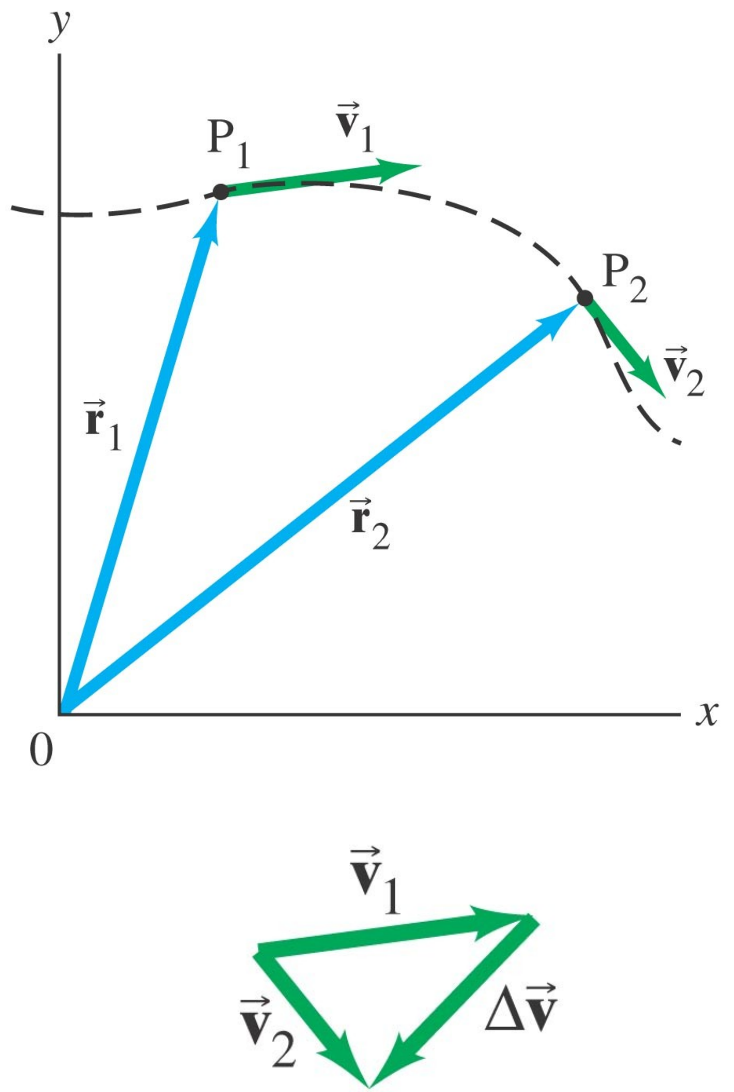
$$\vec{\mathbf{v}} = \lim_{\Delta t \to 0} \frac{\Delta \vec{\mathbf{r}}}{\Delta t} = \frac{d\vec{\mathbf{r}}}{dt}$$



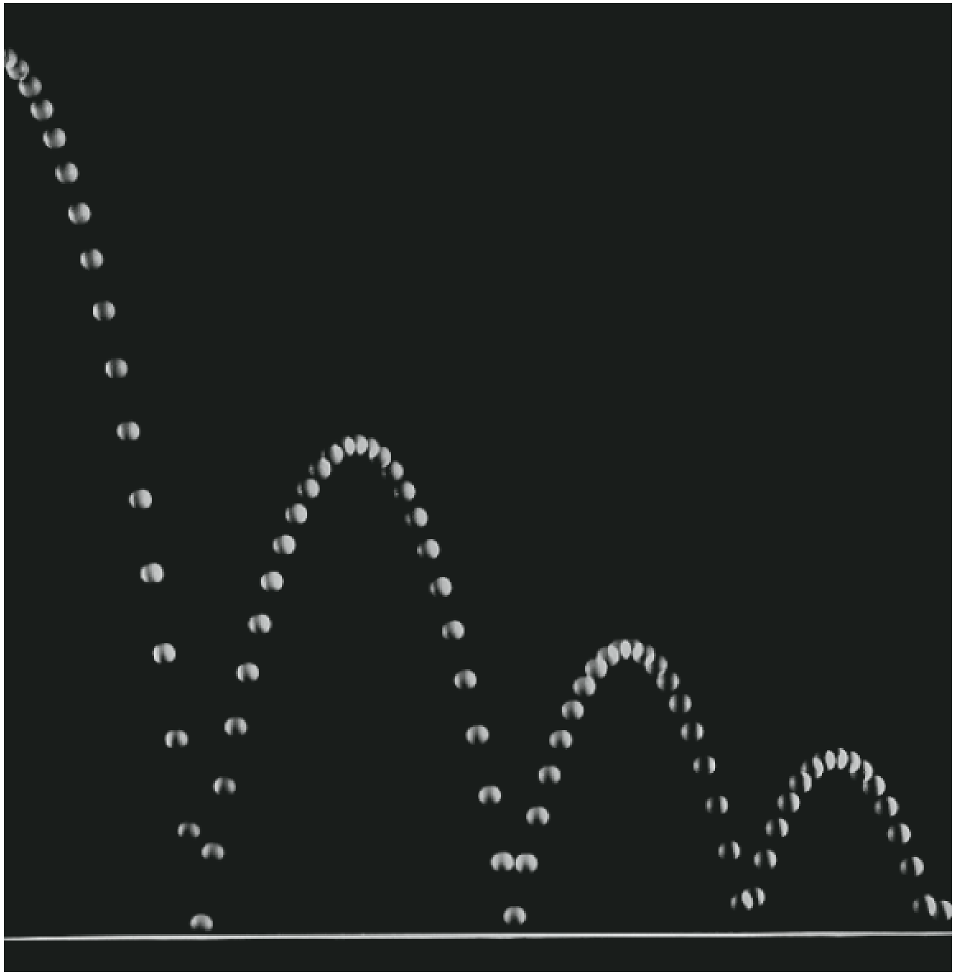


The instantaneous acceleration is in the direction of $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$, and is given by:

$$\vec{\mathbf{a}} = \lim_{\Delta \to 0} \frac{\Delta\vec{\mathbf{v}}}{\Delta t} = \frac{d\vec{\mathbf{v}}}{dt}$$



Projectile Motion



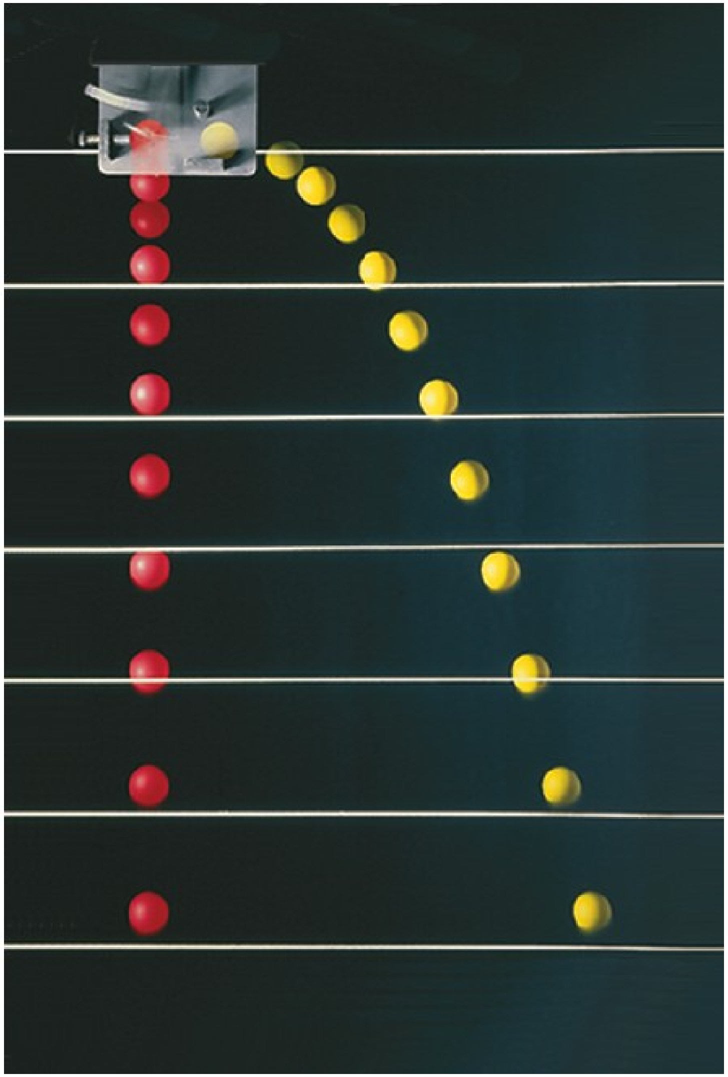
A **projectile** is an object moving in two dimensions under the influence of Earth's **gravity**; its path is a **parabola**.

I-clicker question 3-6:

The acceleration of a particle in projectile motion

A. points along the path of the particle.
B. is directed horizontally.
C. vanishes at the particle’s highest point.
D. is directed down at all times.
E. is zero.

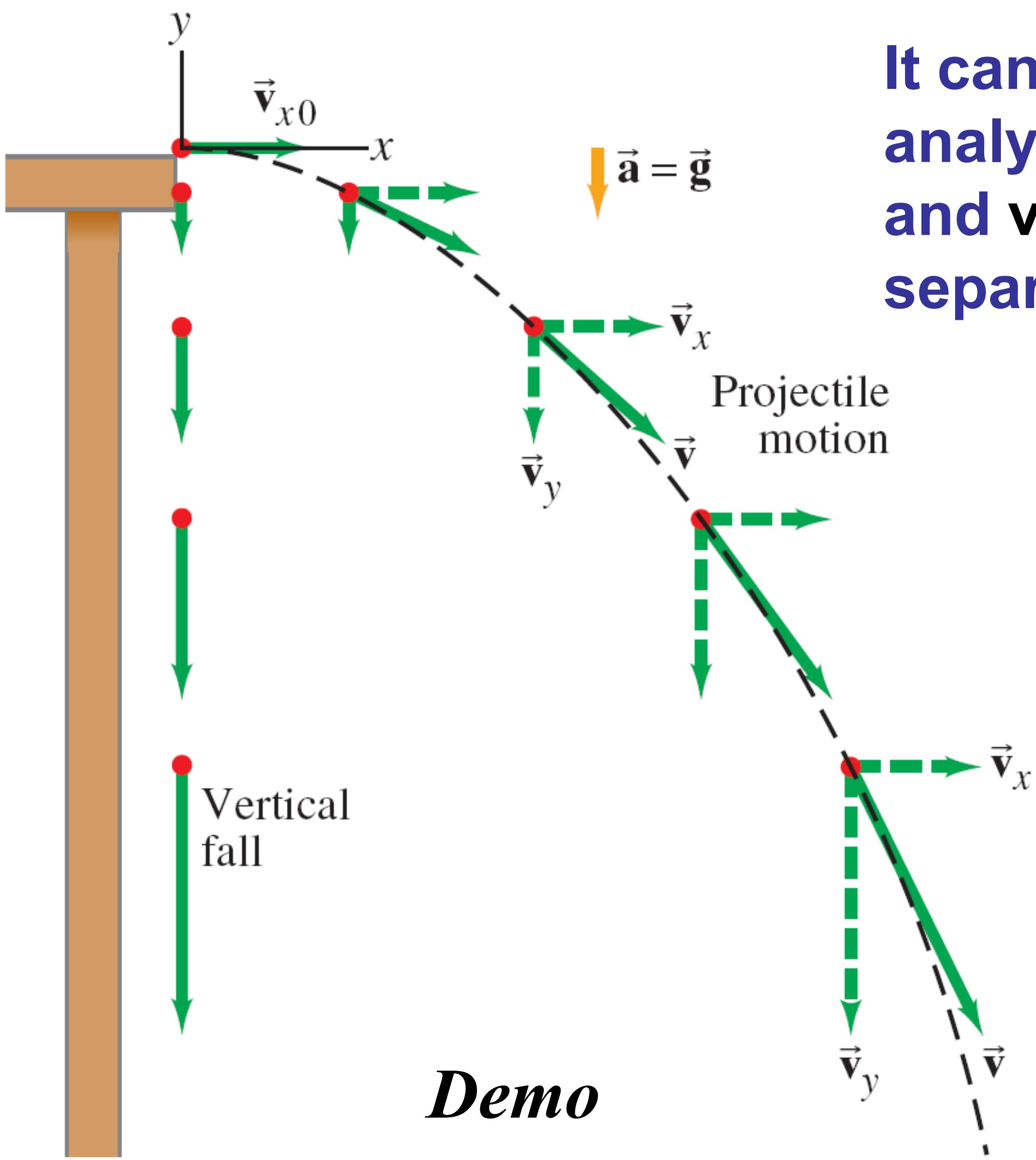
Projectile Motion



The **speed** in the *x*-direction is constant; in the *y*-direction the object moves with constant **acceleration** *g*.

This photograph shows two balls that start to fall at the same time. The one on the right has an initial speed in the *x*-direction. It can be seen that vertical positions of the two balls are identical at identical times, while the horizontal position of the yellow ball increases linearly.

Projectile Motion



It can be understood by analyzing the **horizontal** and **vertical** motions separately. Why?

Demo

Solving Problems Involving Projectile Motion

Projectile motion is motion with constant acceleration in two dimensions, where the acceleration is *g* and is down.

TABLE 3–2 Kinematic Equations for Projectile Motion
(y positive upward; $a_x = 0$, $a_y = -g = -9.80\ \mathrm{m/s^2}$)

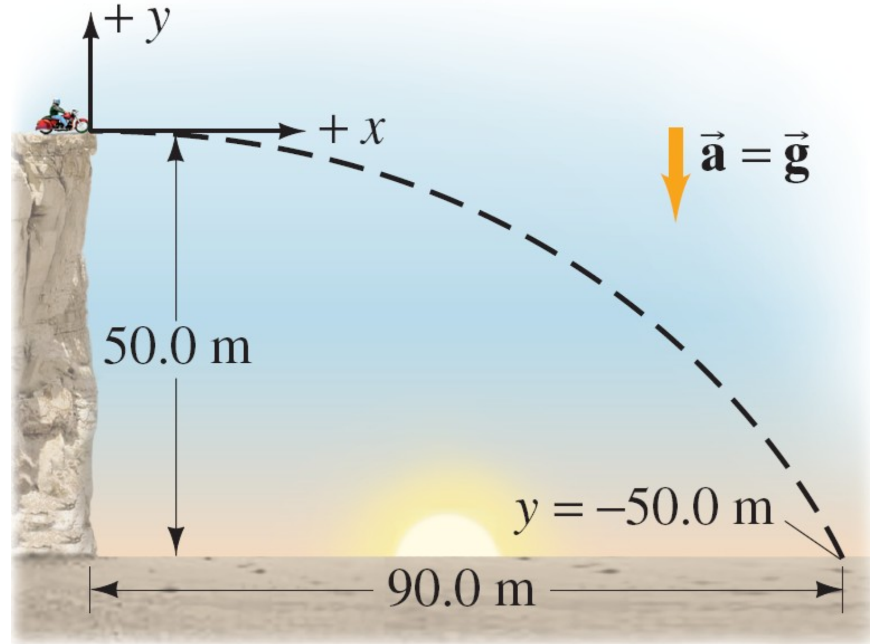
Horizontal Motion ($a_x = 0$, $v_x =$ constant)		Vertical Motion† ($a_y = -g =$ constant)
$v_x = v_{x0}$	(Eq. 2–12a)	$v_y = v_{y0} - gt$
$x = x_0 + v_{x0}t$	(Eq. 2–12b)	$y = y_0 + v_{y0}t - \frac{1}{2}gt^2$
	(Eq. 2–12c)	$v_y^2 = v_{y0}^2 - 2g(y - y_0)$

† If y is taken positive downward, the minus (−) signs in front of g become plus (+) signs.

Example 3-4,5,8,9.

Example 3-4: Driving off a cliff.

A movie stunt driver on a motorcycle speeds horizontally off a 50.0-m-high cliff. How fast must the motorcycle leave the cliff top to land on level ground below, 90.0 m from the base of the cliff where the cameras are? Ignore air resistance.



$$\text{Initial (at } t = 0):$$
$$x_0 = 0, \quad v_{x0} = ?\,\text{(unknown)};$$
$$y_0 = 0, \quad v_{y0} = 0.$$
$$\text{Final (at time } t):$$
$$x = 90.0m,$$
$$y = -50.0m,$$

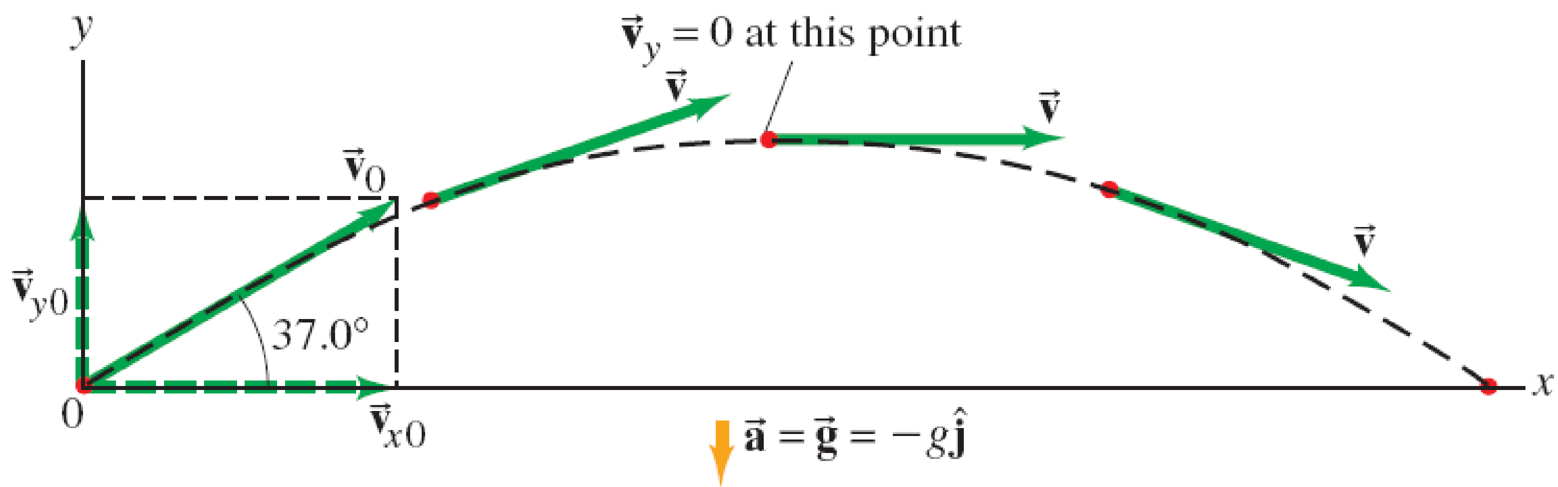
$$x = x_0 + v_{x0}t$$
$$y = y_0 + v_{y0}t - \frac{1}{2}gt^2$$

Two equations, two unknowns, v_{x0} and t.

$$90 = v_{x0}t$$
$$-50 = -\frac{1}{2}(9.8)t^2$$
$$t = 3.19s$$
$$v_{x0} = 28.2m/s$$

Example 3-5: A kicked football.

A football is kicked at an angle θ_0 = 37.0° with a velocity of 20.0 m/s, as shown. Calculate (a) the maximum height, (b) the time of travel before the football hits the ground, (c) how far away it hits the ground, (d) the velocity vector at the maximum height, and (e) the acceleration vector at maximum height. Assume the ball leaves the foot at ground level, and ignore air resistance and rotation of the ball.



$$v_{x0} = v_0 \cos\theta = 20.0 \cos 37° = 16.0 m/s$$

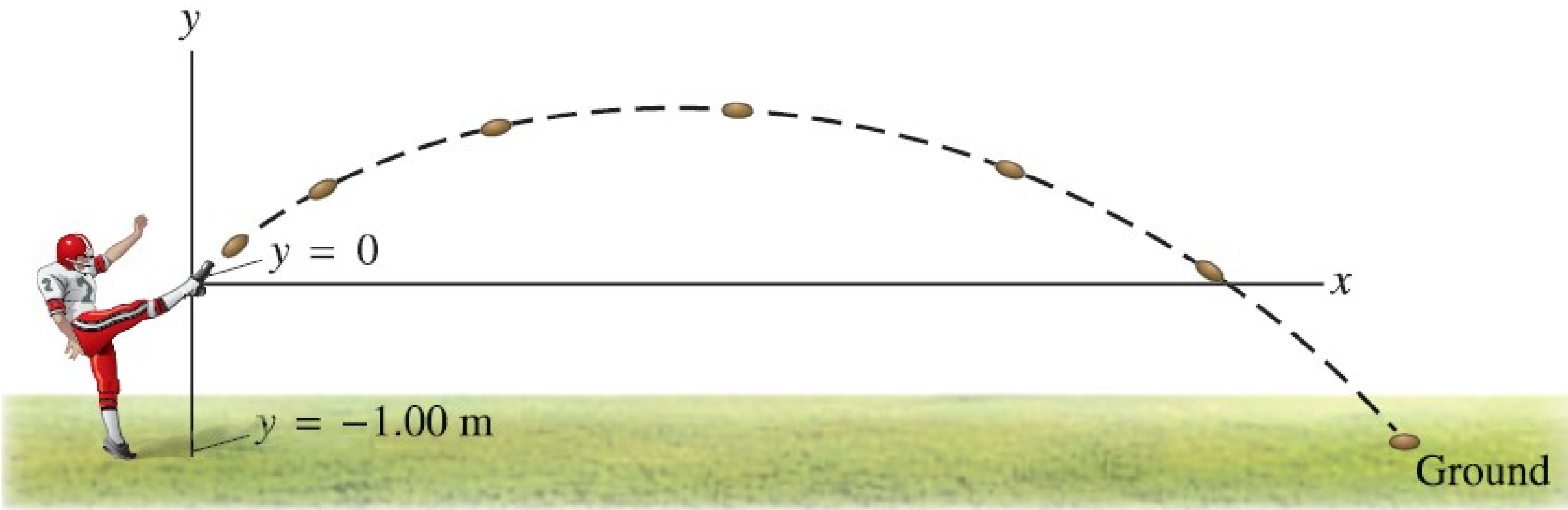
$$v_{y0} = v_0 \sin\theta = 20.0 \sin 37° = 12.0 m/s$$

"Max height" means: $v_y=0$;

"Hits the ground" means: $y=0$.

Example 3-9: A punt.

Suppose the football in Example 3–5 was punted and left the punter's foot at a height of 1.00 m above the ground. How far did the football travel before hitting the ground? Set $x_0 = 0$, $y_0 = 0$.



Now "the ground" means: $y = -1.00m$