**Homework #5 • MATH 462 • More Potential Flow**

- submit your write-up Wednesday 20 February.
- thank you for acknowledging collaborations & assistance from colleagues.
- midterm reminder: Wednesday 27 February.

***) **Memory Sheet** (1/2 pages, pts on midterm) For reference during the midterm, Acheson appendices A1-A6 will be attached to the exam. You will also be allowed to prepare a memory sheet of formulas on the 1/2-page of beige paper handed out in lecture. Rules:

- no microfilm (reasonable size writing please), 1/2-page single-sided.
- no derivations, only basic formulas & ideas.
- memory sheets to be submitted with midterm.

**A) Flow over the Wall** (3 pages, 10pts) Consider the potential flow as defined by the complex potential $\Phi(z) = \sqrt{z^2+1}$ in the upper-half $z$-plane. Manipulate the complex-valued relation

$$\phi + i\psi = \Phi(z) = \sqrt{z^2+1} = \sqrt{(x+iy)^2+1}$$

to obtain a (real-valued) function for $y(x,\psi)$. Explain how this calculation determines the flow streamlines (Matlab's contourplot will fail here unless you know how to deal with the multi-valuedness). Give a formula for the velocity potential (make sure it always gives the proper sign).

Produce a Matlab plot showing equi-spaced contours of streamfunction and velocity potential. Indicate areas of fast/slow flow and high/low pressure. This flow hints at the basic principles that explain how an atomizer works.

**B) Outflow** (2 pages, 10pts) Consider the potential flow as defined by the complex potential

$$\Phi(z) = \frac{M}{2\pi} \ln z$$

for $z \neq 0$. Describe this flow and calculate, in two different ways, the mass flux emanating from the origin.

Then, find the streamlines for the superposition flow ($M > 0$)

$$\Phi(z) = U^\infty z + \frac{M}{2\pi} \ln z$$

beginning from a polar coordinate version of the method used in part **A)**. Which values of *your* streamfunction define the separating streamline? What is the limiting gap between the separating streamlines as $x \to +\infty$? Why might this distance have been guessed without knowing the exact formula for the streamlines? Does the flow case $M < 0$ make sense here?