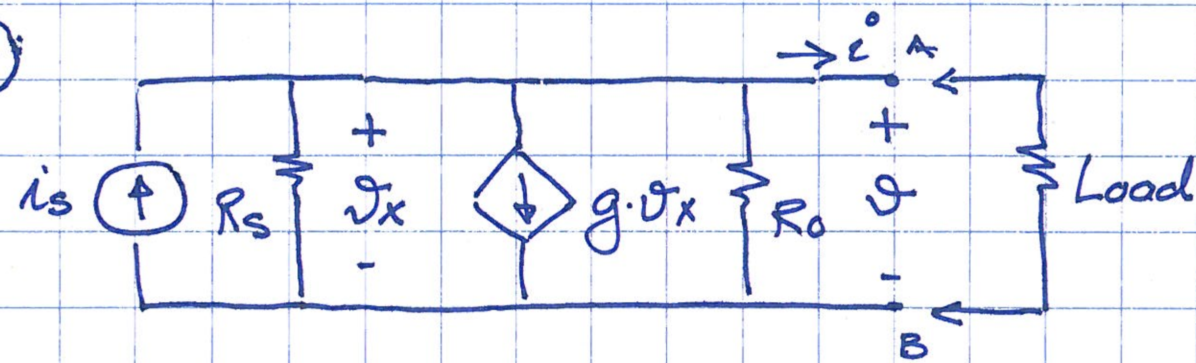
ENSC 220

Fall 2017

Midterm #2

#1:

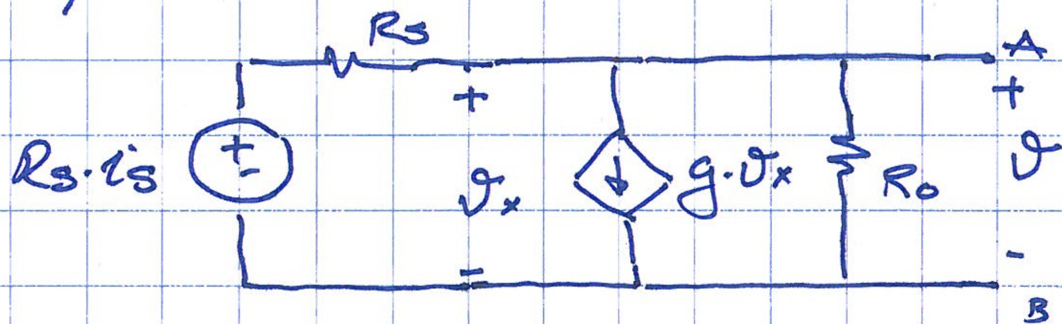


$\vartheta_{Th} = ?$

$R_{Th} = ?$

To find ϑ_{Th} keep node A and B open and find ϑ_{AB}:

Equivalent circuit



Node equation for node A:

$$\frac{\vartheta_A - R_S \cdot i_S}{R_S} + g \cdot \vartheta_x + \frac{\vartheta}{R_o} = 0$$

Note that $\vartheta_x = \vartheta_A$; $\vartheta = \vartheta_A$

Hence,
$$\frac{\vartheta - R_S \cdot i_S}{R_S} + g \cdot \vartheta + \frac{\vartheta}{R_o} = 0$$

$$v \cdot \left(\frac{1}{R_S} + \frac{1}{R_o} + g \right) = i_S$$

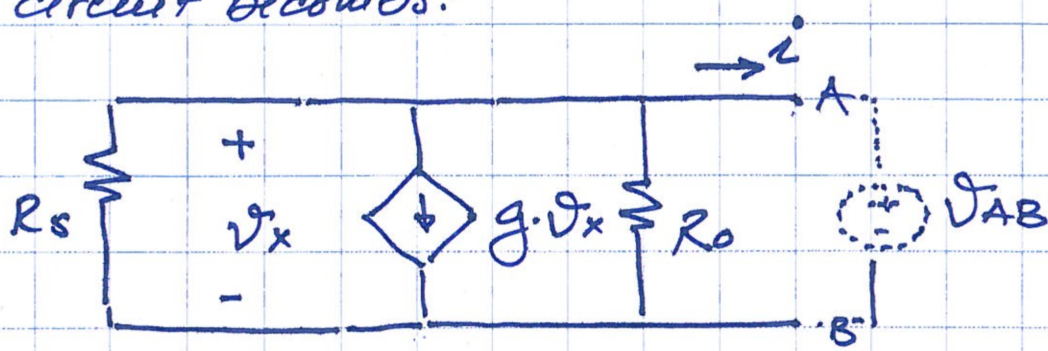
$$v = \frac{i_S}{1/R_S + 1/R_o + g}$$

or: $$v_{Th} = \frac{R_S \cdot R_o \cdot i_S}{R_o + R_S + g \cdot R_o \cdot R_S}$$

To find R_{Th}, apply voltage v_{AB}, disconnect the current source, and find

$$R_{Th} = \frac{v_{AB}}{-i}$$

the circuit becomes:



Write nodal equation again to get:

$$\frac{v_{AB}}{R_S} + g \cdot v_x + \frac{v_{AB}}{R_o} + i = 0$$

Note that $v_x = v_{AB}$

Hence, $$v_{AB} \cdot \left(\frac{1}{R_S} + g + \frac{1}{R_o} \right) = -i$$

Hence,

$$\frac{V_{AB}}{-i} = \frac{1}{\frac{1}{R_S} + \frac{1}{R_o} + g}$$

$$R_{th} = \frac{1}{1/R_S + 1/R_o + g}$$

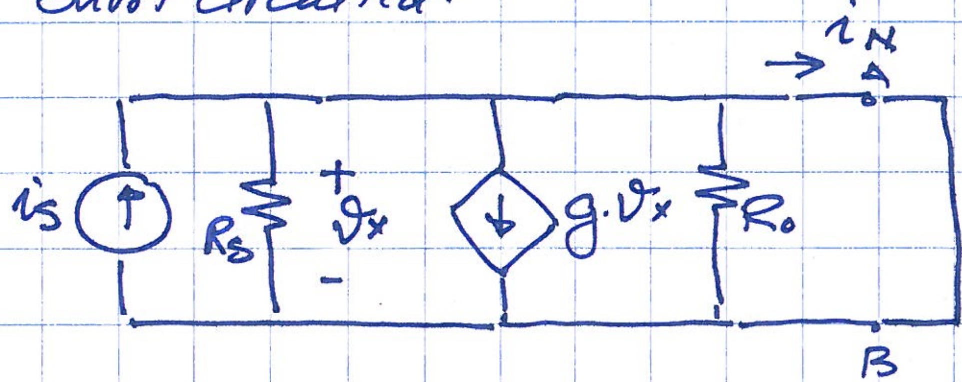
or

$$R_{th} = \frac{R_o \cdot R_S \cdot g}{R_o + R_S + g R_o \cdot R_S}$$

Another approach to find R_{th} is to find the Norton equivalent current i_N. Then,

$$R_{th} = \frac{v_{th}}{i_N}$$

Find the current i when terminals A an B are short circuited:



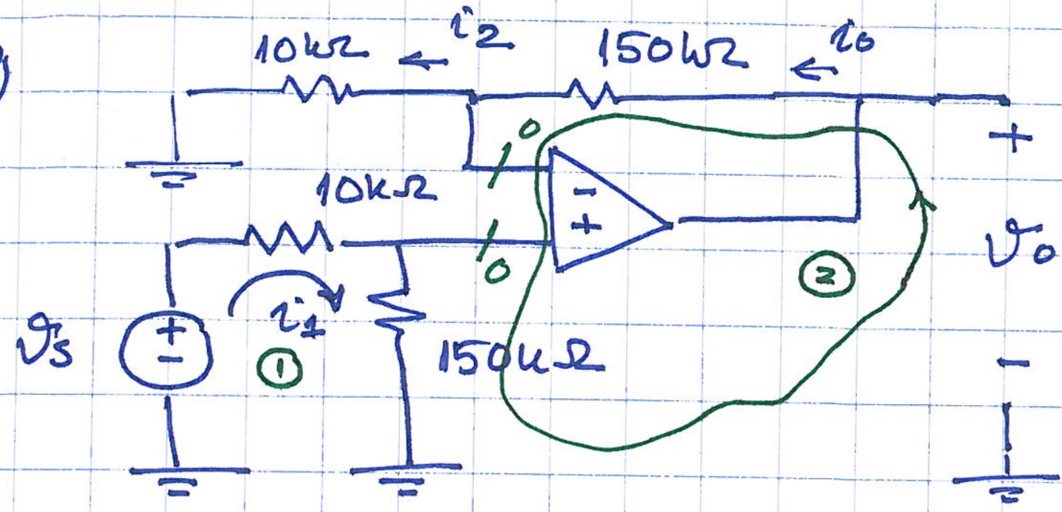
nodal equation (or KCL) gives:

$$i_S - \frac{v_x}{R_S} - g \cdot v_x - i_N = 0 \qquad ; \quad v_x = 0$$

Hence, $i_N = i_S$

$$R_{th} = \frac{V_{AB}}{i_N} \; ; \; R_{th} = \frac{v_{th}}{i_N} \; ; \; R_{th} = \frac{R_S \cdot R_o}{R_o + R_S + g R_o \cdot R_S}$$

#2



Find v_o in terms of v_s

Find i_o for $v_s = 1V$

Ideal op amp: $i_- = 0$; $i_+ = 0$; $v_+ - v_- = 0$

KVL: $v_s - 10\times10^3 \cdot i_1 - 150\times10^3 i_1 = 0$ Loop 1

$$i_1 = \frac{v_s}{160} \times 10^{-3}$$

KCL: $i_2 = i_o$

Note that $v_- = v_+$

Since $v_- = 10\times10^3 \cdot i_2$

$v_+ = 150\times10^3 \cdot i_1$

Then $10\times10^3 \cdot i_2 = 150\times10^3 \cdot i_1$; $i_2 = 15 \cdot i_1$

and $i_o = 15 \cdot i_1$

KVL for loop 2:

$$v_o - 150\times10^3 \cdot i_o - 150\times10^3 \cdot i_1 = 0$$

$$v_o = 150\times10^3 (i_o + i_1)$$

$$v_o = 150\times10^3\cdot(15 i_1 + i_1)$$

Recall that $i_1 = \dfrac{v_s}{160}\times10^{-3}$

Hence,

$$v_o = 150\cdot10^3\cdot16\cdot\frac{v_s}{160}\cdot10^{-3}$$

$$v_o = 15\cdot v_s \qquad (\text{gain} = 15)$$

If $v_s = 1V$

then $i_1 = \dfrac{1\times10^{-3}}{160}$

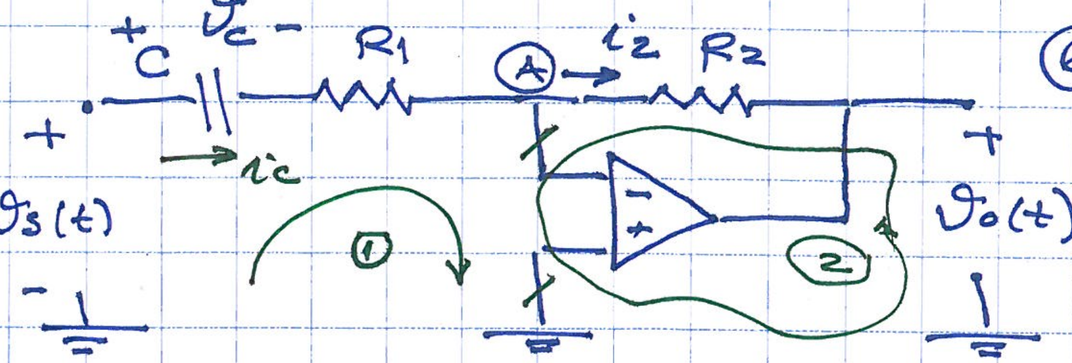
$$i_o = 15\cdot i_1$$

$$i_o = \frac{15}{160}\times10^{-3}A \;,\quad i_o = \frac{3}{32}\times10^{-3}A$$

$$i_o = \frac{3}{32}\,mA$$

or $i_o = 93.75\mu A$

#3



Find relationship between $v_o(t)$ and $v_S(t)$.

Ideal opamp:
$v_- - v_+ = 0$
$v_- = 0$
$v_+ = 0$
$i_- = 0;\ i_+ = 0$

KVL for loop 1:

$$v_S(t) - v_C(t) - v_{R_1}(t) = 0$$

or

$$v_S - v_C - v_{R_1} = 0$$

$$v_{R_1} = R_1 \cdot i_C$$

$$i_C = C \cdot \frac{dv_C}{dt}$$

Hence, $$v_C + R_1 \cdot C \cdot \frac{dv_C}{dt} = v_S$$

Let us express $v_o(t)$ in terms of $v_C(t)$:

KVL for loop 2:

$$v_o(t) + R_2 \cdot i_2 = 0;$$

KCL for node A: $i_C - i_2 = 0;\ i_2 = i_C$

Hence: $$v_o - R_2 \cdot i_C = 0;\ v_o = R_2 \cdot C \cdot \frac{dv_C}{dt}$$

From the two equations:

$$v_C + R_1 C \cdot \frac{dv_C}{dt} = v_S \qquad (1)$$

$$v_o = R_2 C \frac{dv_C}{dt} \qquad (2)$$

we get:

$$v_C = \frac{1}{R_2 C} \cdot \int v_0(x)\,dx \qquad \text{from (2)}$$

Substitute into (1):

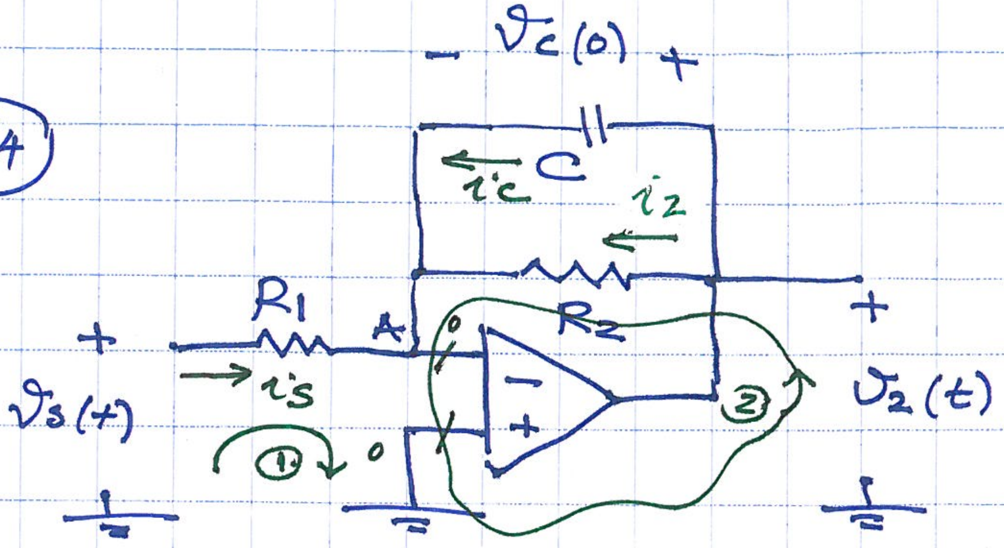
$$\frac{1}{R_2 C} \cdot \int v_0(x)\,dx + R_1 C \cdot \frac{1}{R_2 C} \cdot v_0 = v_S$$

or $$\frac{R_1}{R_2} \cdot v_0 + \frac{1}{R_2 C} \cdot \int v_0(x)\,dx = v_S$$

Find the derivative to avoid having an integral:

$$\frac{R_1}{R_2} \cdot \frac{dv_0}{dt} + \frac{1}{R_2 C} \cdot v_0 = \frac{dv_S}{dt}$$

#4



ideal op amp:

$v_- = v_+$; $v_- = 0$

$v_+ = 0$

$i_- = 0$

$i_+ = 0$

KVL: $v_s - R_1 \cdot i_s = 0$ Loop 1

$$i_s = \frac{1}{R_1} \cdot v_s$$

KVL: $v_2 - R_2 \cdot i_2 = 0$ Loop 2

$$i_2 = \frac{v_2}{R_2}$$

Note that: $v_C = v_2$; $i_2 = \frac{1}{R_2} \cdot v_c$

KCL for node A:

$$i_s + i_c + i_2 = 0$$

Hence, $\frac{1}{R_1} \cdot v_s + i_c + \frac{1}{R_2} \cdot v_c = 0$

Since $i_c = C\frac{dv_c}{dt}$; $C\frac{dv_c}{dt} + \frac{1}{R_2} \cdot v_c = -\frac{1}{R_1} \cdot v_s$

(DAE)

Let $v_s(t) = 2\cdot u(t)$

DAE becomes:

$$C\cdot\frac{dv_c}{dt} + \frac{1}{R_2}\cdot v_c = -\frac{1}{R_1}\cdot 2\cdot u(t)$$

or

$$\frac{dv_c}{dt} + \frac{1}{CR_2}\cdot v_c = -\frac{2}{CR_1} \quad (1) \text{ for } t \geq 0$$

Solution:

$$v_{c_h}(t) = Ke^{st} \qquad \text{Homogeneous solution.}$$

$$\frac{dv_{c_h}}{dt} = Kse^{st}$$

Substitute into (1) gives:

$$K\cdot se^{st} + \frac{1}{CR_2}\cdot Ke^{st} = 0 \quad ; e^{st} \neq 0$$

$K \neq 0$ (non-trivial solution)

$$s = -\frac{1}{CR_2}$$ ↑ natural frequency

$$v_{c_h} = Ke^{-t/CR_2}$$

Particular solution.

$$v_{cp}(t) = A \qquad (\text{because } v_s(t) = 2u(t))$$

$$\frac{dv_{cp}(t)}{dt} = 0$$

Hence

$$0 + \frac{1}{CR_2}\cdot A = -\frac{2}{CR_1} \quad ; A = -2\frac{R_2}{R_1}$$

Total solution is.

$$v_c(t) = Ke^{-\frac{t}{CR_2}} - 2\cdot\frac{R_2}{R_1}$$

Find K from the initial conditions.

$$v_C(0-) = V_0 \quad ; \text{ state variable}$$

$$v_C(0+) = V_0$$

$$V_0 = K e^{-\frac{0}{CR_2}} - 2 \cdot \frac{R_2}{R_1} \quad ; \quad V_0 = K - 2 \cdot \frac{R_2}{R_1}$$

$$K = V_0 + 2 \frac{R_2}{R_1}$$

Total solution is:

$$v_C(t) = \left(V_0 + 2 \frac{R_2}{R_1}\right) e^{-\frac{t}{CR_2}} - 2 \cdot \frac{R_2}{R_1} \qquad \forall t \geq 0$$

It can be also written as:

$$v_C(t) = \left(\left(V_0 + 2 \frac{R_2}{R_1}\right) \cdot e^{-\frac{t}{CR_2}} - 2 \frac{R_2}{R_1}\right) \cdot u(t)$$

Recall that

$$i_S(t) = \frac{1}{R_1} \cdot v_S(t) \quad ; \quad i_S(t) = \frac{1}{R_1} \cdot 2 \cdot u(t)$$

$$i_S(t) = \frac{2}{R_1} \cdot u(t)$$

Graphs: