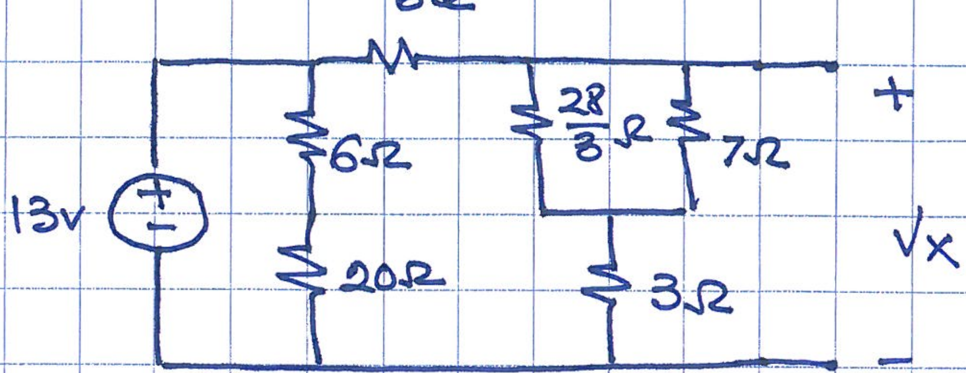
ENSC 220 Fall 2017

Midterm #1

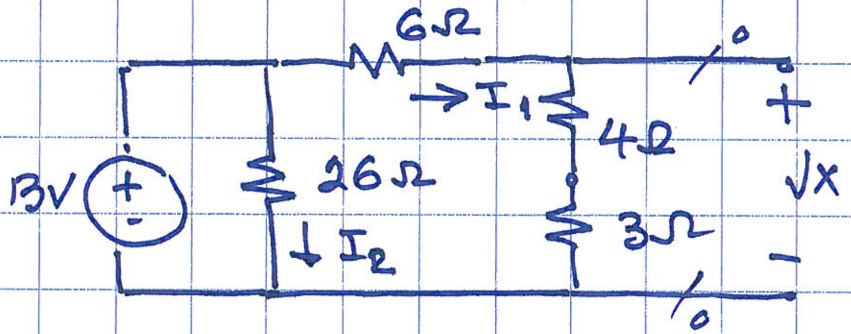
Problem:

1.



$V_x = ?$

Simplify the circuit by combining resistors connected in series and in parallel:



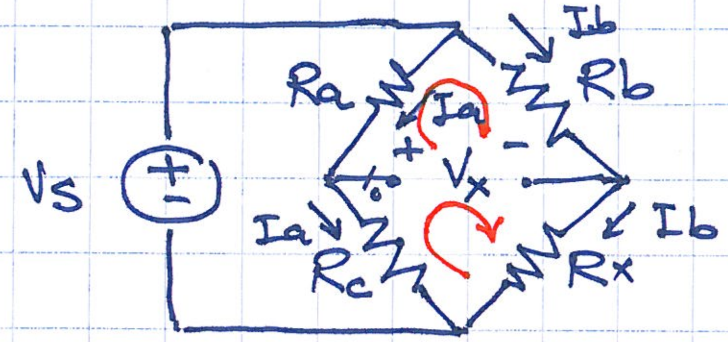
where: $6\Omega + 20\Omega = 26\Omega$

$$\frac{\frac{28}{3}\times 7}{\frac{28}{3}+7} = \frac{28\times 7}{28+3\times 7} = \frac{28\times 7}{49} = 4\Omega$$

Note that: $I_1 = \dfrac{13}{6+4+3}$; $I_1 = 1A$

$V_x = (4+3)\times I_1$; $V_x = 7\times 1$; $V_x = 7V$

Problem:
2



$R_x = ?$ if the bridge is balanced

$$V_x = 0$$

KVL for the top mesh:

$$R_a \cdot I_a - R_b \cdot I_b = 0 \qquad (1)$$

KVL for the bottom mesh:

$$R_c \cdot I_a - R_x \cdot I_b = 0 \qquad (2)$$

Note: No current flows through the terminals where we measure V_x

Hence, based on KCL:

$$I_{Ra} = I_{Rc}$$

$$I_{Rb} = I_{Rx}$$

Hence,

$$R_a I_a = R_b \cdot I_b \qquad (3)$$

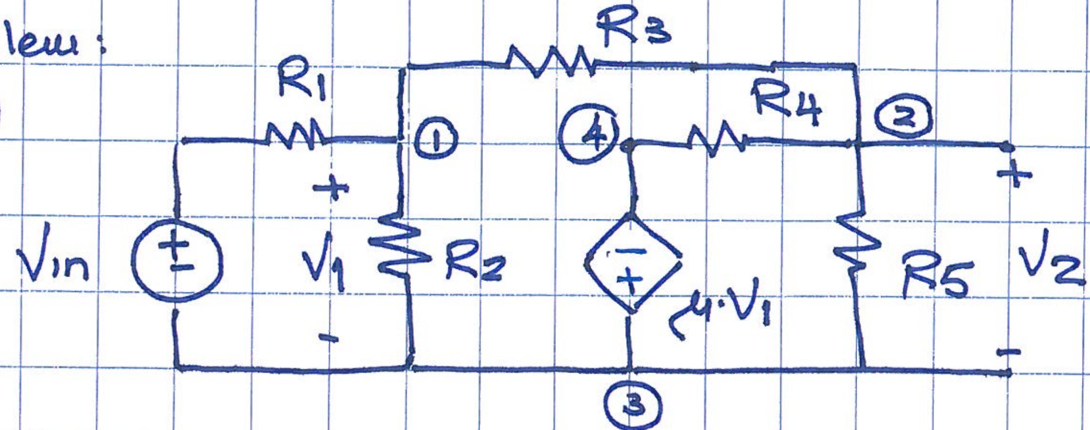
$$R_c \cdot I_a = R_x \cdot I_b \qquad (4)$$

Dividing (3) and (4) gives:

$$\frac{R_a}{R_c} = \frac{R_b}{R_x} \quad \text{or} \quad R_x = \frac{R_a \cdot R_b}{R_c}$$

Problem:

3



Write 2 nodal equations:

1 $$\frac{V_1 - V_{in}}{R_1} + \frac{V_1}{R_2} + \frac{V_1 - V_2}{R_3} = 0 \qquad (1)$$

2 $$\frac{V_2 - V_1}{R_3} + \frac{V_2 - V_4}{R_4} + \frac{V_2}{R_5} = 0 \qquad (2)$$

Note that $V_4 = -\mu \cdot V_1$

Hence, $$V_1 \cdot \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) - \frac{1}{R_3} \cdot V_2 = \frac{1}{R_1} \cdot V_{in} \qquad (3)$$

$$V_2 \cdot \left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right) - \frac{1}{R_3} \cdot V_1 - \frac{1}{R_4} \cdot (-\mu \cdot V_1) = 0 \qquad (4)$$

Or: $$V_1 \cdot \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) - \frac{1}{R_3} \cdot V_2 = \frac{1}{R_1} \cdot V_{in}$$

$$-V_1 \cdot \left(\frac{1}{R_3} - \mu \cdot \frac{1}{R_4}\right) + V_2 \cdot \left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right) = 0$$

Conductance matrix: $$\begin{pmatrix} G_1 + G_2 + G_3 & -G_3 \\ -G_3 + \mu \cdot G_4 & G_3 + G_4 + G_5 \end{pmatrix}$$

If $\mu = 0$ (no controlled source)

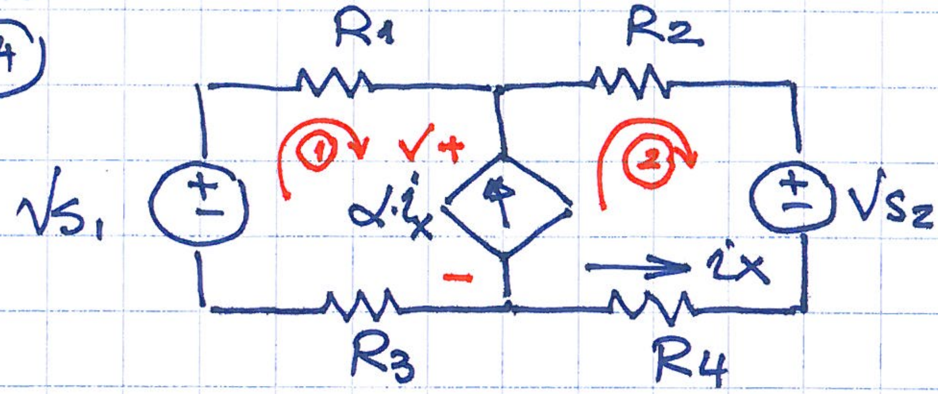
Conductance matrix is:

$$\begin{pmatrix} G_1+G_2+G_3 & -G_3 \\ -G_3 & G_3+G_4+G_5 \end{pmatrix}$$

Properties:
- symmetric
- all diagonal elements are positive
- all off-diagonal elements are negative.

Problem:

4

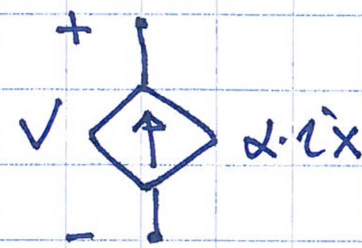


Write two mesh equations:

Mesh currents:
j_1
j_2

(1) $R_1 \cdot j_1 + R_3 \cdot j_1 + V - V_{S_1} = 0$

(2) $R_2 \cdot j_2 + R_4 \cdot j_2 - V + V_{S_2} = 0$



Hence:

$$(R_1 + R_3) \cdot j_1 + V = V_{S1} \qquad (3)$$

$$(R_2 + R_4) \cdot j_2 - V = -V_{S2} \qquad (4)$$

Note that $j_2 - j_1 = \alpha \cdot i_x$

$$i_x = -j_2$$

Hence,

(5) $(R_1 + R_3) \cdot j_1 + V = V_{S1}$

(6) $(R_2 + R_4) \cdot j_2 - V = -V_{S2}$

(7) $j_2 - j_1 = \alpha \cdot (-j_2)$ or $(1+\alpha) \cdot j_2 - j_1 = 0$

Adding (5) and (6) to eliminate V gives:-

(8) $(R_1 + R_3) \cdot j1 + (R_2 + R_4) \cdot j2 = V_{S1} - V_{S2}$

(9) $-j_1 + (1+\alpha) \cdot j_2 = 0$

Resistance matrix:

$$\begin{pmatrix} R_1 + R_3 & R_2 + R_4 \\ -1 & 1+\alpha \end{pmatrix}$$

If $\alpha = 0$, current controlled current source is removed

$j1 = j2$

Matrix degenerates to:

$(R_1 + R_2 + R_3 + R_4) \cdot j_1 = V_{S1} - V_{S2}$

Properties:
- symmetric
- all diagonal elements positive
- all off diagonal elements positive or negative depending on the orientation of mesh currents

Solution:

$(30+20)j_1 + (270+80)j2 = 225 - 15$

$-j_1 + (1-0.5)j2 = 0$

$$50 j_1 + 350 j_2 = 210$$
$$-j_1 + 0.5 j_2 = 0$$

or: $j_2 = 2 j_1$

Hence,

$$50 j_1 + 350 \cdot 2 j_1 = 210$$
$$750 j_1 = 210$$
$$j_1 = \frac{210}{750}$$
$$j_1 = \frac{7}{25} \text{ A}$$
$$j_2 = \frac{14}{25} \text{ A}$$