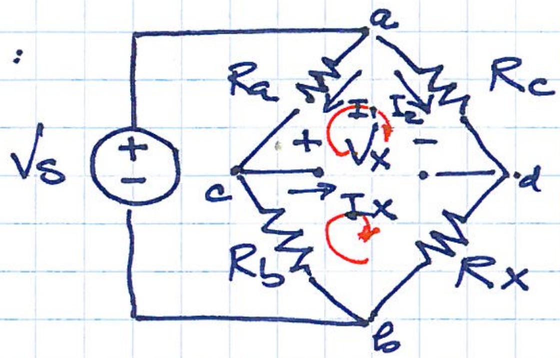
Problem # 1:



Current $I_x = 0$
(open circuit)

Bridge is ballanced when $V_x = 0$

KVL around the top mesh:

$$R_a \cdot I_1 - R_c \cdot I_2 = 0 \qquad (1)$$

KVL around the bottom mesh:

$$R_b \cdot I_1 - R_x \cdot I_2 = 0 \qquad (2)$$

Note: $I_{R_a} = I_{R_b}$; $I_{R_c} = I_{R_x}$

Due to KCL for nodes c and d.

Hence:

$$R_a I_1 = R_c \cdot I_2 \qquad (3)$$

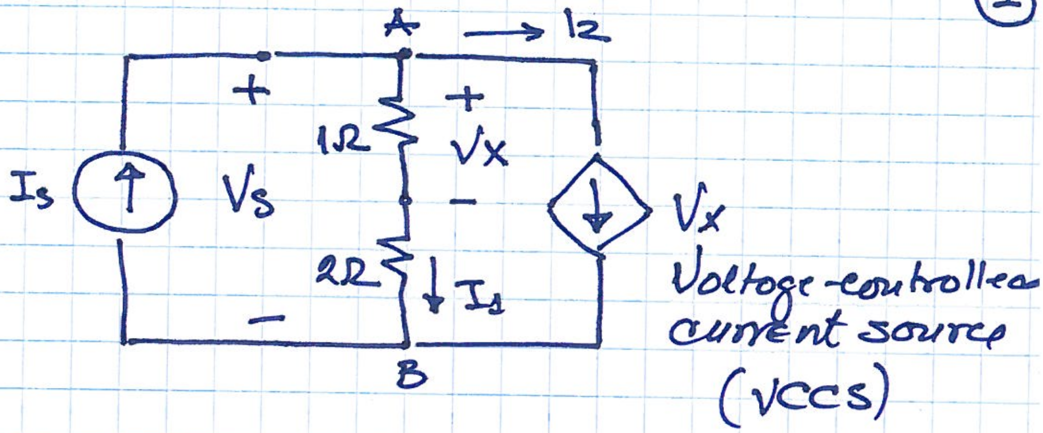
$$R_b \cdot I_1 = R_x \cdot I_2 \qquad (4)$$

Deviding (3) and (4) gives:

$$\frac{R_a}{R_b} = \frac{R_c}{R_x}$$

Hence, $R_x = \dfrac{R_b \cdot R_c}{R_a}$

Problem #2:



equivalent resistance:

$$R_{eq} = \frac{V_s}{I_s}$$

KCL for node A: $I_s - I_1 - I_2 = 0$ (1)

Since: $I_2 = V_x$ (2) VCCS

Ohm's law: $V_x = 1 \times I_1$ (3)

Hence: $I_1 = I_2$

From (1) $I_s = I_1 + I_2$

$I_s = 2 I_1 \Rightarrow I_1 = \frac{1}{2} \cdot I_s$ (4)

KVL involving V_s:

$V_s - 1 \times I_1 - 2 \times I_1 = 0$ (5)

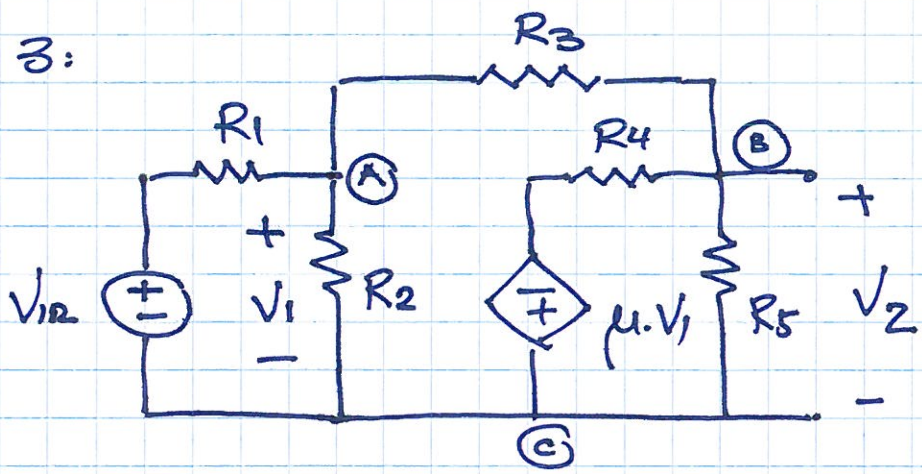
$V_s = 3 \times I_1$

From (4): $V_s = 3 \times \frac{1}{2} \times I_s$

Hence: $\frac{V_s}{I_s} = 3/2$

$$R_{eq} = \frac{3}{2}\,\Omega$$

Problem # 3:



KCL for node ~~A~~ and B: C: Reference node

$$\frac{V_A - V_{in}}{R_1} + \frac{V_A}{R_2} + \frac{V_A - V_B}{R_3} = 0 \qquad (1)$$

$$\frac{V_B + \mu . V_1}{R_4} + \frac{V_B}{R_5} + \frac{V_B - V_A}{R_3} = 0 \qquad (2)$$

Note: $V_A = V_1$

$V_B = V_2$

Hence: $$\frac{V_1 - V_{in}}{R_1} + \frac{V_1}{R_2} + \frac{V_1 - V_2}{R_3} = 0 \qquad (3)$$

$$\frac{V_2 + \mu \cdot V_1}{R_4} + \frac{V_2}{R_5} + \frac{V_2 - V_1}{R_3} = 0 \qquad (4)$$

$$V_1 \cdot \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) - \frac{1}{R_3} \cdot V_2 = \frac{1}{R_1} \cdot V_{in} \qquad (5)$$

$$\left(-\frac{1}{R_3} + \frac{\mu}{R_4}\right) V_1 + V_2 \left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right) = 0 \qquad (6)$$

Using conductances rather than resistances leads to:

$$(G_1+G_2+G_3)\cdot V_1 - G_3\cdot V_2 = G_1\cdot V_{in}$$

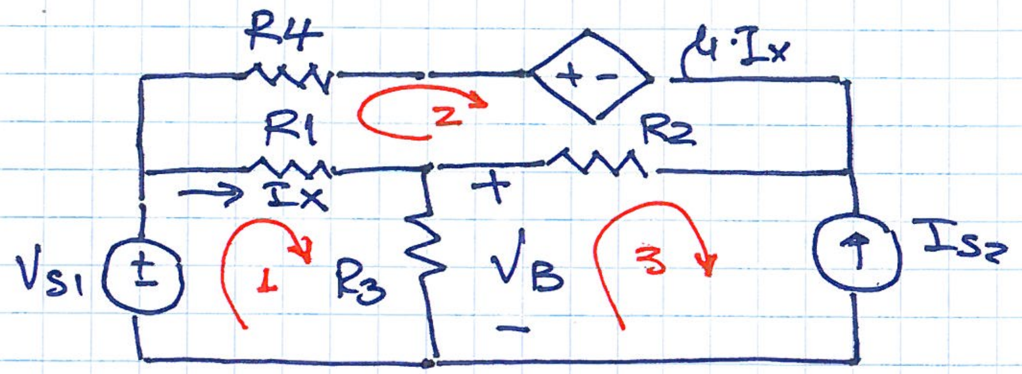
$$(-G_3+\mu\cdot G_4)\,V_1 + (G_3+G_4+G_5)\cdot V_2 = 0$$

Matrix-form:

$$\begin{bmatrix} G_1+G_2+G_3 & -G_3 \\ -G_3+\mu G_4 & G_3+G_4+G_5 \end{bmatrix}\cdot\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}=\begin{bmatrix} G_1\cdot V_{in} \\ 0 \end{bmatrix}$$

Test: If $\mu=0$, the circuit contains resisters only. Hence, matrix G should be symmetric, with all diagonal elements positive.

Problem #4:



There are three meshes. However, we can write two mesh equations that will enable us to solve the circuit equations.

Write KVL equations for mesh 1 and mesh 2:

$$V_{s1} - R_1 \cdot (I_1 - I_2) - R_3 (I_1 - I_3) = 0 \qquad (1)$$

$$\mu \cdot I_x + R_4 \cdot I_2 + R_1 \cdot (I_2 - I_1) + R_2 \cdot (I_2 - I_3) = 0 \qquad (2)$$

Note that: $I_3 = -I_{s2}$ (known current)

$I_x = I_1 - I_2$ (Note: I_x is current flowing through R_1)

Hence:

$$(R_1 + R_3) I_1 - R_1 \cdot I_2 - R_3 \cdot I_3 = V_{s1} \qquad (3)$$

$$\mu (I_1 - I_2) + (R_1 + R_2 + R_4) I_2 - R_1 I_1 - R_2 I_3 = 0 \qquad (4)$$

Then:

$$(R_1 + R_3) I_1 - R_1 I_2 + R_3 I_{s2} = V_{s1} \qquad (5)$$

$$(\mu - R_1) I_1 + (R_1 + R_2 + R_4 - \mu) I_2 + R_2 I_{s2} = 0 \qquad (6)$$

Finally:

$$(R_1+R_3)I_1 - R_1 \cdot I_2 = V_{s1} - R_3 \cdot I_{s2}$$
$$(\mu - R_1)I_1 + (R_1 + R_2 + R_4 - \mu) I_2 = -R_2 \cdot I_{s2}$$

Matrix form:

$$\begin{bmatrix} R_1+R_3 & -R_1 \\ \mu - R_1 & R_1+R_2+R_4-\mu \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_{s1} - R_3 I_{s2} \\ -R_2 I_{s2} \end{bmatrix}$$

Test: If $\mu = 0$, the circuit contains only resistors and, hence, matrix R is symmetrical and contains all positive diagonal entries.

Substituting numerical values gives:

$$\begin{bmatrix} 200+200 & -200 \\ 300-200 & 200+200+100-300 \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 250 - 200 \times 0.75 \\ -200 \times 0.75 \end{bmatrix}$$

or $$\begin{bmatrix} 400 & -200 \\ 100 & 200 \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 100 \\ -150 \end{bmatrix}$$

Simply: $$\begin{bmatrix} 4 & -2 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1.5 \end{bmatrix}$$

$\det = 4 \times 2 - (1)(-2) \therefore \det = 10$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \underbrace{\frac{1}{10} \cdot \begin{bmatrix} 2 & 2 \\ -1 & 4 \end{bmatrix}}_{\text{inverse matrix}} \cdot \begin{bmatrix} 1 \\ -1.5 \end{bmatrix} ; \quad I_1 = -0.1\,A, \quad I_2 = -0.7\,A$$

$V_B = ?$

$V_B = R_3 \cdot (I_1 - I_3)$

$V_B = R_3 \cdot (I_1 + I_{S_2})$

(Recall: $I_3 = -I_{S_2}$)

Hence:

$V_B = 200 \cdot (-0.1 + 0.75)$

$V_B = 130V$