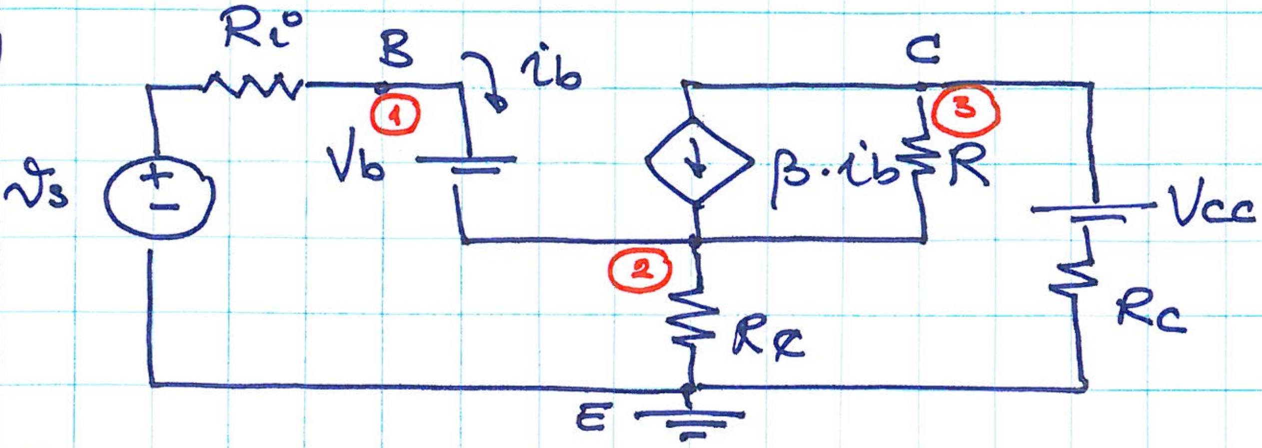
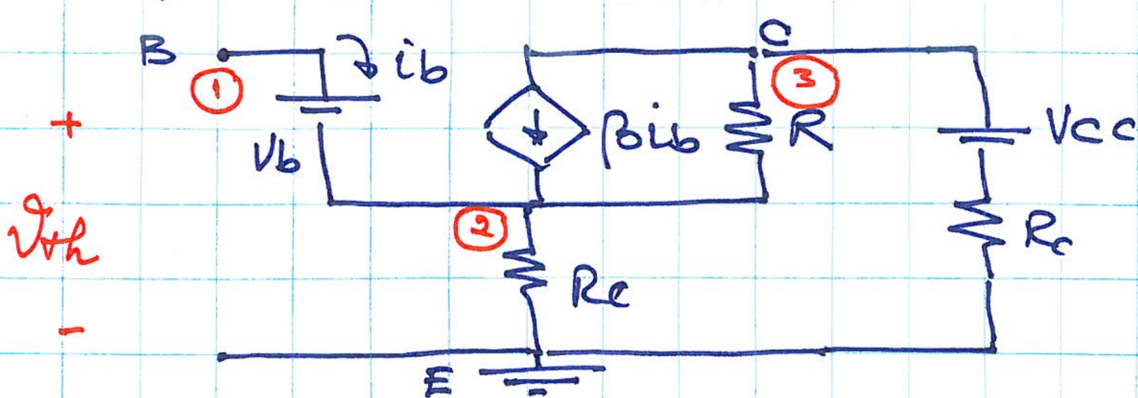
1



Thévenin voltage:

Nodal equations should be written for the circuit.



Nodes B and E are open-circuited.

Hence: $i_b = 0$

Nodal equations:

$$V_1 - V_2 = V_b$$

$$\frac{V_2}{R_e} - \beta \cdot i_b + \frac{V_2 - V_3}{R} = 0$$

$$\beta i_b + \frac{V_3 - V_2}{R} + \frac{V_3 - V_{cc}}{R_c} = 0$$

Since $i_b = 0$:

$$V_1 - V_2 = V_b$$

$$V_2\left(\frac{1}{Re} + \frac{1}{R}\right) - V_3 \cdot \frac{1}{R} = 0 \qquad (1)$$

$$-\frac{1}{R} \cdot V_2 + V_3 \cdot \left(\frac{1}{Rc} + \frac{1}{R}\right) = \frac{Vcc}{Rc} \qquad (2)$$

Thévenin voltage: $V_{Th} = V_1$; It suffices to find V_2 since $V_1 = V_2 + V_b$

From the 1st equation:

$$V_3 = R \cdot \left(\frac{1}{Re} + \frac{1}{R}\right) V_2$$

Substituting into (2) leads to:

$$-\frac{1}{R} \cdot V_2 + R \cdot \left(\frac{1}{Re} + \frac{1}{R}\right) \cdot \left(\frac{1}{Rc} + \frac{1}{R}\right) = \frac{Vcc}{Rc}$$

Hence,

$$V_2 = \frac{Vcc}{Rc} \cdot \frac{1}{-\frac{1}{R} + \frac{R}{Re} \cdot \left(\frac{1}{Rc} + \frac{1}{R}\right) + \frac{1}{Rc} + \frac{1}{R}}$$

$$V_2 = \frac{Vcc}{Rc} \cdot \frac{1}{\frac{R}{Re \cdot Rc} + \frac{1}{Rc} + \frac{1}{Rc}}$$

$$V_2 = \frac{Vcc \cdot Re}{R + Re + Rc}$$

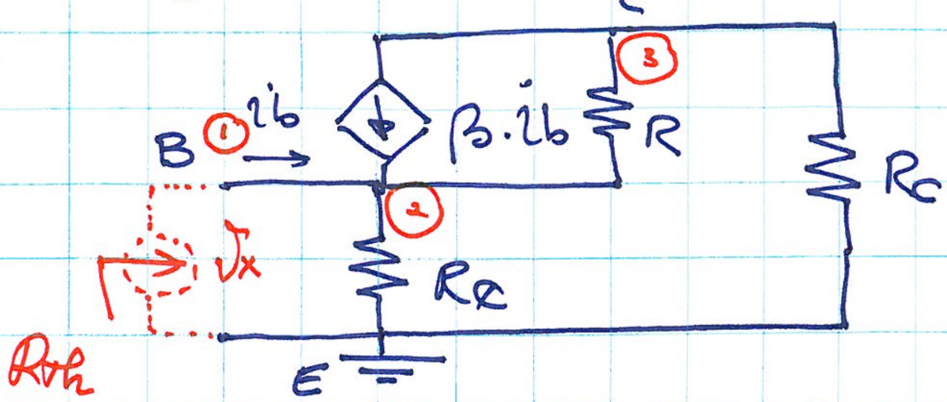
Finally:

$$V_{Th} = V_1$$

$$V_{Th} = V_2 + V_b$$

$$V_{Th} = V_b + Vcc \cdot \frac{Re}{R + Re + Rc}$$

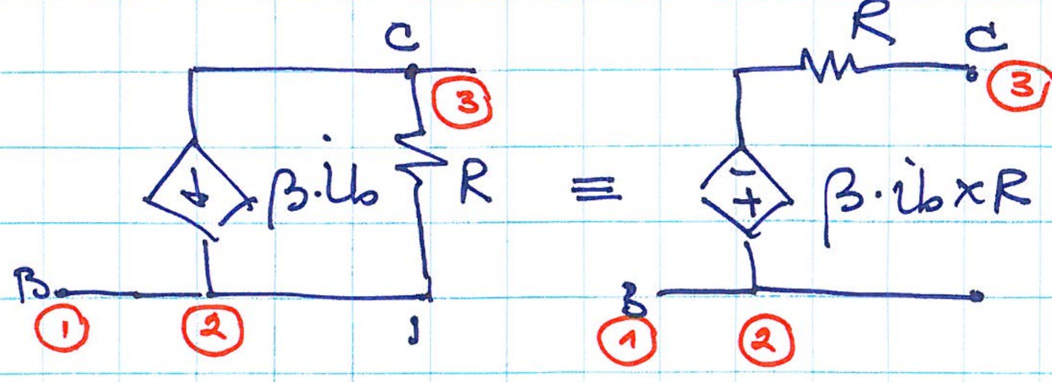
To find Thévenin resistance, we consider the same circuit but with independent sources properly disconnected:



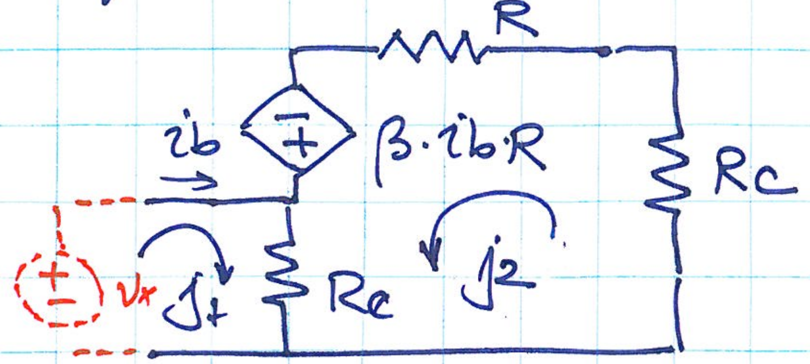
Assume a voltage source V_x is connected between terminals B and E. Then:

$$R_{th} = \frac{V_x}{i_b}$$

Transform the parallel connection of the dependent current source and the resistor:



The equivalent circuit is:



Mesh equations:

$$V_x - R_e(j_1 + j_2) = 0$$

$$\beta \cdot i_b \cdot R - R_e(j_1 + j_2) - (R + R_c) \cdot j_2 = 0$$

Note that $i_b = j_1$

Hence:

$$R_e \cdot j_1 + R_e \cdot j_2 = V_x$$

$$-(R_e - \beta \cdot R) \cdot j_1 - (R + R_e + R_c) \cdot j_2 = 0$$

Since $R_{th} = \frac{V_x}{j_1}$, let us find j_2 in terms of j_1.

$$j_2 = -j_1 + \frac{V_x}{R_e}$$

From the second equation:

$$-(R_e - \beta R) \cdot j_1 - (R + R_e + R_c) \cdot \left(-j_1 + \frac{V_x}{R_e}\right) = 0$$

$$j_1(\beta R + R + R_C) = \frac{R + R_E + R_C}{R_E} \cdot V_x$$

Hence: $$R_{th} = \frac{V_x}{j_1}$$

$$R_{th} = \frac{R_E \cdot (\beta R + R + R_C)}{R + R_E + R_C}$$

or
$$R_{th} = R_E \cdot \frac{(1+\beta)R + R_C}{R + R_E + R_C}$$

Check:

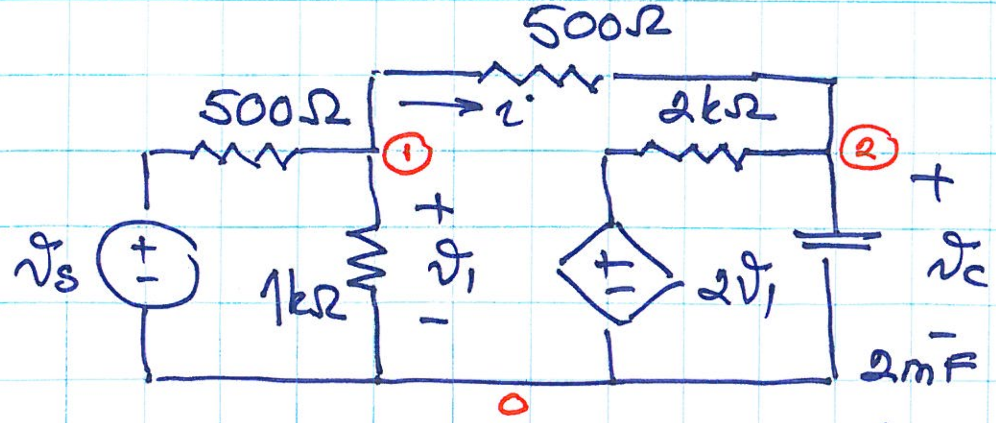
If $\beta \to 0$
$$R_{th} = R_E \cdot \frac{R + R_C}{R + R_E + R_C}$$

From the circuit: R and R_C are connected in series and then in parallel with R_E

If $\beta \to \infty$ $\quad R_{th} = R_E$

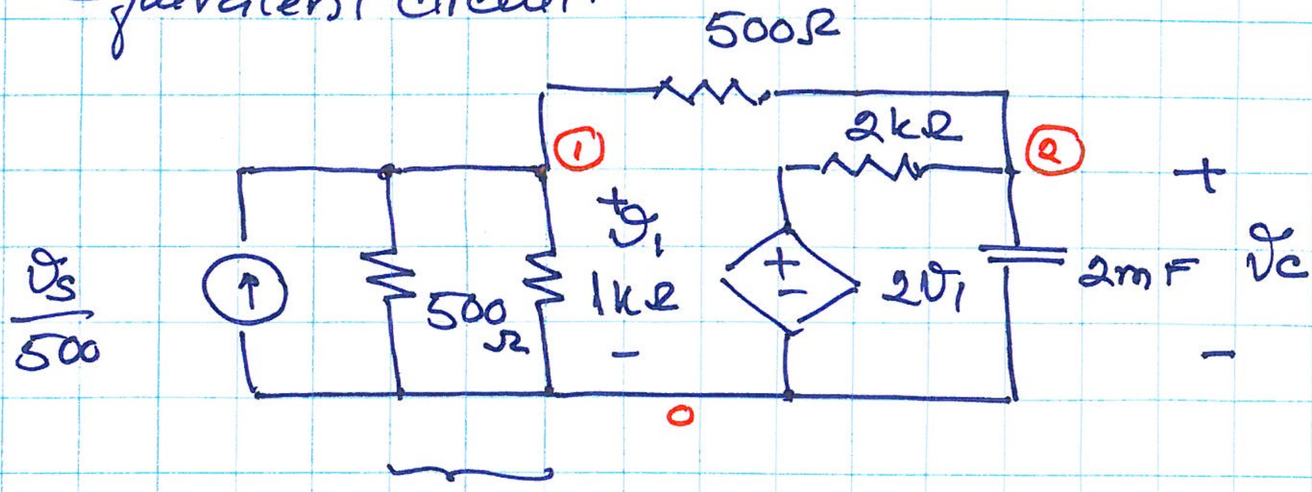
Note that in this case the rest of the circuit is open.

2



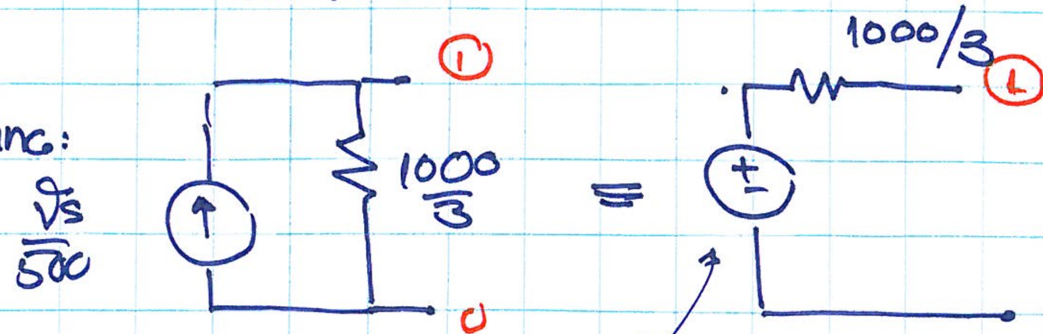
$v_s(t) = 2t\,u(t)$

Equivalent circuit:



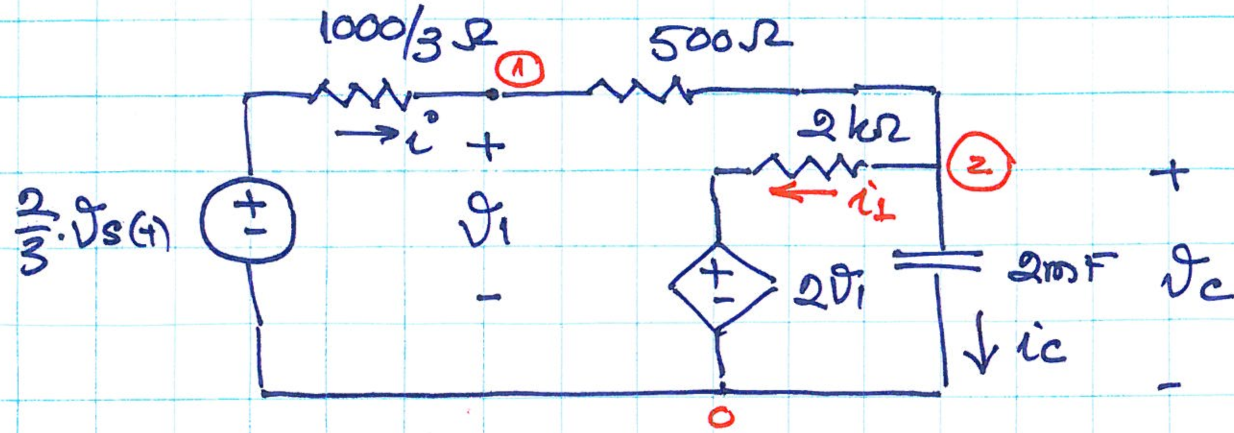
$$R_e = \frac{500 \times 1000}{500 + 1000} \; ; \; R_e = \frac{1000}{3}\ \Omega$$

Since:



$$\frac{v_s}{500} \times \frac{1000}{3} = \frac{2}{3} \cdot v_s$$

The circuit becomes:



KVL: $\frac{2}{3}\cdot v_s(t) - \frac{1000}{3}\cdot i - 500\cdot i - v_c = 0$

$2v_1 + 2\times10^3\cdot i_1 - v_c = 0$

$i_1 = i - i_c$

$i_c = C\frac{dv_c}{dt}$; $i_c = 2\times10^{-3}\cdot\frac{dv_c}{dt}$

Note: $v_s(t) = t\,u(t)$

Hence: $\left(\frac{1000}{3} + 500\right)\cdot i + v_c = \frac{2}{3}\cdot t\,u(t)$

$2v_1 + 2\times10^3\cdot(i - i_c) - v_c = 0$

$i_c = 2\times10^{-3}\cdot\frac{dv_c}{dt}$

Note that: $v_1 - 500\times i - v_c = 0$

Hence: $v_1 = 500i + v_c$

Then: $\frac{2500}{3}\cdot i + v_c = \frac{2}{3}\cdot t$ $\quad u(t) = 1$ for $t > 0$

$2\cdot(500i + v_c) + 2\times10^3(i - i_c) - v_c = 0$

From the 1^{st} equah:

$$i^0 = \frac{3}{2100} \cdot \left(\frac{2}{3}t - v_c\right)$$

Equation 2 can be arranged as:

$$\left(2 \times 500 + 2 \times 10^3\right) i^0 + 2 \cdot v_c - 2 \times 10^3 \cdot i_c - v_c = 0$$

$$3000 \cdot i^0 + v_c - 2 \times 10^3 \cdot 2 \times 10^{-3} \frac{dv_c}{dt} = 0$$

or: $$30\cancel{00} \times \frac{3}{21\cancel{00}} \cdot \left(\frac{2}{3}t - v_c\right) + v_c - 4 \cdot \frac{dv_c}{dt} = 0$$

$$-4 \frac{dv_c}{dt} + v_c\left(1 - \frac{30 \times 3}{25}\right) = -\frac{3 \times 30}{25} \times \frac{2}{3} \cdot t$$

$$-4 \frac{dv_c}{dt} - \frac{13}{5} \cdot v_c = -\frac{12}{5} \cdot t \quad / : (-4)$$

$$\frac{dv_c}{dt} + \frac{13}{20} v_c = \frac{3}{5} \cdot t$$

1^{st} order differential equah:

Characters equ: $s + \frac{13}{20} = 0$; $s = -\frac{13}{20}$

$$v_{ch} = k e^{-13/20 \cdot t} \; ; \; v_c(t) = v_{ch}(t) + v_{cp}(t)$$

given: $v_c(0-) = 0$; $v_c(0+) = 0$

$v_{cp}(t) = at + b$ (Recall that: input is a ramp func.)

$\frac{dv_{cp}}{dt} = a$; Hence from DAE: $a + \frac{13}{20} \cdot (at + b) = \frac{3}{5} t$

$$a+\frac{13}{20}\cdot at+\frac{13}{20}b=\frac{3}{5}t \quad \forall t$$

$$a+\frac{13}{20}b=0$$

$$\frac{13}{20}a=\frac{3}{5}\ ; \quad a=\frac{12}{13}$$

$$\frac{13}{20}b=-a\ ; \quad b=-\frac{20}{13}\times\frac{12}{13}$$

$$b=-\frac{240}{169}$$

$$v_{cp}(t)=\frac{12}{13}t-\frac{240}{169}$$

$$v_c(t)=ke^{-13/20t}+\frac{12}{13}t-\frac{240}{169}$$

$$v_c(0+)=k-\frac{240}{169}\ ; \quad v_c(0+)=0 \quad ; k=\frac{240}{169}$$

$$v_c(t)=\left(\frac{240}{169}\cdot e^{-\frac{13}{20}t}+\frac{12}{13}t-\frac{240}{169}\right)u(t)$$

Current $i(t)$ through the $500\,\Omega$ resister:

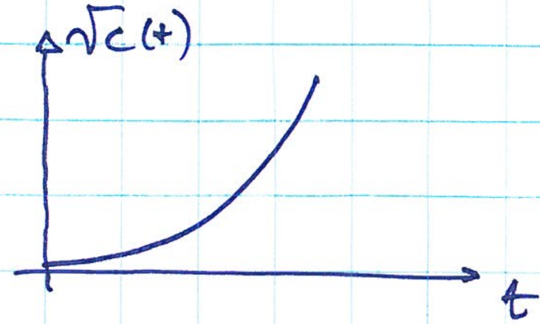
$$i=\frac{3}{2500}\cdot\left(\frac{2}{3}t-v_c\right)$$

$$i=\frac{3}{2500}\times\frac{2}{3}\cdot t-\frac{3}{2500}\times\left(\frac{240}{169}e^{-13/20t}+\frac{12}{13}t-\frac{240}{169}\right)$$

$$i=-\frac{1}{13\times 250}\cdot t-\frac{36}{125\times 169}\cdot e^{-13/20t}+\frac{36}{125\times 169}$$

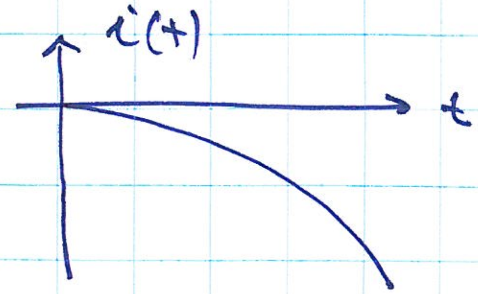
At $t=0$: $v_C(0^+)=0$

$t\to\infty$ $v_C(t)\to\infty$

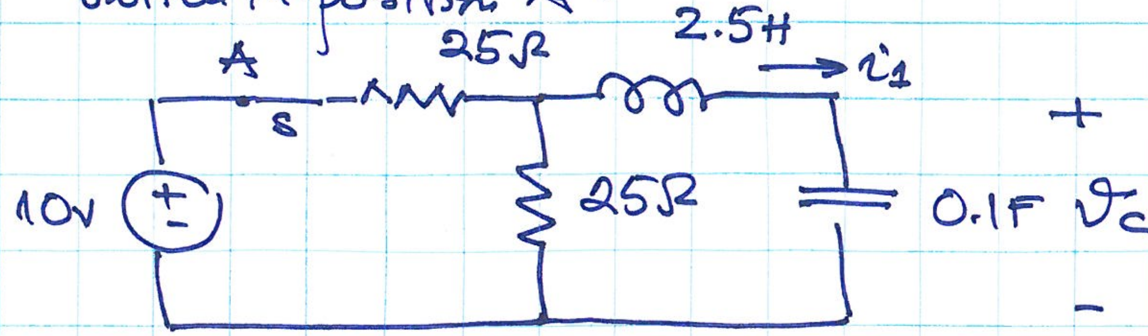


At $t=0$: $i(0^+)=0$ (From the solution for $i(t)$)

$t\to\infty$ $i(t)\to-\infty$



3 Switch in position A:



After a long time: $i_L(0-) = 0$ (capacitor is open)

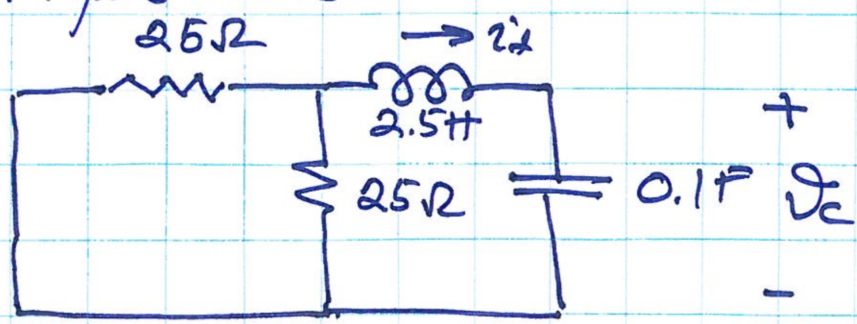
$V_C(0-) = \frac{10}{25+25} \times 25$

voltage across the 25 Ω resistor

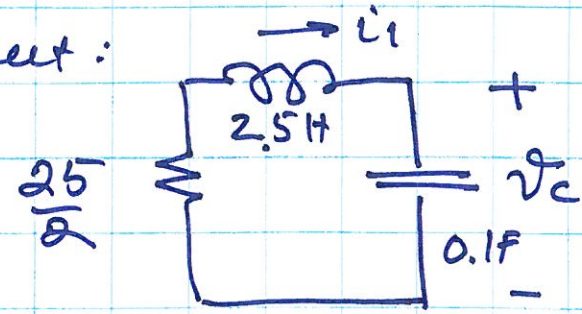
Hence: $i_L(0+) = 0$ A

$V_C(0+) = 5$ V

Switch in position B:



Equivalent circuit:



This is a second order circuit (series RLC connection)

KVL:

$$\frac{25}{2}\times i + v_L + v_C = 0$$

$$v_L = L\frac{di}{dt} \qquad ; v_L = 2.5\times\frac{di}{dt}$$

$$i = C\frac{dv_C}{dt} \qquad i = 0.1\cdot\frac{dv_C}{dt}$$

Hence: $\frac{25}{2}\times 0.1\cdot\frac{dv_C}{dt} + 2.5\cdot\frac{di}{dt} + v_C = 0$

$$i = 0.1\cdot\frac{dv_C}{dt}$$

OR:

$$\left.\begin{aligned} &1.25\cdot\frac{dv_C}{dt} + 2.5\cdot\frac{di}{dt} + v_C = 0 \\ &0.1\frac{dv_C}{dt} - i = 0 \end{aligned}\right\}$$

Substitute $i = 0.1\cdot\frac{dv_C}{dt}$ into the 1st equation:

$$1.25\cdot\frac{dv_C}{dt} + 2.5\cdot\left(0.1\cdot\frac{d^2v_C}{dt^2}\right) + v_C = 0$$

$$0.25\cdot\frac{d^2v_C}{dt^2} + 1.25\cdot\frac{dv_C}{dt} + v_C = 0$$

Characteristic equation:

$$0.25s^2 + 1.25s + 1 = 0 \quad /\cdot 4$$

$$s^2 + 5s + 4 = 0 \qquad s_1 = -1$$

$$(s+1)(s+4) = 0 \qquad s_2 = -4$$

$$v_C(t) = k_1e^{-t} + k_2e^{-4t}$$

Note: $v_{Cp}(t) = 0$ (no external sources)

$$i(t) = k_3e^{-t} + k_4e^{-4t}$$

Find constants:

$$v_C(0)=5$$
$$i_L(0+)=0$$

Hence: $k_1+k_2=5$
$k_3+k_4=0$

Note: $\frac{dv_c}{dt}=k_1(-1)e^{-t}+k_2(-4)e^{-4t}$

$\frac{di}{dt}=k_3(-1)e^{-t}+k_4(-4)e^{-4t}$

at $t=0+$: $\left.\frac{dv_c}{dt}\right|_{0+}=-k_1-4k_2$

$\left.\frac{di}{dt}\right|_{0+}=-k_3-4k_4$

From the differential equation:

$$\left.\begin{aligned}&1.25\cdot\frac{dv_c}{dt}+2.5\cdot\frac{di}{dt}+V_c=0\\&0.1\cdot\frac{dv_c}{dt}-i=0\end{aligned}\right\}$$

At $t=0+$:

$$1.25\cdot(-k_1-4k_2)+2.5\cdot(-k_3-4k_4)+5=0$$
$$0.1\cdot(-k_1-4k_2)-0=0$$

or:

$$1.25(k_1+4k_2)+2.5\cdot(k_3+4k_4)=5$$
$$-k_1-4k_2=0$$

Equations to be solved:

$$k_1 + k_2 = 5$$
$$k_3 + k_4 = 0$$
$$1.25 \cdot (k_1 + 4k_2) + 2.5(k_3 + 4k_4) = 5$$
$$k_1 + 4k_2 = 0$$

From the 1st and 4th equation:

$$\begin{array}{l} k_1 + k_2 = 5 \\ k_1 + 4k_2 = 0 \end{array} \Rightarrow k_2 - 4k_2 = 5$$

$$k_2 = -\frac{5}{3}$$

$$k_1 = \frac{20}{3}$$

Also: $k_3 + k_4 = 0$

$$1.25 \cdot (k_1 + 4k_2) + 2.5 \cdot (k_3 + 4k_4) = 5$$

note from the previous calculation that $k_1 + 4k_2 = 0$

Hence:

$$\begin{array}{l} k_3 + k_4 = 0 \\ 2.5 \cdot (k_3 + 4k_4) = 5 \end{array} \quad \text{or} \quad \begin{array}{l} k_3 + k_4 = 0 \\ k_3 + 4k_4 = 2 \end{array}$$

$$3k_4 = 2; \; k_4 = \frac{2}{3}$$

$$k_3 = -\frac{2}{3}$$

Finally: $v_C(t) = \left(\frac{20}{3} e^{-t} - \frac{5}{3} e^{-4t}\right) u(t)$

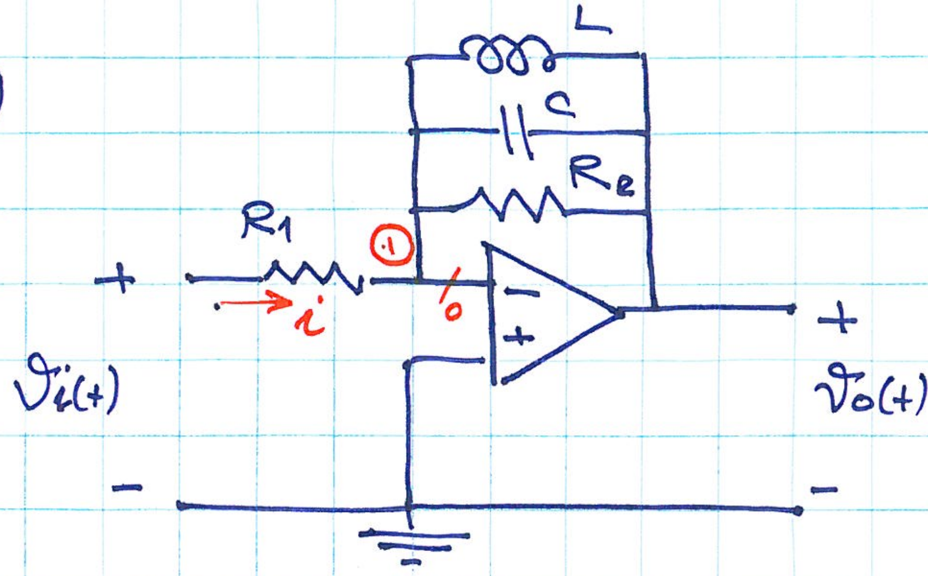
$$i(t) = \left(-\frac{2}{3} e^{-t} + \frac{2}{3} e^{-4t}\right) u(t)$$

At $t \to \infty$

$v_C(t) = 0$

$i_L(t) = 0$ as expected.

4



The op-amp is ideal. Hence:

$$v_1 = 0$$

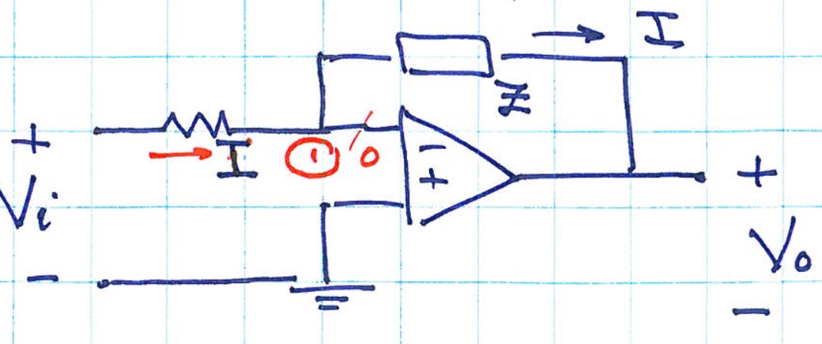
Phasor: $V_1 = 0$

$$V_i - R_1 \cdot I - V_1 = 0$$

or

$$\underline{I} = \frac{\underline{V_i}}{R_1}$$

$\underline{V_i}$ = phasor of $v_i(t)$



$$Z = \frac{1}{Y} \; ; \; Y = \frac{1}{j\omega L} + j\omega C + \frac{1}{R_2}$$

(admittance of three parallel branches)

$$V_o + Z \cdot I = 0$$

$$V_o = -Z \cdot \frac{V_i}{R_1}$$

$$V_o = -\frac{1}{Y} \cdot \frac{V_i}{R_1}$$

$$V_o = -\frac{1}{\frac{1}{j\omega L} + j\omega C + \frac{1}{R_2}} \times \frac{1}{R_1} \times V_i$$

$$V_o = -\frac{j\omega L \times R_2}{R_2 - R_2 \cdot \omega^2 LC + j\omega L} \times \frac{1}{R_1} \cdot V_i$$

$$V_o = -\frac{R_2}{R_1} \cdot \frac{j\omega L}{R_2(1-\omega^2 LC) + j\omega L} \cdot V_i$$

Ratio: $$\frac{V_o}{V_i} = -\frac{R_2}{R_1} \cdot \frac{j\omega L}{R_2(1-\omega^2 LC) + j\omega L}$$

if $\omega^2 LC = 1$ or $\omega = 1/\sqrt{LC}$

$$\frac{V_o}{V_i} = -\frac{R_2}{R_1} \cdot \frac{j\omega L}{j\omega L} \quad \text{or} \quad \frac{V_o}{V_i} = -\frac{R_2}{R_1}$$

Then: $V_o = -\frac{R_2}{R_1} \cdot V_i$ phasors

or

$v_o(t) = -\frac{R_2}{R_1} \cdot v_i(t)$ Time domain