Math 818, Fall 2024, Dr. Honigs
Homework 0
Due Fri. Sept. 6

Instructions: You are encouraged to work in groups, but your final written solutions must be in your own words. At the top of your paper, write down the names of anyone you have worked with on the problem set.

Complete the following textbook exercises and questions.

Exercises:

In Gathmann's "Commutative Algebra" course notes: **1.9, 1.13, 1.19, 2.9, 2.23**

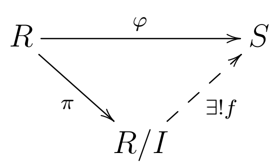
Possible hint for 1.9: Let $I := (x_1, \ldots, x_n)$. Consider I/I^2 as a vector space.

Questions:

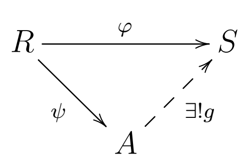
1. Assume all rings are commutative.

   Let R be a ring and I an ideal of R. Let $\pi : R \to R/I$ be the quotient ring homomorphism. The universal property of the quotient ring is the following:

   For any ring homomorphism $\varphi : R \to S$ such that $\varphi(x) = 0$ for all $x \in I$, there exists a unique homomorphism $f : R/I \to S$ so that $f \circ \pi = \varphi$. That is, the following diagram commutes:



   (a) Let $\psi : R \to A$ be a ring homomorphism so that $\psi(x) = 0 \; \forall x \in I$ and suppose that ψ satisfies the same universal property as above. That is, for any ring homomorphism $\varphi : R \to S$ such that $\varphi(x) = 0 \; \forall x \in I$, there exists a unique homomorphism $g : A \to S$ so that $g \circ \psi = \varphi$:



   Show that $A \cong R/I$.

   (b) Use the universal property of quotients to complete Exercise 1.22 in Gathmann's "Commutative Algebra" notes.