Order conditions for general linear methods

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Abstract

We describe the derivation of order conditions, without restrictions on stage order, for general linear methods for ordinary differential equations. This derivation is based on the extension of Albrecht approach proposed in the context of Runge-Kutta and composite and linear cyclic methods. This approach was generalized by Jackiewicz and Tracogna to two-step Runge-Kutta methods, by Jackiewicz and Vermiglio to general linear methods with external stages of different orders, and by Garrappa to some classes of Runge-Kutta methods for Volterra integral equations with weakly singular kernels. This leads to general order conditions for many special cases of general linear methods such as diagonally implicit multistage integration methods, Nordsieck methods, and general linear methods with inherent Runge-Kutta stability. Examples of high order methods with some desirable stability properties are also presented.

Keywords: General linear methods; order conditions; Nordsieck methods; two-step Runge-Kutta formulas

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