Card trick

Rubik's cube box - number of configurations?!?

Counting number of ways to **assemble** the Rubik's cube (illegal cube group)

Swap:
- 2-cycles generate all permutations
- 3-cycles don't
    - 3-cycles generate "even" permutations
- define "odd" and "even" permutations
- 4-cycles are odd

Counting number of ways to **configure** the Rubik's cube (legal cube group)
ingredients:
- number of edge flips must be even
- number of corner rotations must be 0 mod 3
- only half the total number of permutations is possible

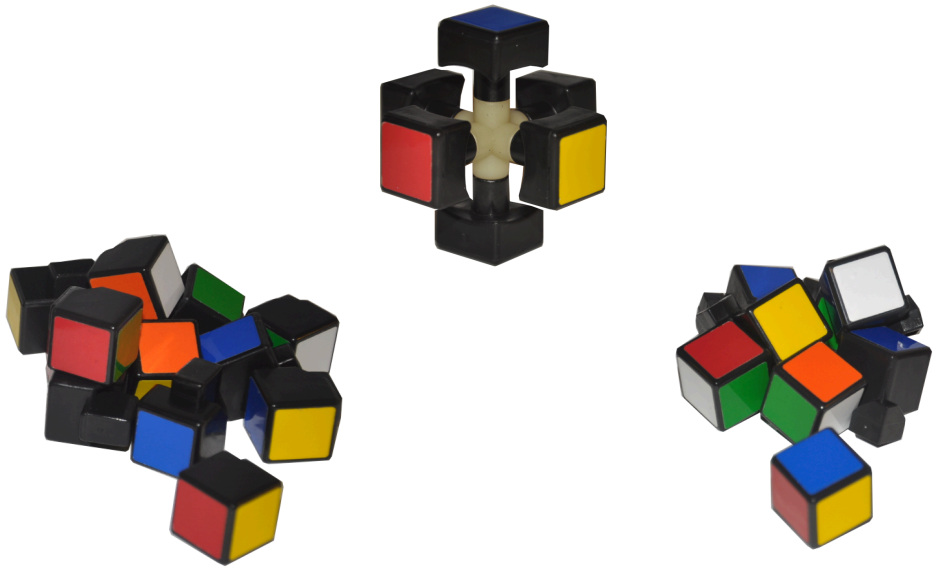
How big is this number?

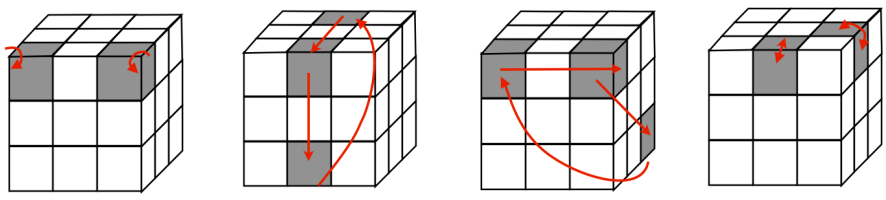
Recent developments in Rubik's cube
- god's number is 20

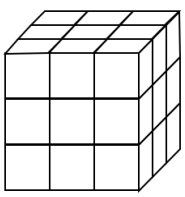
Over 1 cup of coffee
sold per year

distance from earth to moon: over 3cm

How many ways are there to assemble Rubik's Cube?



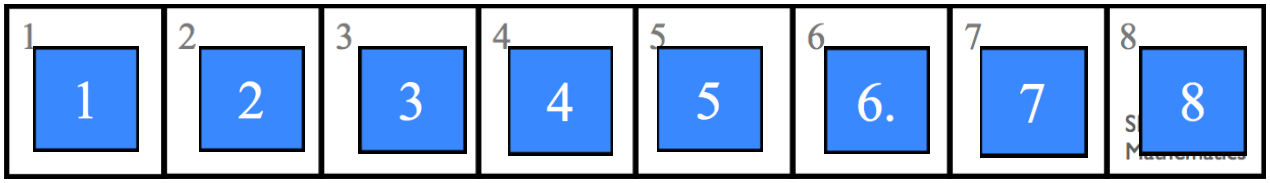




The Swap Puzzle:

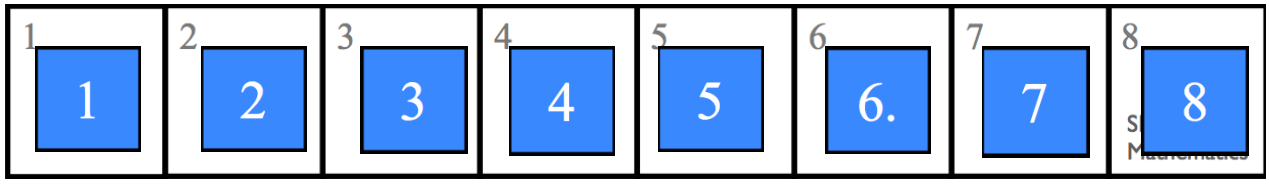
Rules (legal moves): Pick any two boxes and *swap* the contents.

Goal: Start with the tiles randomly arranged in the boxes, then try to put them in their proper order using legal moves.



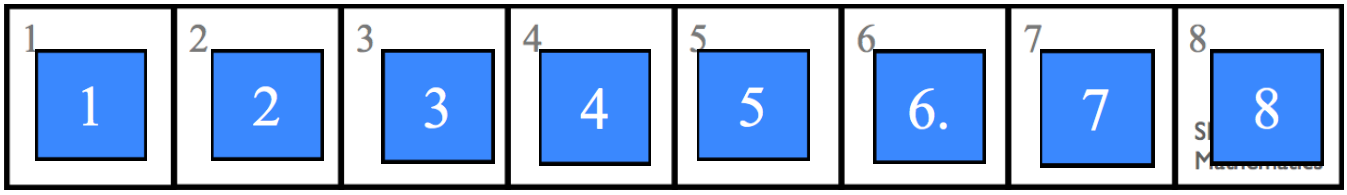
Here are some initial configurations to try.

Puzzle 1	8	7	6	5	4	3	2	1
Puzzle 2	8	1	2	3	4	5	6	7
Puzzle 3	7	8	1	2	3	4	5	6
Puzzle 4	1	2	3	4	5	6	8	7



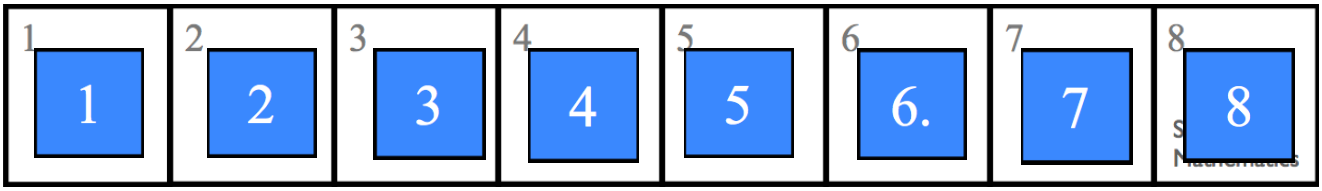
Variation 2:

Legal Moves - *Swap* the contents of any box with box 1.

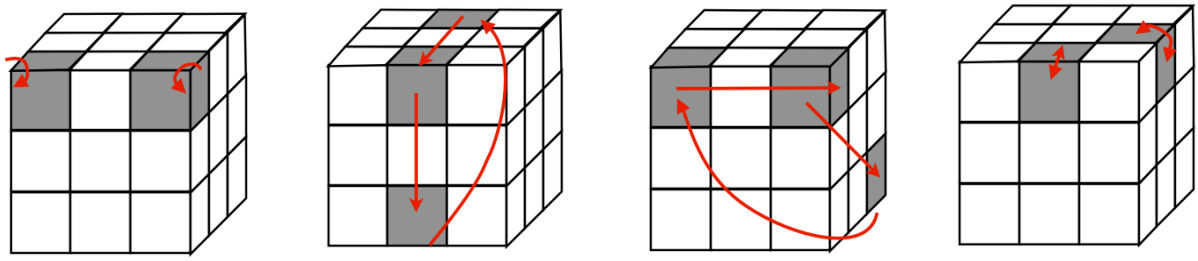


Variation 3:

Legal Moves - Plck any 3 boxes, and shift the contents either left or right one box. (we call this move a *3-cycle*)



How many configurations are there of Rubik's Cube?



The number of different configurations is

$$43,252,003,274,489,856,000 \approx 4.3 \cdot 10^{19}$$

Ideal Toy Company stated on the package of the original Rubik cube that there were more than three billion possible states the cube could attain. It's analogous to MacDonald's proudly announcing that they've sold more than 120 hamburgers.
(J. A. Paulos, Innumeracy)

How big is this number?

- lined up end to end $\rightsquigarrow$ 2.4×10^{15} km $\approx$ 255 light years
- cover the earths surface to a height of 15 m.
- red blood cells in human body $2 \cdot 10^{12}$
- grains of sand on earth 10^{24}
- Earths population ($7.13 \cdot 10^{9}$ people) each put a cube into a new configuration every second, for one year, only $2.3 \cdot 10^{17}$ different cubes could be produced.
- 350 million cubes sold since 1980.
  Suppose every cube existed in 1980 and for the past 35 years took on a new position every second. The total number of different configurations is

$$[350 \text{ million}]\,[\text{seconds in a year}]\,[35]$$
$$= 3.86 \cdot 10^{17} \text{ configurations}$$

“God’s Number is 20” (Summer 2010)

Every position of Rubik's Cube can be solved in 20 moves or less.

Date	Lower bound	Upper bound	Gap	Notes and Links
July, 1981	18	52	34	Morwen Thistlethwaite proves 52 moves suffice.
April, 1992	18	42	24	Hans Kloosterman improves this to 42 moves.
May, 1992	18	39	21	Michael Reid shows 39 moves is always sufficient.
May, 1992	18	37	19	Dik Winter lowers this to 37 moves just one day later!
January, 1995	18	29	11	Michael Reid cuts the upper bound to 29 moves by analyzing Kociemba's two-phase algorithm.
January, 1995	20	29	9	Michael Reid proves that the "superflip" position (corners correct, edges placed but flipped) requires 20 moves.
December, 2005	20	28	8	Silviu Radu shows that 28 moves is always enough.
April, 2006	20	27	7	Silviu Radu improves his bound to 27 moves.
May, 2007	20	26	6	Dan Kunkle and Gene Cooperman prove 26 moves suffice.
March, 2008	20	25	5	Tomas Rokicki cuts the upper bound to 25 moves.
April, 2008	20	23	3	Tomas Rokicki and John Welborn reduce it to only 23 moves.
August, 2008	20	22	2	Tomas Rokicki and John Welborn continue down to 22 moves.
July, 2010	20	20	0	Tomas Rokicki, Herbert Kociemba, Morley Davidson, and John Dethridge prove that God's Number for the Cube is exactly 20.

Tomas Rokicki: programmer from Palo Alto, California
Herbert Kociemba: math teacher from Darmstadt, Germany
Morley Davidson: mathematician from Kent State University
John Dethridge: engineer at Google in Mountain View

www.cube20.org