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MARGINAL UTILITY AND MRS (detailed notes)

Knowing about utility, a natural question is by how much a consumer's utility would increase if she consumes one more unit of some good. This increment in utility is called *marginal utility*.

Definition: Marginal Utility (MU) - the change in utility associated with a small change in the amount of one of the goods consumed holding the quantity of the other good fixed.

There are two important things above:

1. First, notice that marginal utility measures the rate of change in utility when we vary the quantity of a good consumed. Thus it basically measures the "slope" of the utility function with respect to changes in this good. However the utility function has two arguments so there will be two "slopes" i.e. when we talk about marginal utility we should always specify with respect to which good.
2. Notice that the quantity of one of the goods is always held constant when computing the marginal utility with respect to the other.

Given the above definition we can write the marginal utility with respect to good 1 (MU_1) as the ratio:

$$MU_1 = \frac{\triangle U}{\triangle x_1} = \frac{u(x_1 + \triangle x_1, x_2) - u(x_1, x_2)}{\triangle x_1} \tag{1}$$

that measures the rate of change in utility ($\triangle U$) associated with a small change in the amount of good 1 ($\triangle x_1$). Thus to calculate the change in utility resulting from a change in consumption in good 1 we should just multiply the marginal utility (MU_1) by the change in consumption:

$$\triangle U = MU_1 \triangle x_1 \tag{2}$$

the marginal utility thus measures the rate at which consumption units are converted into utility units, i.e. the "price" of one more unit of consumption in terms of utility units.

Marginal utility is even easier to understand using simple calculus - notice that the above expression (1) is exactly the *partial derivative* of u with respect to x_1 when we let the change $\triangle x_1$ go to zero i.e. become infinitesimally small. Thus more formally we can write:

$$MU_1 = \frac{\partial u(x_1, x_2)}{\partial x_1} \text{ and } MU_2 = \frac{\partial u(x_1, x_2)}{\partial x_2}$$

Remember what partial derivatives are: you differentiate the function with respect to one of the variables holding the other fixed (i.e. treating it as a constant).

Some examples of marginal utilities:

1. perfect substitutes (the blue/red pencil example) : $u(x_1, x_2) = x_1 + x_2$.

since utility is just the total number of pencils you have, one more pencil increase your utility by exactly 1 - thus we must have $MU_1 = MU_2 = 1$. This is very easy to verify using the partial derivative definition.

2. Cobb-Douglas: $u(x_1, x_2) = x_1^c x_2^d$. It is hard to figure out the marginal utilities without derivatives. But with little calculus we have:

$$MU_1 = \frac{\partial x_1^c x_2^d}{\partial x_1} = x_2^d c x_1^{c-1}$$

notice that x_2^d is treated like a constant. Similarly,

$$MU_2 = \frac{\partial x_1^c x_2^d}{\partial x_2} = x_1^c d x_2^{d-1}$$

Marginal Utility and the MRS

We see from the above dervations that the marginal utility depends on the actual form of the utility function chosen to represent the preferences. Thus if we take a monotonic transformation of the utility function this will affect the marginal utility as well - i.e. by looking at the value of the marginal utility we cannot make any conclusions about behavior, about how people make choices. However this doesn't mean that marginal utility is useless - even if each of MU_1 and MU_2 cannot describe behavior it turns out that their ratio can.

To see that suppose that we have a consumer who is at some bundle (x_1, x_2) and we consider a change in this bundle $(\triangle x_1, \triangle x_2)$, i.e. a move to a point $(x_1 + \triangle x_1, x_2 - \triangle x_2)$ such that the consumer is kept at the same indifferent curve - i.e. at the same utility level. To be kept at the same utility level we must have that the change in utility resulting from the increase of good 1 is exactly offset by the decrease of utility resulting from the decrease in good 2. Thus we must have:

$$\triangle U_{good1} + \triangle U_{good2} = 0$$

or,

$$MU_1 \triangle x_1 + MU_2 \triangle x_2 = 0$$

which is equivalent to:

$$MRS = \frac{\triangle x_2}{\triangle x_1} = -\frac{MU_1}{MU_2}$$

But what is the left hand side of the above inequality (the rate of change at which you're willing to substitute good 1 for good 2) - it is the MRS! Thus we obtain that

The marginal rate of substitution is equal to the ratio of the marginal utilities with a minus sign.

Thus even though the marginal utilities have no behavioral content their ratio does - it measures the rate at which a consumer is willing to substitute between the two goods.

Calculus derivation (optional)

The above relationship between the MUs and the MRS can be derived also using calculus. Remember that the MRS is just the slope of the indifference curve. But an indifference curve is just a relationship giving us a value of x_2 as a function of x_1 (draw an IC in the x_1,x_2 plane to convince yourself of this). So we can write an IC as the function $x_2(x_1)$ - for every quantity x_1 we know the corresponding value of x_2 such that the bundle (x_1, x_2) belongs to the given IC. This is just a notation - you can call $x_2=f(x_1)$ if you want.

The slope of the IC then would be just the first derivative of this function:

$$MRS = slope\ of\ IC = \frac{dx_2(x_1)}{dx_1}$$

Now consider the definition of the indifference curve - a set of points that have the same utility, e.g. the set of points that satisfy

$$u(x_1, x_2) = k$$

Thus the function $x_2(x_1)$ if representing the indifference curve must satisfy:

$$u(x_1, x_2(x_1)) = k$$

for any x_1. Differentiate the above with respect to x_1 using the chain rule:

$$\frac{\partial u(x_1, x_2(x_1))}{\partial x_1} + \frac{\partial u(x_1, x_2(x_1))}{\partial x_2}\frac{dx_2(x_1)}{dx_1} = 0$$

Above we used that u is first a direct function of x_1 and then a function of x_1 through x_2. From above we get:

$$MRS = \frac{dx_2(x_1)}{dx_1} = -\frac{\frac{\partial u(x_1,x_2(x_1))}{\partial x_1}}{\frac{\partial u(x_1,x_2(x_1))}{\partial x_2}} = -\frac{MU_1}{MU_2}$$

which is what we had before.