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UNIT 11: ENERGY CONSERVATION

In order to understand the equivalence of mass and energy, we must go back to two conservation principles which ... held a high place in pre-relativity physics. These were the principle of the conservation of energy and the principle of the conservation of mass.

Albert Einstein

OBJECTIVES

1. To understand the concepts of potential energy and kinetic energy.

2. To investigate the conditions under which mechanical energy is conserved.

3. To relate conservative and non-conservative forces to the net work done by a force when an object moves in a closed loop.

OVERVIEW

The last unit on work and energy culminated with a mathematical proof of the work-energy theorem for a mass falling under the influence of the force of gravity. We found that when a mass starts from rest and falls a distance y, its final velocity can be related to y by the familiar kinematic equation

$$v_f^2 = v_i^2 + 2gy$$

or

$$gy = \frac{1}{2}\left(v_f^2 - v_i^2\right)$$

[Equation 11-1]

where v_f is the final velocity and v_i is the initial velocity of the mass.

We believe this equation is valid because: (1) you have derived the kinematic equations mathematically using the *definitions* of velocity and constant acceleration, and (2) you have verified experimentally that masses fall at a constant acceleration.

We then asked whether the transformation of the mass from a speed v_i to a speed v_f is related to the work done on the mass by the force of gravity as it falls.

The answer is mathematically simple. Since $F_g = mg$, the work done on the falling object by the force of gravity is given by

$$W_g = F_g\, y = mg\, y$$ [Equation 11-2]

But according to Equation 11-1,
$gy = (1/2)\, v_f^2 - (1/2)\, v_i^2$,so we can re-write equation Equation 11-2 as

$$W_g = mgy = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

[Equation 11-3]

The $(1/2)\, mv_f^2$ is a measure of the motion resulting from the fall. If we *define* it as the energy of motion, or, more succinctly, the *kinetic energy*, we can define a work-energy theorem for falling objects:

$$W = \Delta E_{kin} = E_{kin,f} - E_{kin,i}$$ [Equation 11-4]

or, the work done on a falling object by the earth is equal to the change in its kinetic energy as calculated by the difference between the final and initial kinetic energies.

If external work is done on the mass to raise it through a height y (a fancy phrase meaning "if someone picks up the mass"), it now has the potential to fall back through the distance y, gaining kinetic energy as it falls. Aha! Suppose we define *potential energy* to be *the amount of external work,* W_{ext}, *needed to move a mass at constant velocity through a distance y against the force of gravity*. Since this amount of work is positive while the work done by the gravitational force has the same magnitude but is negative, this definition can be expressed mathematically as

$$E_{pot} = W_{ext} = -W_g = -mgy \qquad \text{[Equation 11-5]}$$

Note that when the potential energy is a maximum, the falling mass has no kinetic energy. As it falls, the potential energy becomes smaller and smaller as the kinetic energy increases. The kinetic and potential energy are considered to be two different forms of mechanical energy. What about the *total mechanical energy*, consisting of the sum of these two energies? Is the total mechanical energy constant during the time the object falls? If it is, we might be able to hypothesize a law of conservation of mechanical energy as follows: *In some systems, the sum,* E_{tot}*, of the kinetic and potential energy is a constant.* This hypothesis can be summarized mathematically by the following statement.

$$E_{tot} = E_{kin} + E_{pot} = \text{constant} \qquad \text{[Equation 11-6]}$$

The idea of mechanical energy conservation raises a number of questions. Does it hold quantitatively for falling masses? How about for masses experiencing other forces, like those exerted by a spring? Can we develop an equivalent definition of potential energy for the mass-spring system and other systems and re-introduce the hypothesis of conservation of mechanical energy for those systems? Is mechanical energy conserved for masses experiencing frictional forces, like those encountered in sliding?

In the first session you will explore whether or not the mechanical energy conservation hypothesis is valid for a falling mass. You will then spend the second session investigating energy conservation for a sliding mass. During the second session you will also explore the validity of the definition of conservative forces as those that do no work when an object moves in a closed path.

SESSION ONE: CONSERVATION OF MECHANICAL ENERGY

Is Mechanical Energy Conserved for a Falling Mass?
Is the mechanical energy conservation hypothesis stated above valid for a falling mass? In other words, is mechanical energy conserved within the limits of uncertainty? In Unit 6 you recorded data for the vertical position of a ball that was tossed in the laboratory as a function of time. This should give you the information you need to calculate the position and average velocity of the ball at each time. You can then calculate the kinetic and potential energy for the tossed ball in each frame and see if there is any relationship between them.

✍ **Activity 11-1: Mechanical Energy for a Falling Mass**
Before you actually do the calculations using data, let's do a little predicting:

(a) Where is E_{pot} a maximum? a minimum?

E_{pot} is a max at the greatest height. Its is a min at the lowest height.

(b) Where is E_{kin} a maximum? a minimum?

E_{kin} is a maximum where the speed is the greatest which is the lowest height. Its a min when at the max height where the speed is a minimum.

(c) If mechanical energy is conserved what should the sum of $E_{kin}+ E_{pot}$ be for any point along the path of a falling mass?

It should be constant, equal to the total mechanical energy.

Now you can proceed to use data for a ball being tossed like the one you observed at the beginning of Unit 6 to test our mechanical energy conservation hypothesis as follows:

Set up a spreadsheet with columns for the basic data including elapsed time (t) and vertical position (y). In other columns calculate the average velocity at each time (v), the kinetic energy (E_{kin}), and the potential energy (E_{pot}) at each time. In the last column, calculate the total mechanical energy $E_{tot}= (E_{kin}+ E_{pot})$ at each time.

To find the average velocity at a given time you should take the change in position from the time just before and the time just after the time of interest in a kind of leap frog game as shown in the diagram below.

	Vertical Toss Data			Mass of Ball = .321 kg			
	y(m)						
	t(s)		y(m)	v(m/s)	E kin	E pot	E Total
to	0.000	yo	2.134	no value available			
t1	0.033	y1	2.188	.=(y2-y0)/(t2-t0)			
t2	0.067	y2	2.235	.=(y3-y1)/(t3-t1)			
t3	0.100	y3	2.262	.=(y4-y2)/(t4-t2)			
t4	0.133	y4	2.289	.=(y5-y3)/(t5-t3)			
t5	0.167	y5	2.295	no value available			

> You need to play leap frog to find the average velocity at time t1 from the positions at times t2 and t0.

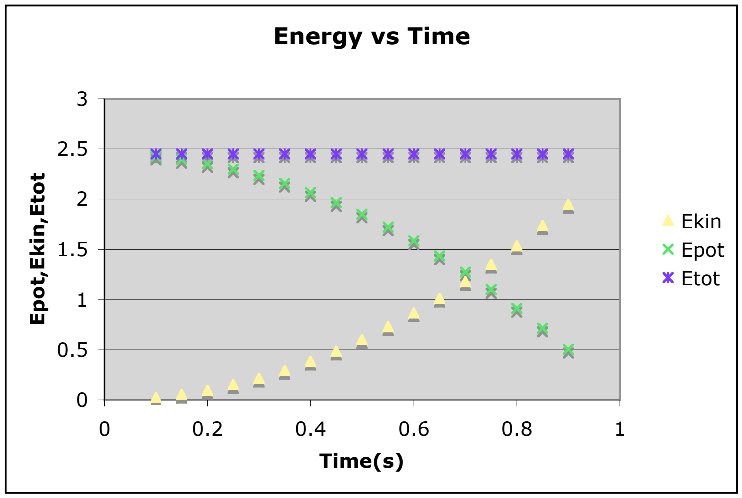
Figure 11-1: A schematic with suggestions about how to find the velocity at each time. Suppose there were only 5 data points. Then no average velocity can be calculated directly for the first and the last frames using position data from adjoining frames. The average velocity at the time of the second frame (t1=0.033 s) is given by

$$\langle v_1 \rangle = \frac{y_2 - y_0}{t_2 - t_0} = \frac{2.235 - 2.134}{0.067 - 0.033}$$

✍ Activity 11-2: Energy Analysis for a Falling Mass

(a) Submit your spreadsheet with the requested data, along with the calculations for E_{kin}, E_{pot} and E_{total} to WebCT.

(b) Create an overlay plot of E_{kin}, E_{pot} and E_{tot}, as a function of time for all but the first and the last times and sketch the graph in the space below.



(c) Do the maximum and minimum values for each agree with what you predicted?

Yes

(d) Is the sum of E_{kin} and E_{pot} what you predicted?

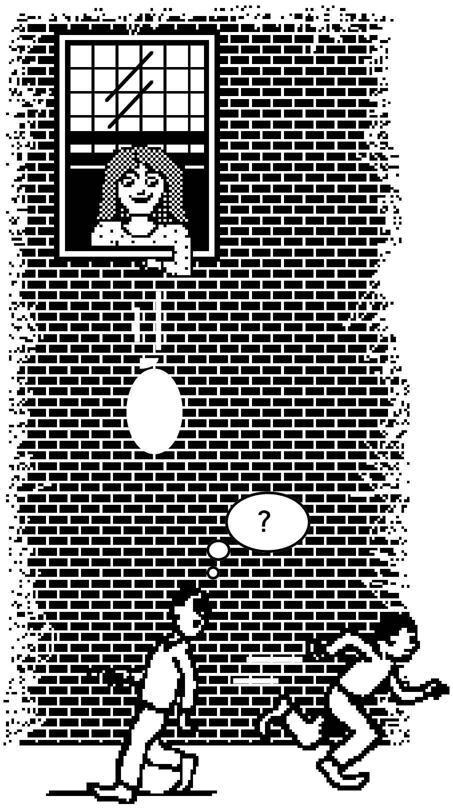
Yes

Mechanical Energy Conservation

How do people in different reference frames near the surface of the Earth view the same event with regard to mechanical energy associated with a mass and its conservation? Suppose the president of some college drops a 2.0 kg water balloon from the second floor of the administration building (10.0 metres above the ground). The president takes the origin of his or her vertical axis to be even with the level of the second floor. A student standing on the ground below considers the origin of his co-ordinate system to be at ground level. Have a discussion with your classmates and try your hand at answering the questions below.

✍ **Activity 11-3: Mechanical Energy and Co-ordinate Systems**

(a) What is the value of the potential energy of the balloon before and after it is dropped according to the president?



According to the student? Show your calculations and don't forget to include units!

The president's perspective is that y = 0.0 m at t = 0 s and that y = -10.0 m when the balloon hits the student) :

$E_{pot,i}$ = 0

$E_{pot,f}$ = (2.0kg)(9.8m/s^2)(-10.0m)= -196 J

The student's perspective is that y = 10.0 m at t = 0 s and that y = 0.0 m when the balloon hits) :

$E_{pot,i}$ = 196 J

$E_{pot,f}$= 0

Note: If you get the same potential energy value for the student and the president you are on the wrong track!

(b) What is the value of the kinetic energy of the balloon before and after it is dropped (but before it hits the ground) according to the president? According to the student? Show your calculations. **Hint**: Use a kinematic equation to find the velocity of the balloon at ground level.

President's perspective: $E_{kin,i}$= 0

v = sqrt(2(-9.8m/s^2)(-10.0m))= - 14m/s

$E_{kin,f}$ = 1/2(2.0kg)(-14m/s)^2= 196 J

Student's perspective: $E_{kin,i}$ = 0

$E_{kin,f}$ = 196 J

Note: If you get the same values for both the student and the president for values of the kinetic energies you are on the *right* track!

(c) What is the value of the total mechanical energy of the balloon before and after it is dropped according to the president? According to the student? Show your calculations. **Note:** If you get the same values for both the student and the president for the total energies you are on the wrong track!!!!!

Presidents perspective: $E_{tot,i}$= 0 + 0 = 0

$E_{tot,f}$= -196 J + 196 J = 0

Student's perspective: $E_{tot,i}$ = 196 J + 0 = 196 J

$E_{tot,f}$ = 0 + 196 J = 196 J

(d) Why don't the two observers calculate the same values for the mechanical energy of the water balloon?

Because they set Epot = 0 at different heights.

(e) Why do the two observers agree that mechanical energy is conserved?

Because energy conservation does not depend on the reference point for Epot.

SESSION TWO: CONSERVATIVE AND NON-CONSERVATIVE FORCES

Is Mechanical Energy Conserved for a Sliding Object?

In the last session you should have found that mechanical energy is conserved for a freely falling object. Let's investigate whether mechanical energy is conserved when an object slides down an inclined plane in the presence of a friction force.

In this activity you will raise the incline just enough to allow the block to slide at a constant velocity. By measuring the velocity you can determine if mechanical energy is conserved.

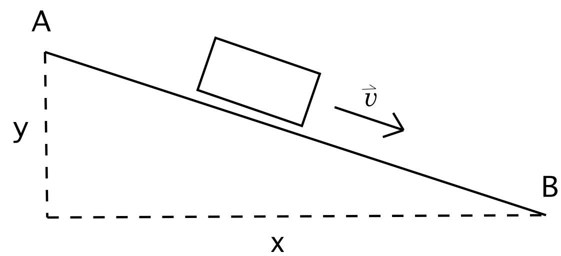


Figure 11-2: Sliding a block down an incline at a constant velocity

For this investigation you will need the following items:

- A block
- An adjustable inclined plane w/ a smooth surface
- A metre stick
- A balance
- A stop watch

✍ **Activity 11-4: Is Mechanical Energy Conserved for a Sliding Block?**

(a) Raise the incline until the block slides at a constant velocity once it is pushed gently to overcome the static friction force. What is the potential energy of the block at point A relative to the bottom of the ramp? Show your data and calculation.

For y = 0.30 m, m = 0.500 kg:

Epot = (0.500kg)(9.8m/s^2)(0.30m) = 1.47 J

(b) What is the velocity of the block as it travels down the incline? Show your data and calculation.

$v = \Delta x/\Delta t = (0.50\text{ m})/(0.40\text{s}) = 1.25\text{ m/s}$

(c) What is the kinetic energy of the block at the bottom of the incline? Does it change as the block slides?

Ekin = 1/2(0.500kg)(1.25 m/s)^2 = 0.391 J
It does not change because the speed is constant.

(d) What is the potential energy change, ΔE_{pot}, of the block when it reaches point B? Explain.

ΔE_{pot} *= Epotf - Epoti = 0 - 1.47 J = -1.47 J (it decreases)*

(e) Assuming the initial kinetic energy (just after your starter shove) is the same as the final kinetic energy, what is the kinetic energy change, ΔE_{kin}, of the block when it reaches point B?

$\Delta E_{kin} = 0$

(f) Is mechanical energy conserved as a result of the sliding? Cite the evidence for your answer.

No, its not conserved. The total mechanical energy changes value because the potential energy decreases, but the kinetic energy remains constant.

As you examine the activities you just completed you should be conclude that the conservation of mechanical energy will probably only hold in situations where there are no friction forces present.

Conservative and Non-Conservative Forces
Physicists have discovered that certain conservative forces like such as gravitational forces and spring forces do no total work on an object when it moves in a closed loop. Other forces involving friction are not conservative and hence the total work these forces do on an object moving in

a closed loop is not zero. In this next activity you will try to determine the validity of this assertion.

Figure 11-3: A diagram showing a closed loop

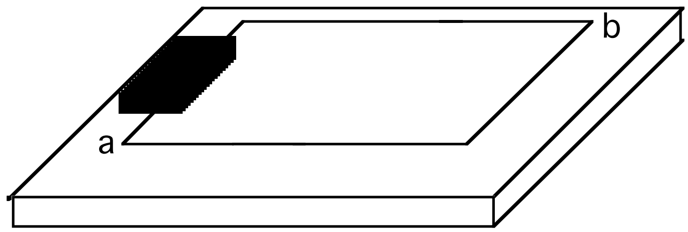
It is not hard to see that a gravitational force does no net work when an object moves at a constant speed through a complete round trip. In Figure 11-3, it takes negative external work to lower a mass from point a to point b, as the force of gravity takes care of the work. On the other hand, raising the mass from point b to point a requires positive external work to be done against the force of gravity, and thus the net work done by the gravitational force for the complete trip is zero.

When a friction force is present it always does net work on an object as it undergoes a round trip. For example, when a block slides from point a to point b on a horizontal surface, it takes work to overcome the friction force that opposes the motion. When the block slides back from point b to point a, the friction force *still opposes the motion of the block* so that net work is done.

Let's make this idea more concrete by sliding a block around a horizontal loop on your lab table in the presence of a friction force and computing the work it does. Then you can raise and lower the block around a similar vertical loop and calculate the work the gravitational force does.

✍ Activity 11-5: Are Friction Forces Conservative According to the Loop Rule?

(a) Use the spring scale and pull the wooden block around a rectangular path on your tabletop at constant velocity. Draw arrows along the path for the direction of motion and the direction of the force you exert on the block. List the measured forces and distances for each of the four segments of the path to calculate the work you do around the entire path a to b to a as shown.



(b) What is the change in E_{kin} as you progress through the loop?

$\Delta E_{kin} = 0$

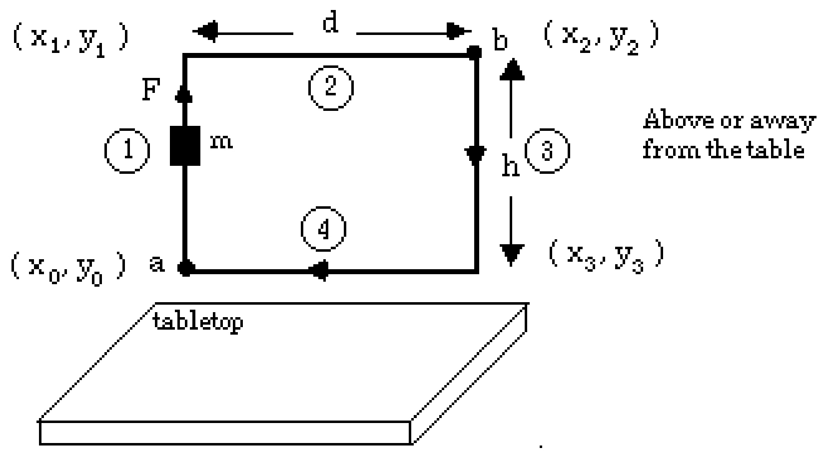
(c) Is the friction force conservative or non-conservative (i.e. is the total work done by the friction force zero or non-zero)? Explain.

The friction force is non-conservative. The total work done by the friction force is non-zero around a closed path.

(d) Is mechanical energy conserved? Explain.

Mechanical energy is conserved here because both the potential and kinetic energies remain constant. This is because the net work done by both friction and the pulling force combined is zero.

Let's return once again to our old friend the gravitational force and apply an external force to move the same block in a loop above the table so the block doesn't slide. *Be very, very careful to pay attention to the direction of the gravitational force relative to the direction of the motion.* Remember that work is the dot product of this force and the distance in each case so that the work done by the gravitational force when the block moves up and when it moves down are not the same. What happens to the work when the gravitational force is perpendicular to the direction of motion, as is the case in moving from left to right and then later from right to left?



✍ **Activity 11-6: Are G-Forces Conservative?**
(a) Use the spring scale and raise and lower the wooden block around a rectangular path above your tabletop at a constant speed without allowing it to slide at all. Draw arrows along the path for the direction of motion and the direction of the gravitational force exerted on the block. Use your measured distances, the measurement of the mass of the block and the dot product notation to calculate the work done by the gravitational force on the block over the entire path a to b to a as shown. *Be careful not to let the block slide on the table or rub against it.* Don't forget to specify the units! **Hints**: (1) In path 1 $\Delta x = x_1 - x_o$, $\Delta y = y_1 - y_o$ etc. (2) Keep track of the signs. For which paths is the work negative? Positive?

Path 1: $F_x = 0$ $\Delta x = 0$ $F_y =$ $\Delta y =$ $W_1 = F_x \Delta x + F_y \Delta y =$	Path 3: $F_x = 0$ $\Delta x = 0$ $F_y =$ $\Delta y =$ $W_3 = F_x \Delta x + F_y \Delta y =$
Path 2: $F_x = 0$ $\Delta x =$ $F_y =$ $\Delta y = 0$ $W_2 = F_x \Delta x + F_y \Delta y =$	Path 4: $F_x = 0$ $\Delta x =$ $F_y =$ $\Delta y = 0$ $W_4 = F_x \Delta x + F_y \Delta y =$

$$W_{net} = \sum_{i=1}^{4} W_i = W_1 + W_2 + W_3 + W_4 =$$

_______0__________

(c) Is the gravitational force conservative or non-conservative according to the loop rule? Explain.

The gravitational force is conservative. The total work done around a closed loop by the gravitational force alone is zero.

The Conservation of "Missing" Energy

We have seen in the case of the sliding block that the Law of Conservation of Mechanical Energy does not seem to hold for forces involving friction. The question is, where does the "missing" energy ΔE_{tot} go when friction forces are present? All the energy in the system might not be potential energy or kinetic energy. If we are clever and keep adding new kinds of energy to our collection, we might be able to salvage a Law of Conservation of Energy. If we can, this law has the potential to be much more general and powerful than the Law of Conservation of Mechanical Energy.

✍ Activity 11-7: What Happens to the Missing Energy?

(a) Rub the sliding block back and forth vigourously against your hand? What sensation do you feel?

It feels hot.

(b) How might this sensation account for the missing energy?

The energy is being converted into heat.

Physicists call the energy which is lost by a system as a result of work done against frictional forces *thermal energy*. This thermal energy may lead to an increase in the system's *internal energy*. Using the symbol ΔE_{int} to represent the change in internal energy of a system that experiences friction forces allows us to express the Law of Conservation of Total Energy mathematically with the expression

$$E_{tot} = E_{pot} + E_{kin} + \Delta E_{int} = \text{constant}$$
[Equation 11-7]

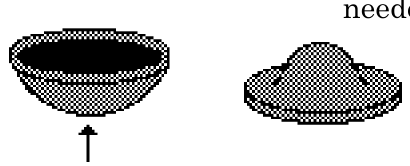
We are not prepared in this part of the course to consider the nature of internal energy or its actual measurement, so the Law of Conservation of Energy will for now remain an untested hypothesis. However, we will re-consider the concept of internal energy later in the course when we deal with heat and temperature.

Let's assume for the moment that the law of conservation of energy has been tested and is valid. We can have some fun applying it to the analysis of motion of a little device called the blue popper.

The Popper – Is Mechanical Energy Conserved?
A popular toy consists of part of a hollow rubber sphere that pops up when inverted and dropped. A home-made version of this toy can be made from the bottom 1/3rd of a racquetball. We call it the "popper". We would like you to use your knowledge of physics to determine whether or not mechanical energy is conserved as it "pops" up. Use a hard smooth floor or table top to make your observations. You will need the following items:

- A popper
- A balance
- A 2 metre stick
- A 10 kg scale
- A ruler

✍ **Activity 11-8: The Work Needed to Cock a Popper**
(a) Use some appropriate crude measurements to estimate the work needed to cock the popper. **Hint:** Use the 10 kg scale to measure the force needed to cock the popper. Over what distance does that force need to be applied? Try cocking the popper several times with your hands to determine this distance, as the maximum force is not needed over the whole distance.



(b) What is the work needed to cock the popper?

W = F Δx cosθ = (25 N)(0.01m) = 0.25 J

(c) Assuming mechanical energy is conserved determine the maximum speed of the popper as it "pops" up off the floor. **Hint:** You will need to measure the mass of your popper for this part.

0.25 J = 1/2(0.008kg)v^2   v = 7.91 m/s

(d) Calculate the highest distance the popper can possibly rise into the air in the presence of the gravitational force, if mechanical energy is conserved both during the "pop" itself and during the rise of the popper against the force of gravity.

Epot = mgh

0.25 J = (0.008kg)(9.8m/s^2)(h)   h = 3.2 m

or

v^2 =v_0^2 +2gh

0 = (7.91m/s)^2 + 2(-9.8 m/s^2)(h)   h = 3.2 m

(e) Measure the greatest height that the popper rises as a result of measurements from at least 5 good solid pops. Record your measurement below.

h = 1.7 m

(f) Is mechanical energy conserved as the cocked popper releases the energy stored in it? If not, assuming the law of conservation of energy holds, what is the internal energy gained by the popper during the cocking and popping process? Show your reasoning and calculations. **Hint:** You need to compare the work needed to cock the popper with the gravitational potential energy it gains after popping. Is anything lost?

No, its not conserved. It should go up to 3.2 m, but it only goes up to 1.7 m. The popper loses:

E = (0.008kg)(9.8m/s^2)(1.5m) = 0.12 J

This energy is lost to internal energy and probably also to heat generated by air resistance.

HOMEWORK AFTER SESSION ONE (MON)

Before Wednesday November 16th:

- Read Chapter 10 in the Textbook *Understanding Physics*
- *Work Textbook Problems 10-4, 10-10 & 10-15*
- *Work Supplemental Problem SP11-1 listed below*

SP11-1) [20 pts] **Frames of Reference:** Different observers can choose to use different co-ordinate systems. Some observers might be moving relative to others. Some simply choose a different origin. Let's simulate the case of the devilish college president dropping a balloon on a hapless student as in Activity 11-3. You can use the movie of the ball toss for this analysis. Simply pretend that (1) the falling ball is a water balloon of mass .321 kg, and (2) the president's floor is at the highest point the tossed ball reaches. You should do the following:

(1) Use Logger Pro to examine the movie.
(2) Calibrate assuming the meter stick is 2 m long.
(3) Set the origin at the president's floor level i.e. at the top of the toss. Take data for position vs. time and save it.
(4) Reset the origin at the student's floor level (i.e. the lab floor level) and recalculate the data with the new origin.
(5) Set up spreadsheets to calculate the total energy of the balloon (potential and kinetic in each case). Be sure to use the leap-frog method for determining the velocities. See Figure 11-1.

(a) What is the total mechanical energy that the president observes? Does the president see a constant total energy? (b) What is the total mechanical energy that the student observes? Does the student see a constant total energy? (d) Do they agree on the value of the total mechanical energy? (e) Do they agree that energy is conserved? Explain.

UNIT 11 HOMEWORK AFTER SESSION TWO (WED)

Before Friday November 18th:

- *Work Textbook Problems 10-46, 10-54 & 10-73*
- *Work Supplemental Problem SP11-2 listed below*

SP11-2) A 5.0-kg block travels on a rough, horizontal surface and collides with a spring. The speed of the block *just before* the collision is 3.0 m/s. As the block rebounds to the left with the spring uncompressed, its speed as it leaves the spring is 2.2 m/s. If the coefficient of kinetic friction between the block and surface is 0.3, determine (a) the work done by friction while the block is in contact with the spring and (b) the maximum distance the spring is compressed.

• *Complete Unit 11 entries in the Activity Guide*