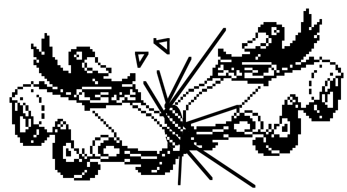
Name ______________________ Date(YY/MM/DD) ______/_________/_______
St.No. __ __ __ __ __-__ __ __ __ Section__________

UNIT 9: TWO-DIMENSIONAL COLLISIONS



It is difficult even to attach a precise meaning to the term "scientific truth." Thus, the meaning of the word "truth" varies according to whether we deal with a fact of experience, a mathematical proposition, or a scientific theory.
A. Einstein

OBJECTIVES

1. To explore the applicability of conservation of momentum to the mutual interactions among objects that experience no external forces (so that the system of objects is *isolated*).

2. To *calculate* momentum changes for an isolated system consisting of two very unequal masses and to *observe* momentum changes for a system consisting of two equal masses.

3. To devise a mathematical definition of the *centre of mass* of an isolated system so that the total momentum of the system (which we now know is constant) can be easily determined during interactions.

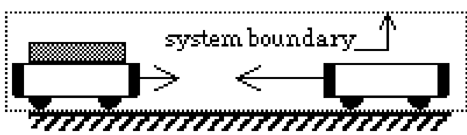
4. To understand why, by definition, the centre of mass of a system of interacting objects that experiences no outside forces will always move with a constant velocity if its momentum is conserved.

5. To learn how to find the centre of mass of extended objects (which are not just mathematical points).

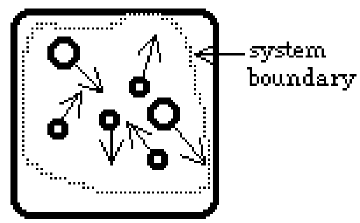
6. To use centre-of-mass concepts to verify experimentally that the Law of Conservation of Momentum holds for two-dimensional collisions in isolated systems.

OVERVIEW

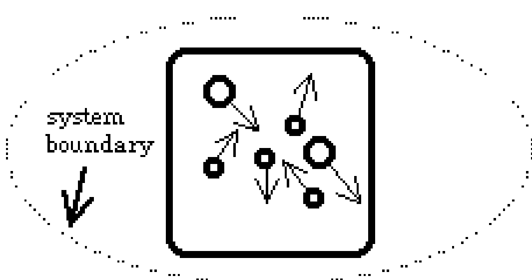
You have tested Newton's third law under different conditions in the last two units. It always seems to hold. The implications of that are profound, because whenever an object experiences a force, another entity must also be experiencing a force of the same magnitude. A single force is only half of an interaction. Whenever there are interactions between two or more objects, it is often possible to draw a boundary around a system of objects and say there is no net *external* force on it. A closed system with no external forces on it is known as an *isolated* system. Some examples of isolated systems are shown in the diagrams below.



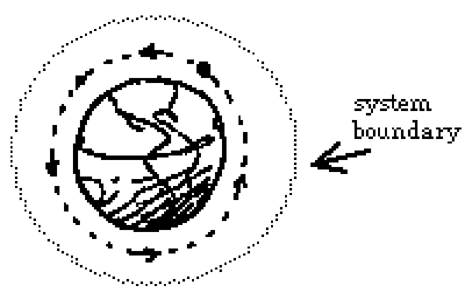
Carts with almost frictionless bearings interact. (Frictional forces from the track are considered negligible.)



Pucks riding on a cushion of air on an air table interact with each other before hitting the walls of the table. (Friction forces with the surface of the table are negligible.)



Gas molecules interact with each other and with the walls of their container. (Other forces, such as those of the table holding up the container and gravity are considered to have a negligible effect on the motions of the molecules and the container.)



An orbiting satellite and the Earth interact. (Forces on these objects due to other objects such as the sun and the moon are considered negligible.)

Figure 9-1: Examples of isolated systems in which the influence of outside forces is negligible.

As a consequence of Newton's laws, momentum is believed to be conserved in isolated systems. This means that, no matter how many internal interactions occur, the total momentum of each of the systems pictured above should remain constant. When one of the objects gains some momentum another part of the system must lose the same amount of momentum. If momentum doesn't seem to be conserved then we believe that there is an outside force acting on the system. Thus, by extending the boundary of the system to include the source of that force we can save our Law of Momentum Conservation. The ultimate isolated system is the whole universe. Most astrophysicists believe that momentum is conserved in the universe!

You will begin this unit by examining a situation in which it appears that momentum is not conserved and then seeing how the Law of Conservation of Momentum can hold when the whole isolated system is considered. In the next activity you will make qualitative observations using two carts of equal mass moving toward each other at the same speed. You will observe momentum changes for several types of interactions, including an elastic and inelastic collision and an explosion.

Next, a new quantity, called the *centre of mass* of a system, will be introduced as an alternative way to keep track of the momentum associated with a system or an extended body. You will use this concept to demonstrate that the Law of Conservation of Momentum holds for both one-dimensional and two-dimensional interactions in isolated systems. Several other attributes of the centre of mass of a system will be studied.

SESSION ONE: MOMENTUM CONSERVATION AND CENTRE OF MASS

Review of Homework Assignment

Come prepared to ask questions about the homework assignments on momentum conservation.

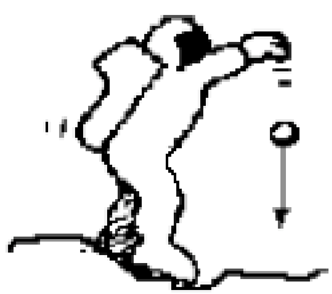
When an Irresistible Force Meets an Immovable Object

Let's assume that a superball and the moon (with an astronaut on it) are the objects in a closed system. (The pull of the Earth doesn't affect the falling ball, the astronaut, or the moon nearly as much as they affect each other.) Suppose that the astronaut drops the superball and it falls toward the moon so that it rebounds at the same speed it had just before it hit. If momentum is conserved in the interaction between the ball and the moon, can we notice the moon recoil?

✍ Activity 9-1: Wapping the Moon with a Superball

(a) Suppose a small ball is dropped and falls toward the surface of the moon so that it hits the ground and rebounds with the same speed. According to the Law of Conservation of Momentum, about how big is the velocity of recoil of the moon?

(b) Will the astronaut notice the jerk as the moon recoils from him? Why or why not?



(c) Consider the ball and the moon as an interacting system with no other outside forces. Why might the astronaut (who hasn't taken physics yet!) have the illusion that momentum isn't conserved in the interaction between the ball and the moon?

(d) Why might an introductory physics student here on Earth have the impression when throwing a ball against the floor or a wall that momentum isn't conserved?

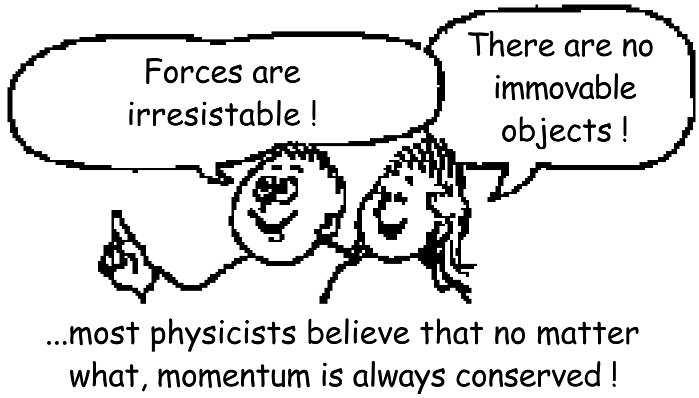


Figure 9-2: The moral of the moon and superball story.

Collisions with Equal Masses: What Do You Know?

Let's use momentum conservation to predict the results of some simple collisions. The diagrams below show objects of equal mass moving toward each other. If the track exerts negligible friction on them then the two cart system is isolated. Assume that the carts have opposite velocities so that $\vec{v}_{1,i} = -\vec{v}_{2,i}$ and observe what actually happens. You can use relatively frictionless carts with springs, magnets, and Velcro. You'll need:

- 2 dynamics carts with equal masses (outfitted with springs, magnets, and Velcro)
- A track for the carts

✍ Activity 9-2: Predictions of the Outcome of Collisions

(a) Sketch a predicted result of the *interaction* between two carts that bounce off each other so their speeds remain unchanged as a result of the collision. Use arrows to indicate the direction and magnitude of the velocity of each object *after* the collision.

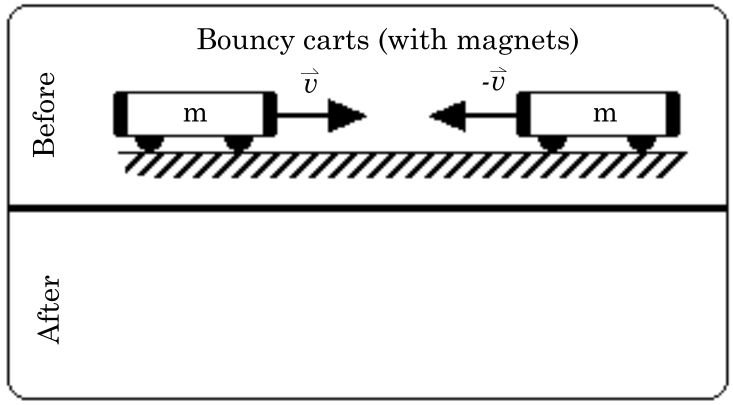


Figure 9-3: Elastic collision

(b) Observe a bouncy collision (also known as an *elastic collision*) and discuss whether or not the outcome was what you predicted it to be. If not, draw a new sketch with arrows indicating the magnitudes and directions of the velocities. What is the apparent relationship between the final velocities $\vec{v}_{1,f}$ and $\vec{v}_{2,f}$? How do their magnitudes compare to those of the initial velocities?

(c) Sketch the predicted result of the *interaction* between two objects that stick together. Use arrows to indicate the direction and magnitude of the velocity of each object *after* the collision.

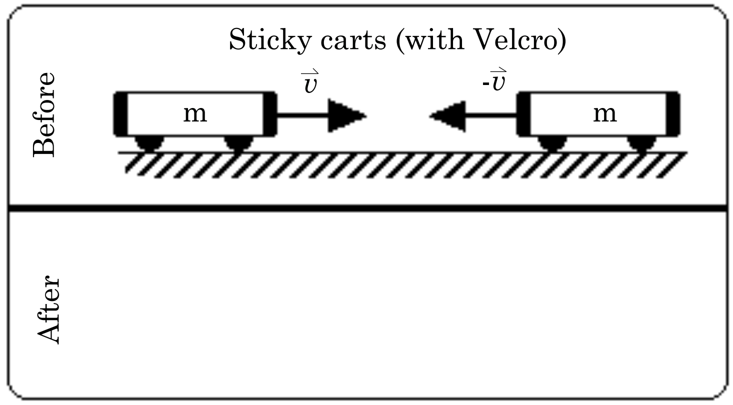


Figure 9-4: Inelastic collision

(d) Observe a sticky collision (also known as an *inelastic collision*) and discuss whether or not the outcome was what you predicted it to be. If not, draw a new sketch with arrows indicating the magnitudes and directions of the velocities. What is the apparent relationship between the final velocities $\vec{v}_{1,f}$ and $\vec{v}_{2,f}$? How do their magnitudes compare to those of the initial velocities?

(e) Sketch a predicted result of the *interaction* between two objects that collide and then explode. Use arrows to indicate the direction and magnitude of the velocity of each object *after* the collision.

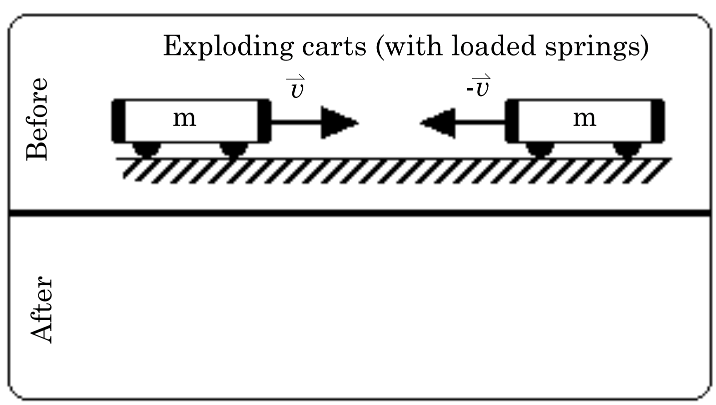


Figure 9-5: Exploding collision

(f) Observe an exploding or "superelastic" collision and discuss whether or not the outcome was what you predicted it to be. If not, draw a new sketch with arrows indicating the magnitudes and directions of the velocities. What is the apparent relationship between the final velocities $\vec{v}_{1,\mathrm{f}}$ and $\vec{v}_{2,\mathrm{f}}$? How do their magnitudes compare to those of the initial velocities?

(g) What is the total momentum (i.e. the *vector* sum of the initial momenta) before the collision or explosion in all three situations?

(h) Does momentum appear to be conserved in each case? Is the final total momentum the same as the initial total momentum of the two cart system?

Defining a Centre for a Two Particle System

What happens to the average position of a system in which two moving carts having the same mass interact with each other? That is, what happens to

$$\langle x \rangle = \frac{x_1 + x_2}{2}$$

as time goes by? What might the motion of the average position have to do with the total momentum of the system? To study this situation you will need:

- A video analysis system
- A track

- Two dynamics carts with magnets attached
- Digital movie of 1D collision from Unit 8

In making these observations you'll need to look at the pattern of data points which you place over the frames. You will not need to create graphs or work with numbers.

✍ Activity 9-3: Motion of the Average Position

(a) Imagine interactions between identical carts moving toward each other at the same speed as described in Activity 9-2. Does the average position of the carts move before, during, or after the collision or explosion in each case? Might this have anything to do with the fact that the total momentum of such a system is zero?

(b) Let's use video analysis to study a real situation in which the total momentum of the system is *not* zero. Do the following:

> 1. Make a movie of cart 1 colliding with cart 2 where cart 2 begins at rest. (Be sure to save a copy of the movie file as you will need to look at it again later in the unit.)
> 2. Using the Add Point function, step through the movie one frame at a time and click on the position average (i.e. halfway between the centres of the two carts).

How does the position average appear to move? Might this motion have anything to do with the fact that the total momentum of the system is directed to the right along the positive x-axis?

(c) You should have found that if the momentum of the carts is constant then the average position moves at a constant rate also. Suppose the masses of the carts are unequal? How does the average position of the two objects move then? Lets have a look at a collision between unequal masses. Open the movie you made at the end of Unit 8. Once again track the motion of the average position by clicking halfway between the centres of the two carts. Is the motion of this average position uniform?

(d) You should have found that the average position of a system of two unequal masses does not move at a constant velocity. We need to define a new quantity called the *centre of mass* which is at the centre of two equal masses but somewhere else when one of the masses is larger. Use the video analysis system and the movie you just analysed to find a "centre-of-mass location". The centre of mass is a location in the isolated system that moves at a constant velocity before, during, and after the collision.

Note: You should be able to make some intelligent guesses. Describe what you tried and the outcomes in the space below.

Defining Centre of Mass in One Dimension

In the last activity you should have discovered that the average position of a system of two carts having equal masses moves at a constant rate. However, if the carts have different masses, we cannot calculate a simple average position and expect it to move at a constant rate. You will find that it is convenient to define a new quantity called the centre of mass which always moves at a steady rate in an isolated system of particles. Let's turn to the Law of Conservation of Momentum for hints on how to develop the idea of centre of mass. We can start with the special case of two particles of different masses moving along a line at different velocities and perhaps colliding with each other. They experience no outside forces.

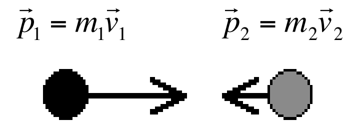


Figure 9-6: Two particles about to collide.

In homework assignment SP-2 you will prove that the vector sum of the momentum of the two particles can be treated as a constant that is characterized as being caused by a mass equal to the sum of the individual masses ***(M)*** moving at a constant velocity v_{cm}, where v_{cm} is the velocity of the centre of mass:

$$\vec{p} = \vec{p}_1 + \vec{p}_2 = \frac{d(m_1x_1 + m_2x_2)}{dt}\hat{i} = M\vec{v}_{cm}$$

We can easily extend the definition of centre of mass to two and three dimensions. Now we'll turn to exercises involving the calculation of the centre of mass of a system.

Centre of Mass of a Simple 1-D Particle System
Let's apply the definition of the centre of mass to some real systems available in the classroom that are made up of "point-like" particles. For this activity you will need:

- modelling clay
- A bamboo skewer
- A balance
- A ruler

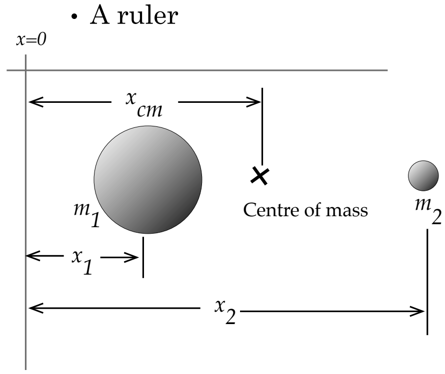


Figure 9-7: Notation for the mass and position of a two-mass system

For two masses m_1 and m_2 that are a distance x_1 and x_2 from the x-axis, respectively, the x coordinate of the centre of mass is given by the equation

$$x_{cm} = \frac{(m_1 x_1 + m_2 x_2)}{M}$$

where M is the total mass of the system ($M = m_1 + m_2$).

Create a two-mass system with two clay balls and a skewer like that shown in Figure 9-7. Make the balls a reasonable size. You can measure the masses of the balls and calculate the x-value of the centre of mass (CM) for your two-mass system.

✍ Activity 9-4: Calculating the CM for Two Masses

(a) Determine the total mass of the system, M. Then pull the more massive ball of clay off the end of the skewer. Determine its mass m_1. Next determine the mass of the lighter ball and its attached skewer, m_2. Assume that the mass of the skewer is small compared to the masses of the balls and ignore it. Record the values below.

$M =$ $m_1 =$ $m_2 =$

(b) Set $x_1 = 10$ cm, measure the distance between the masses, and calculate x_2 from the distance between the masses d.

$x_1 =$ $d =$ $x_2 =$

(c) Calculate the centre of mass, X_{cm}, of the system.
Note: Remember that neither of the masses is at the origin of our coordinate system.

(d) Determine the centre of mass of the two ball system in the designated coordinate system by finding its balance point and record it below. We'll explain more about this balance method later.

Hint: Don't forget that we placed the massive ball at x_1=10 cm.

(e) How does the measured value of CM compare to that which you calculated? Are there any sources of systematic error in your measurements or calculations? What influence does the mass of the skewer have? Explain.

SESSION TWO: CENTRE OF MASS AND MOMENTUM CONSERVATION IN TWO DIMENSIONS

Review of Homework Assignment
Come prepared to ask questions about the homework assignment.

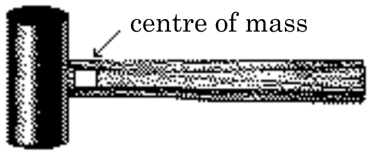
Defining the Centre of Mass for Extended Objects
So far, we have been studying the motion of "point" masses or objects that are quite symmetric like spheres and blocks. Sometimes we have ignored the shape of objects because our observations were not very precise or detailed. However, the real world is made up of some very oddly shaped objects. For example, there are systems of small particles such as atoms and water molecules as well as extended objects such as gorillas, pipe wrenches, DNA molecules, and binary stars. How can we study the motion of such strange objects? For example, what happens when a linear force is applied to different parts of an extended object? Will it rotate or not? What happens when an object or system changes shape during an interaction with various forces?

To continue our study of the centre-of-mass concept, let's observe the motion of a rigid object with a complicated shape that doesn't become deformed while it is moving. For this observation you will need:

- A rubber mallet
- A piece of tape (wrapped around the handle of the mallet at its centre of mass)
- A couple of people to play catch
- A video analysis system

Suppose the rubber mallet is lobbed from one person to another in such a way that you are able to see the complex motions of the hammer as it travels. What does the path of the hammer look like? What does the path of the tape on the handle look like?

✍ Activity 9-5: Examining the Motion of a Tossed Mallet
(a) Use the iSight camera and G-Cam to take a video of the mallet as it is tossed in the front of the classroom. We will take one video for the whole class and share because there isn't enough room for everyone to toss mallets. Sketch below the approximate path you observed as the mallet was tossed.



(b) Using the Add Point function, step through the movie one frame at a time and click on the tape wrapped around the handle of the mallet. Reproduce the graph of *y* vs *x* below.

(c) Describe the difference, if any, between your observation of the motion of the whole mallet and the apparent motion of just the tape.

(d) The tape was placed at the centre of mass of the mallet. If there is a difference between the two apparent motions, what is happening to the actual motion of the centre of mass of the mallet or to other parts of the mallet when it is lobbed?

(e) Try to balance the mallet on your finger with it standing straight up (vertically). Then balance it with it lying on its side (horizontally). How are the balance points related to the location of the tape on the mallet? What does this suggest about one of the characteristics of the CM? Include sketches, if that's helpful.

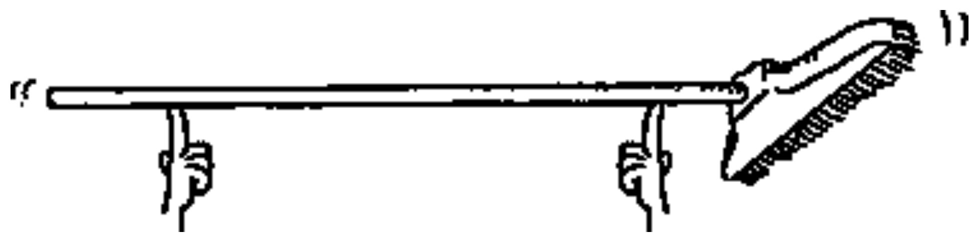
For the purpose of studying motion, *the centre of mass of an extended object or system of particles is defined as the point which appears to move as if all the mass of the object or system of particles were concentrated at that point.*

Centre of Mass Demonstrations

In order to perform the demonstrations outlined below, you will need:

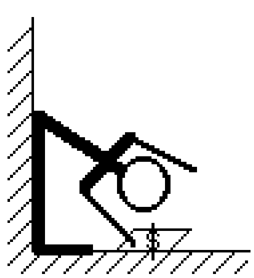
- 1 broom
- 1 $20 bill

(1) Moving Your Hands Together on a Broom



The outcome of slowly sliding your hands closer and closer together on a broom is surprising, but you can explain it by using an understanding of centre of mass concepts and the characteristics of static and kinetic friction.

(2) Picking up $20



Everyone loves to pick up extra money. Your instructors are betting that you can't stand with your heels touching both a wall, the floor and *each other*, and then bend over (without bending your knees!) and pick up a $20 dollar bill that's lying in front of you without moving your heels away from the floor and the wall. (No fair using a wall with a baseboard either!) You must be able to resume your up-right position again without having moved your heels. We're sure enough that this task is very difficult to stake money on it! All of you taking calculus-based introductory physics this semester who can perform this task before the end of the class period under the sharp eye of a bona fide instructor can share the $20 with any others taking the course who can also do the "pickup" job!

✍ Activity 9-6: Can You Pick Up the Money?

(a) What is the necessary balance condition for you to be able to pick up the money? What does this have to do with your CM?

(b) Who in the class do you predict will be good at this task? For example, would you bet on someone with narrow hips and big shoulders or someone with wide hips and small shoulders? Suppose someone has long legs and a short upper body or vice versa?

(c) Watch the attempts of your classmates. What were the physical characteristics of the students who were especially good at this? Especially bad at this?

(d) Did you expect to be good at this task? Why or why not? Were you any good? What personal physical traits entered into your good performance or lack thereof?

2-D Collisions – Intelligent Guesses & Observations
Conservation of momentum can be used to solve a variety of collision and explosion problems. So far we have only considered momentum conservation in one dimension, but real collisions lead to motions in two and three dimensions. For example, air molecules are continually colliding in space and bouncing off in different directions.

You probably know more about two dimensional collisions than you think. Draw on your prior experience with one-dimensional collisions to anticipate the outcome of several two dimensional collisions. Suppose you were a witness to several accidents in which you closed your eyes at the moment of collision each time two vehicles heading toward each other crashed. Even though you couldn't stand to look, can you predict the outcome of the following accidents?

You see car A enter an intersection at the same time as car B coming from its left enters the intersection. Car B is the same make and model as car A and is travelling at the same speed. The two cars collide inelastically and stick together. What happens? Hint: You can use a symmetry argument, your intuition or a quick analysis of 1-D results. For example, you can pick a coordinate system and think about two separate accidents: the x accident in which car B is moving at speed v_{bx} and car A is standing still, and the y accident in which car A is moving at speed $v_{ay} = v_{bx}$ and car B is standing still.

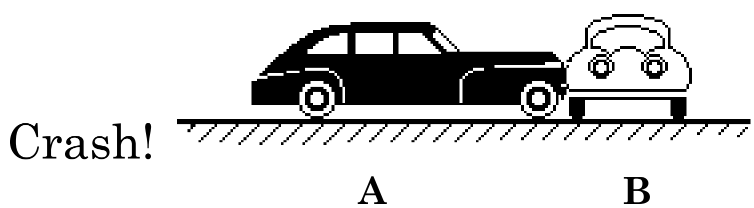


Figure 9-8: Two Identical Cars that Collide

The diagram below shows an aerial view of several possible two-dimensional accidents that might occur. The first is a collision at right angles of two identical cars.

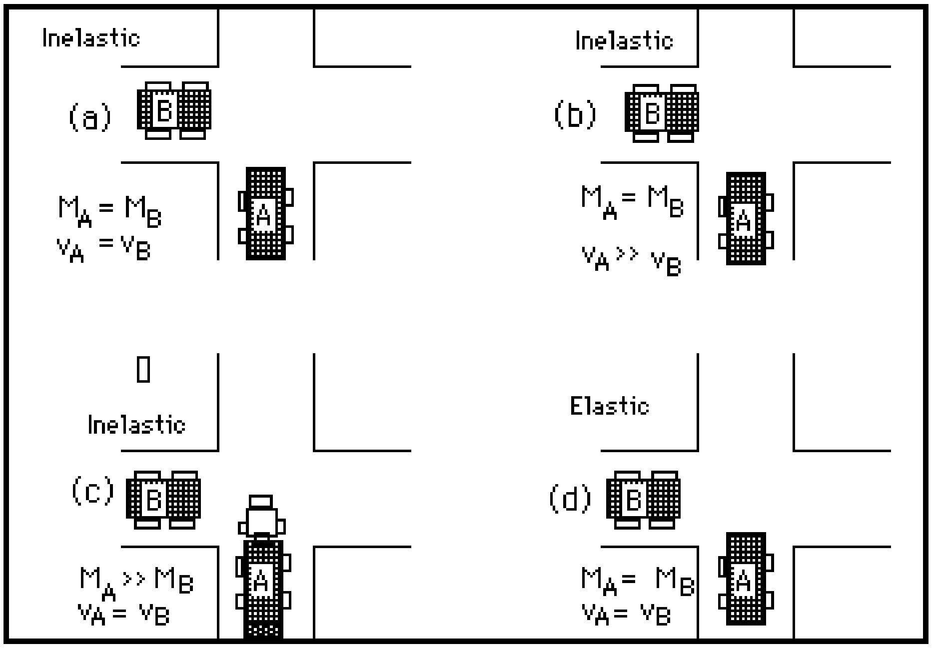


Figure 9-9: Several Types of Two Dimensional Collisions

For the observations associated with these predictions you'll need:

- 2 large air pucks
- weights
- masking tape
- Clay for inelastic collisions

✍ Activity 9-7: Qualitative 2-D Collisions

(a) Using the diagram in Figure 9-9, draw a dotted line in the direction you think your two cars will move after a collision between cars with equal masses and velocities. Explain your reasoning in the space below.

(b) Draw a dotted line for the direction the cars might move if car A were travelling at a speed much greater than that of car B. Explain your reasoning in the space below.

(c) If instead of a car, the vehicle A were a large truck travelling at the same speed as car B, in what direction will the vehicles move? Draw the dotted lines. Explain your reasoning in the space below.

(d) Now suppose that the two vehicles are army tanks travelling at the same speed. We all know colliding tanks don't stick together; in what direction would the two tanks move after the collision, if they undergo an elastic collision? Explain your reasoning in the space below.

(e) Finally, set up these types of collisions using the air pucks in the front of the classroom. Observe each type of collision several times. Draw solid lines in the diagram above for the results. How good were your predictions? Explain your reasoning in the space below.

(f) What rules have you devised to predict more or less what is going to happen as the result of a two dimensional collision?

Is Momentum Conserved in Two Dimensions?

During the last few sessions we have placed a lot of faith in the power of Newton's second and third laws to predict that momentum is always conserved in collisions. We have shown mathematically and experimentally for a number of one-dimensional collisions that if momentum is conserved the centre of mass of a system will move at a constant velocity regardless of how many internal interactions take place. Let's see whether the mathematical prediction that the centre of mass of an isolated three body system will move at a constant velocity is correct within the limits of experimental uncertainty. Consider three pucks moving on an air table which are free to move in two dimensions. You can make a video of the collisions of the three pucks and do a frame-by-frame analysis of the movie.

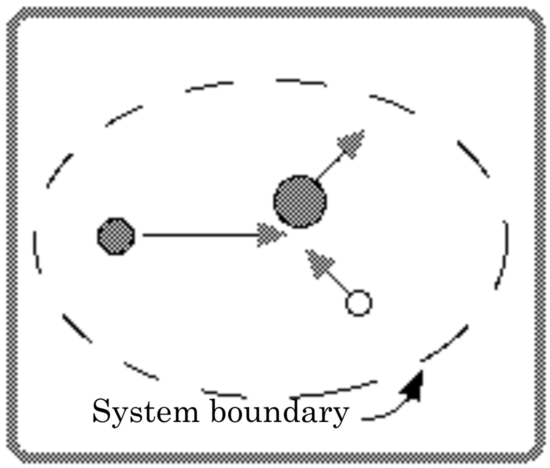


Figure 9-10: Three particles interacting on an air table.

For this experiment you will need the following equipment:

- 3 air pucks with at least one having a *different* mass
- A metre stick
- A balance
- A video analysis system

Take several movies of three bodies colliding in a complex way on an air table in *instances where the air pucks do not touch the walls of the air table.* Pick one of the movies to analyse, and find the coordinates of each air puck before, during, and after collisions that occur in the centre of the air table. You can then find the x- and y-components of centre of mass of the system and graph them as a function of time.

✍ Activity 9-8: Tracking the Centre-of-Mass Motion

(a) Determine the masses of the pucks in your system and record them in the space below.

(b) Analyse the locations of each of your pucks on a frame-by-frame basis. Create a spreadsheet with the coordinates of the pucks in each frame. Use the spreadsheet to calculate the x and y values of the centre of mass of the systems for each of the frames that you analysed. Be sure the data listed below is included in your spreadsheet.

For each puck (A, B, and C):

Mass of the puck

Frame number and elapsed time (seconds) and then at each time:

x (m)

y (m)

The x and y values in metres of the centre of mass of the system

(c) Now create an overlay plot of four functions: the measured y vs. x values for each of the three pucks and the calculated y vs. x values for the centre of mass of the system. Upload a copy of your plot to WebCT assignments. (You can sketch copies of the four plots in the space below. Be sure to label them appropriately.)

Puck A:

Puck B:

Puck C:

Centre of Mass:

(d) Interpret the graphs by drawing arrows indicating the directions of motion of each of the pucks and of the centre of mass of the system. Within the limits of experimental uncertainty is the centre of mass of the system moving at a constant velocity? What is the evidence for your conclusions?