# Magnetic Dipoles (33.5)



**(a)** Cross section through the current loop    **(b)** The current loop seen from the right

The field emerges from the center of the loop.

The field returns around the outside of the loop.

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- To determine the magnetic field direction in a loop either

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- To determine the magnetic field direction in a loop either
  1. Point your right thumb in the direction of the current and curl your fingers around the loop. Your fingers point in the direction the field leaves the loop.

# Magnetic Dipoles (33.5)



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The field emerges from the center of the loop.

The field returns around the outside of the loop.

- To determine the magnetic field direction in a loop either
  1. Point your right thumb in the direction of the current and curl your fingers around the loop. Your fingers point in the direction the field leaves the loop.
  2. Curl the fingers of your right hand around the loop in the direction of the current. Your thumb is then pointing in the direction in which the field leaves the loop.
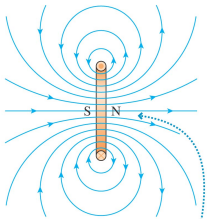
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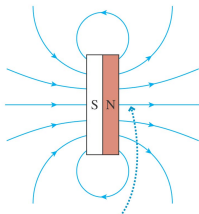- In fact, a current loop **is** a magnetic dipole! It is an electromagnet



**(a)** Current loop

Whether it's a current loop or a permanent magnet, the magnetic field emerges from the north pole.

**(b)** Permanent magnet

Whether it's a current loop or a permanent magnet, the magnetic field emerges from the north pole.

# Magnetic Dipoles

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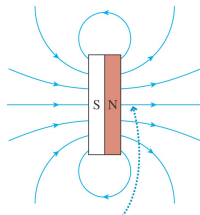- In fact, a current loop **is** a magnetic dipole! It is an electromagnet



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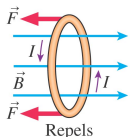Whether it's a current loop or a permanent magnet, the magnetic field emerges from the north pole.

- The magnetic field emerges from the north face of each.

# Magnetic Dipoles
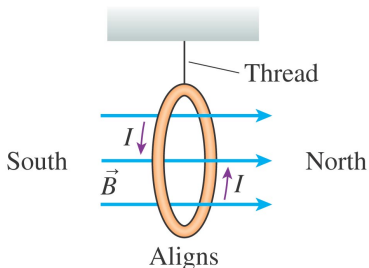


Thread

South    North

$I$    $\vec{B}$    $I$

Aligns

$\vec{F}$    $I$    $\vec{B}$    $I$    $\vec{F}$    Repels

N    S

$\vec{F}$    $I$    $\vec{B}$    $I$    $\vec{F}$    $\vec{F}$    Attracts

$\vec{F}$    S    N
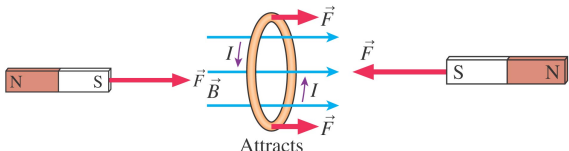
# Magnetic Dipole Moment

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$$B_{loop} \approx \frac{\mu_0}{2} \frac{IR^2}{z^3} = \frac{\mu_0}{4\pi} \frac{2(\pi R^2)I}{z^3} = \frac{\mu_0}{4\pi} \frac{2AI}{z^3}$$

where $A = \pi R^2$ is the area of the loop.

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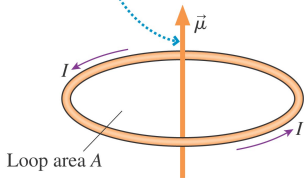
where $A = \pi R^2$ is the area of the loop.

- This actually works even if the loop is not circular (of course $A$ changes).

- We define the magnetic dipole moment of a current loop enclosing area $A$ to be

$$\vec{\mu} = AI, (\textit{direction from right hand rule})$$

The magnetic dipole moment is perpendicular to the loop, in the direction of the right-hand rule. The magnitude of $\vec{\mu}$ is $AI$.

$\vec{\mu}$

$I$

$I$

Loop area $A$

On axis field is:

$$\vec{B}_{dipole} = \frac{\mu_0}{4\pi} \frac{2\vec{\mu}}{z^3}$$

- We could use the Biot-Savart Law to calculate any magnetic field we want just as you can use Coulomb's Law to calculate any electric field through superimposing a large number of charges.
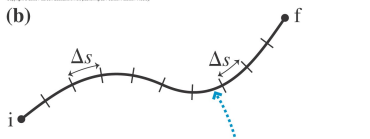
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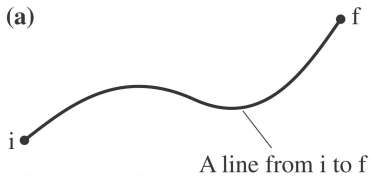- But that would be hard.

# Ampère's Law and Solenoids (33.6)

- We could use the Biot-Savart Law to calculate any magnetic field we want just as you can use Coulomb's Law to calculate any electric field through superimposing a large number of charges.
- But that would be hard.
- In the case of electric field we were able to solve problems with complicated shapes by noticing the symmetries of the problem and applying Gauss' Law.
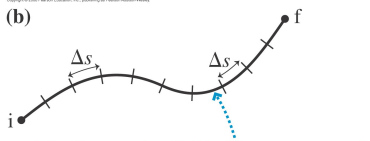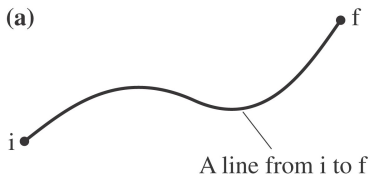
- We could use the Biot-Savart Law to calculate any magnetic field we want just as you can use Coulomb's Law to calculate any electric field through superimposing a large number of charges.
- But that would be hard.
- In the case of electric field we were able to solve problems with complicated shapes by noticing the symmetries of the problem and applying Gauss' Law.
- We can do something similar for magnetic fields using Ampère's Law

**(a)**

f

i

A line from i to f

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**(b)**

$\Delta s$     $\Delta s$

f

i

The line can be divided into many small segments. The sum of all the $\Delta s$'s is the length $l$ of the line.

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- To apply Ampère's Law we will need to do some line integrals

# Line Integrals

**(a)**



A line from i to f

**(b)**



The line can be divided into many small segments. The sum of all the $\Delta s$'s is the length $l$ of the line.
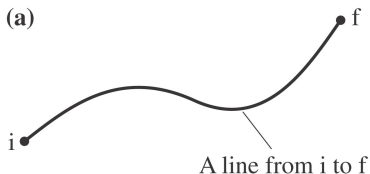
- To apply Ampère's Law we will need to do some line integrals
- Imagine that, instead of integrating along the x-axis, you want to integrate along some arbitrary line. Maybe the line isn't even straight!

# Line Integrals

**(a)**



A line from i to f

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**(b)**



The line can be divided into many small segments. The sum of all the $\Delta s$'s is the length $l$ of the line.

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- To apply Ampère's Law we will need to do some line integrals
- Imagine that, instead of integrating along the x-axis, you want to integrate along some arbitrary line. Maybe the line isn't even straight!
- Take the line on the left and divide it into little segments, then sum over the segments

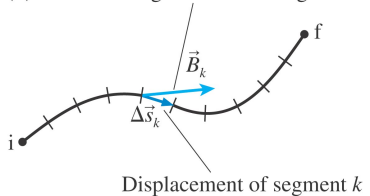$$l = \sum_k \Delta s_k \rightarrow \int_i^f ds$$
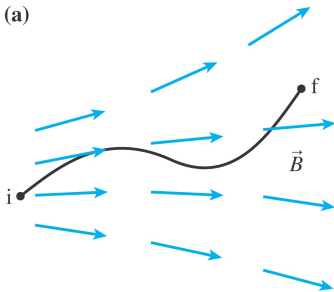
# Line Integrals

**(a)**



The line passes through a magnetic field.

- Imagine we have a magnetic field passing through the line.

**(b)** Magnetic field at segment $k$



Displacement of segment $k$

# Line Integrals



**(a)**

i

f

$\vec{B}$

The line passes through a magnetic field.

**(b)** Magnetic field at segment $k$

f

$\vec{B}_k$

$\Delta\vec{s}_k$

i

Displacement of segment $k$
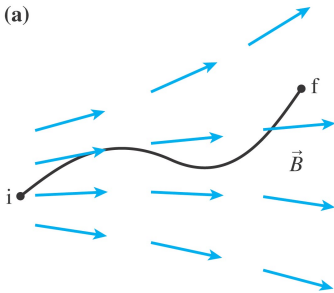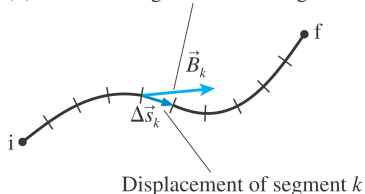
- Imagine we have a magnetic field passing through the line.
- We could take each segment of the line and calculate the dot product of $\vec{B}$ with a vector along the segment, then sum them up:

$$\sum_k B_k \cdot \Delta s_k \rightarrow \int_i^f \vec{B} \cdot d\vec{s}$$

# Line Integrals
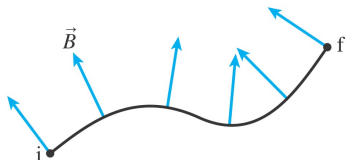


- As in Gauss' Law, we confine ourselves to 2 simple cases:

# Line Integrals



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- As in Gauss' Law, we confine ourselves to 2 simple cases:
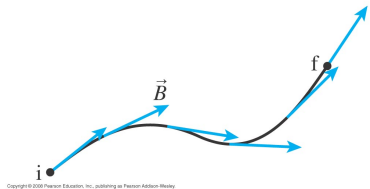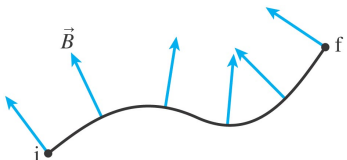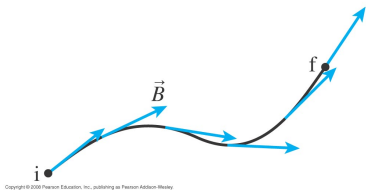
  1. $\vec{B}$ is everywhere perpendicular to the line

$$\int_i^f \vec{B} \cdot d\vec{s} = 0$$

# Line Integrals



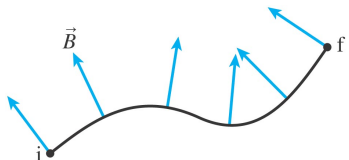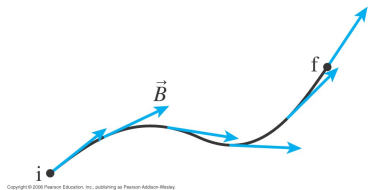- As in Gauss' Law, we confine ourselves to 2 simple cases:

  1. $\vec{B}$ is everywhere perpendicular to the line

  $$\int_i^f \vec{B} \cdot d\vec{s} = 0$$

  2. $\vec{B}$ is everywhere tangent to the line

  $$\int_i^f \vec{B} \cdot d\vec{s} = Bl$$

The integration path is a circle of radius $d$.

The integration starts and stops at the same point.

$\vec{B}$

$d$

$I$
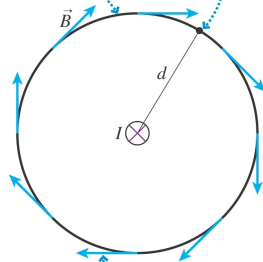
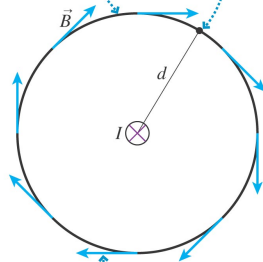$\vec{B}$ is everywhere tangent to the integration path and has constant magnitude.

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- If we draw a circle of radius $d$ around a wire carrying current $I$, we know the magnetic field will be tangent to the circle at every point.

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$\vec{B}$ is everywhere tangent to the integration path and has constant magnitude.

- If we draw a circle of radius $d$ around a wire carrying current $I$, we know the magnetic field will be tangent to the circle at every point.
- We also know that the magnetic field will have the same magnitude everywhere around the circle.

# Ampère's Law



The integration path is a circle of radius $d$.

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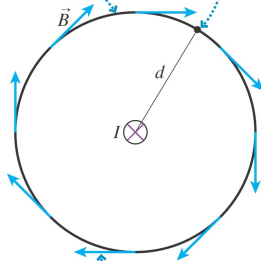$\vec{B}$ is everywhere tangent to the integration path and has constant magnitude.

- If we draw a circle of radius $d$ around a wire carrying current $I$, we know the magnetic field will be tangent to the circle at every point.
- We also know that the magnetic field will have the same magnitude everywhere around the circle.
- So, how about we integrate around that circle?
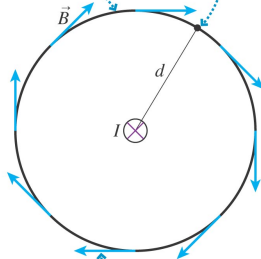
$$\oint \vec{B} \cdot d\vec{s}$$

(note it is a closed curve)

# Ampère's Law



The integration path is a circle of radius $d$.

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$\vec{B}$

$d$

$I$

$\vec{B}$ is everywhere tangent to the integration path and has constant magnitude.

- This is one of those easy cases

$$\oint \vec{B} \cdot d\vec{s} = BI = B(2\pi d)$$

# Ampère's Law

The integration path is a circle of radius $d$.

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$\vec{B}$

$d$

$I$

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- This is one of those easy cases

$$\oint \vec{B} \cdot d\vec{s} = Bl = B(2\pi d)$$

- We know $B$, it is

$$B = \frac{\mu_0 I}{2\pi d}$$

# Ampère's Law

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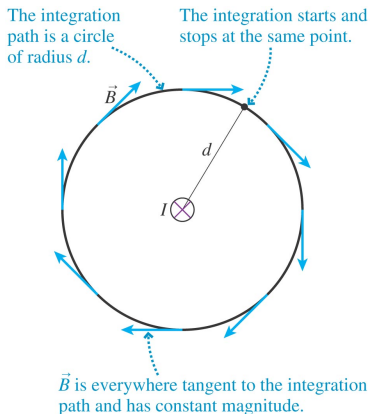
$\vec{B}$

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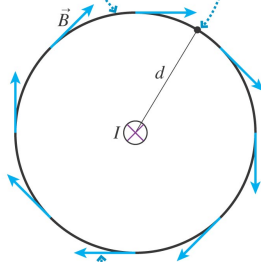Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison Wesley

- Substituting $\vec{B}$ into the expression gives Ampère's Law

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I$$

The integration path is a circle of radius $d$.

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$d$

$I \otimes$

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- Notice it does not depend on the radius of the circle! (just like Gauss' law and flux)

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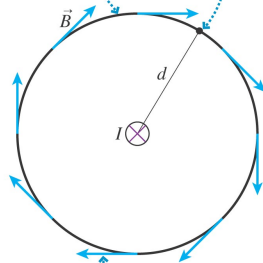
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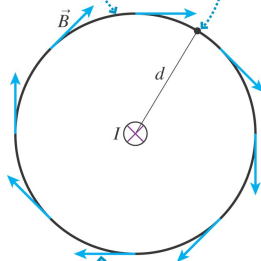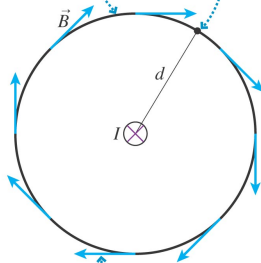- Substituting $\vec{B}$ into the expression gives Ampère's Law

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- We could also show that
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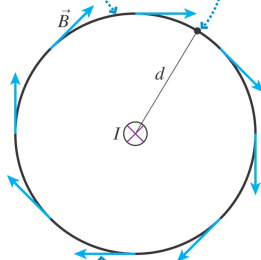- Substituting $\vec{B}$ into the expression gives Ampère's Law

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- Notice it does not depend on the radius of the circle! (just like Gauss' law and flux)
- We could also show that
  - it does not depend on the shape of the curve
  - it does not depend on where the current is within the curve

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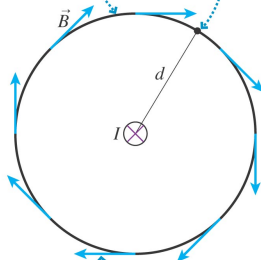- Substituting $\vec{B}$ into the expression gives Ampère's Law

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- Notice it does not depend on the radius of the circle! (just like Gauss' law and flux)
- We could also show that
  - it does not depend on the shape of the curve
  - it does not depend on where the current is within the curve
  - it depends only on the amount of current flowing inside the curve
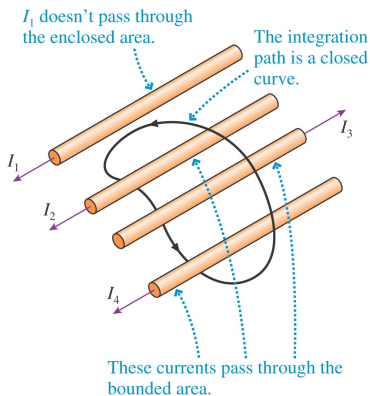
$I_1$ doesn't pass through the enclosed area.

The integration path is a closed curve.

$I_1$

$I_2$

$I_3$

$I_4$

These currents pass through the bounded area.

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- We do need to understand if the current is positive or negative.

# Ampère's Law



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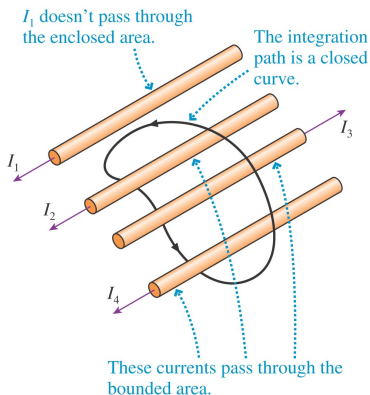
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- We do need to understand if the current is positive or negative.
- Place the fingers of your right hand along the direction of the integral. The direction in which your thumb is pointing is positive, the opposite is negative.

# Ampère's Law



$I_1$ doesn't pass through the enclosed area.

The integration path is a closed curve.

$I_1$
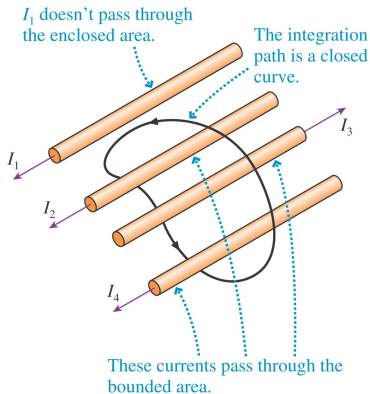
$I_2$

$I_3$

$I_4$

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- We do need to understand if the current is positive or negative.
- Place the fingers of your right hand along the direction of the integral. The direction in which your thumb is pointing is positive, the opposite is negative.
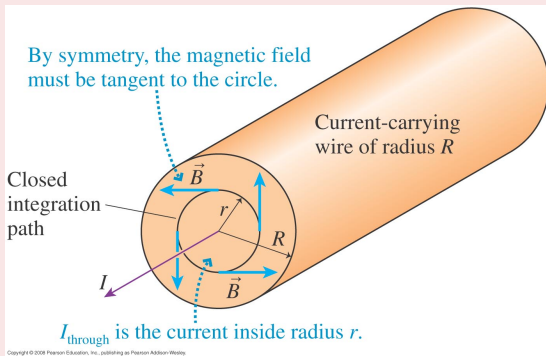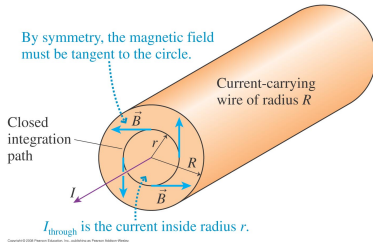- So, in the figure on the left, the total current through the curve is

$$I = I_2 - I_3 + I_4$$

# Example 33.8

## Example 33.8

A wire of radius $R$ carries current $I$. Find the magnetic field inside the wire at distance $r < R$ from the axis.
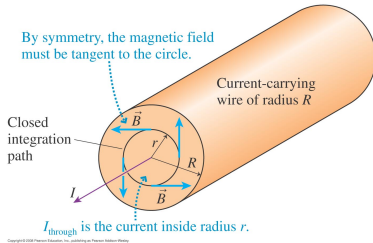


By symmetry, the magnetic field must be tangent to the circle.

Current-carrying wire of radius $R$

Closed integration path

$\vec{B}$

$r$

$R$

$I$

$\vec{B}$

$I_{\text{through}}$ is the current inside radius $r$.

# Example 33.8



By symmetry, the magnetic field must be tangent to the circle.

Current-carrying wire of radius $R$

Closed integration path

$B$

$r$

$R$

$I$

$B$

$I_{through}$ is the current inside radius $r$.

- This wire has cylindrical symmetry and the magnetic field is tangent to circles concentric with the wire.

# Example 33.8



By symmetry, the magnetic field must be tangent to the circle.

Current-carrying wire of radius $R$

Closed integration path

$\vec{B}$

$r$

$R$

$I$

$\vec{B}$

$I_{through}$ is the current inside radius $r$.

- This wire has cylindrical symmetry and the magnetic field is tangent to circles concentric with the wire.
- Assuming the current density is uniform across the wire, the current through the circle is

$$I_{through} = JA_{circle} = \pi r^2 J$$

where $J$ is current density and is given by

$$J = \frac{I}{A} = \frac{I}{\pi R^2}$$

# Example 33.8

- The current through a circle of radius $r$ is then

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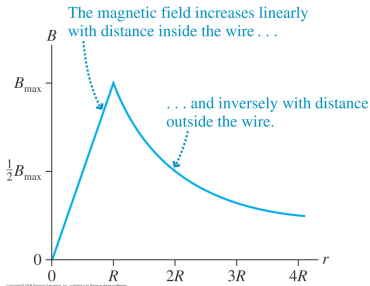
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- $\vec{B}$ is tangent to the circle so

$$\oint \vec{B} \cdot d\vec{s} = Bl = 2\pi r B$$

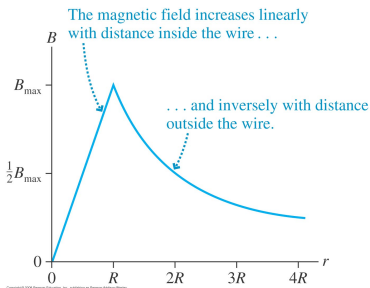# Example 33.8



The magnetic field increases linearly with distance inside the wire . . .

. . . and inversely with distance outside the wire.

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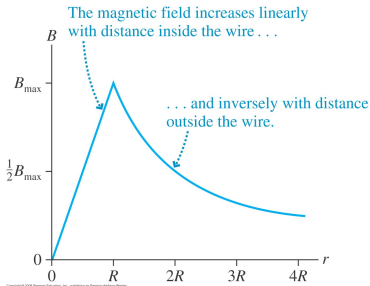
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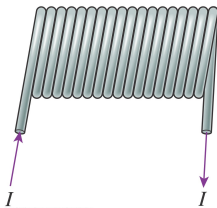$$2\pi r B = \frac{\mu_0 r^2}{R^2} I$$

$$B = \frac{\mu_0 I}{2\pi R^2} r$$

- The magnetic field strength increases linearly with distance until the outer edge of the wire, then follows the Biot-Savart Law.

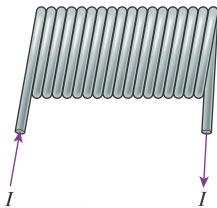# The Magnetic Field of a Solenoid
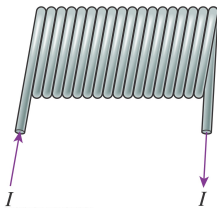


$I$          $I$

- When studying electric fields we talked a lot about parallel plate capacitors because they created a nice uniform field.

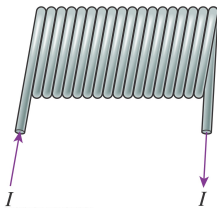# The Magnetic Field of a Solenoid



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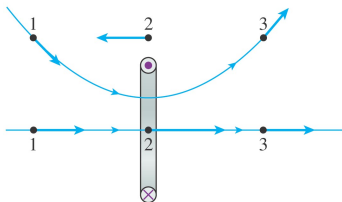- The equivalent for magnetic fields is a solenoid.

# The Magnetic Field of a Solenoid



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- A solenoid is a helical coil of wire with the same current $I$ passing through each of the loops. Each loop is often referred to as a turn.

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- The equivalent for magnetic fields is a solenoid.
- A solenoid is a helical coil of wire with the same current $I$ passing through each of the loops. Each loop is often referred to as a turn.
- Of course, we have already studied a single loop of wire and know that superposition works for $\vec{B}$, so...

# The Magnetic Field of a Solenoid

**(a)** A single loop



**(b)** A stack of three loops

The fields of the three loops nearly cancel here. $\vec{B}_3$

$\vec{B}_2$

$\vec{B}_1$

$\vec{B}_1$

$\vec{B}_2$

$\vec{B}_3$

The fields reinforce each other here.

**(a)** Current loop



Whether it's a current loop or a permanent magnet, the magnetic field emerges from the north pole.

- The superposition of fields inside the loop is reinforcing and gives a field roughly parallel to the axis.

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S  N

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Whether it's a current loop or a permanent magnet, the magnetic field emerges from the north pole.

- The superposition of fields inside the loop is reinforcing and gives a field roughly parallel to the axis.

- The superposition outside the loop is cancelling.

- An ideal solenoid has a strong uniform field inside and no field outside!