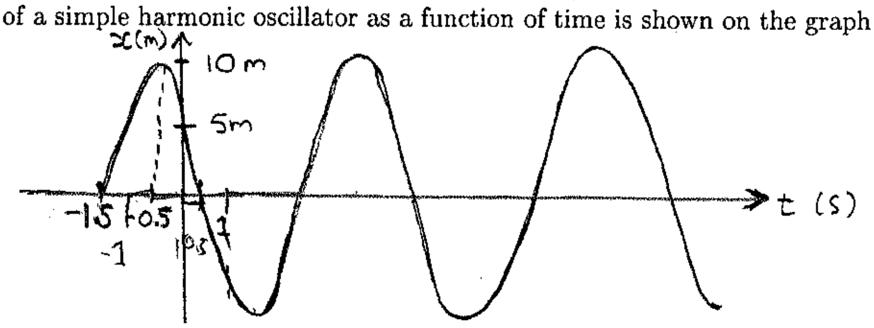
Physics 120
Term Test #2
Friday November 19, 2004

Only a basic, non-programmable calculator is allowed. You have approximately 50 minutes to complete this test. Your work must be neat and easy to follow. If what you are doing is not clear, you will not get full marks even if you did the question correctly. Please explain all steps carefully and simplify as much as possible. When you are finished, please place this question sheet inside your answer booklet and hand everything in. Good luck.

1. The position of a simple harmonic oscillator as a function of time is shown on the graph below.

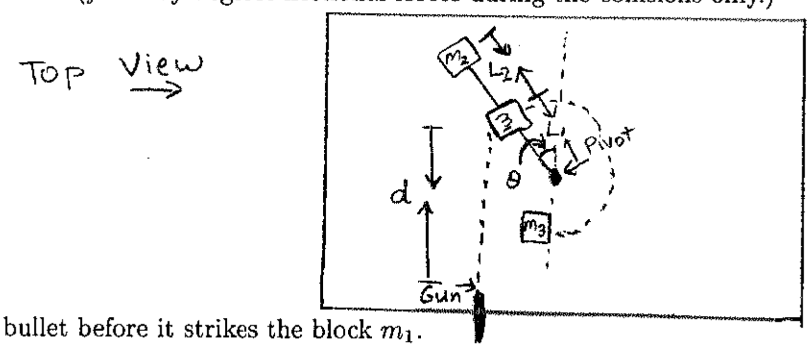


   a) Carefully sketch the corresponding velocity-time and acceleration time graphs in your answer booklet. Make sure to label the axes carefully.
   b) What are the amplitude, period, frequency (linear) and phase constant (what we called δ or ϕ_0 in the class notes) of this oscillator (give the numerical values)?
   c) The oscillator is subject only to conservative forces. Sketch the potential energy and kinetic energy as a function of *time* over several oscillations. Again, please label the axes carefully. (y-axis arbitrary)

2. Consider a spherical neutron star with a mass equal to that of the sun ($M = 1.98 \times 10^{30}$ kg) and a radius of 10 km. The period of rotation of the star is 1.0s.
   a) Calculate the speed of a point on the equator of the star.
   b) Calculate the acceleration due to gravity on the surface of the star.
   c) How many revolutions per minute are made by a satellite orbiting in a circular orbit 1.0 km above the star's surface?
   d) A circular astrosynchronous orbit is one where the orbiting satellite is always above the same point on the surface of the star (ie. it rotates with the star). Calculate the height above the equator of the star of an astrosynchronous orbit.
   e) Explain briefly, using physical principles, why the above satellite's orbit will remain in a plane passing through the star's equator.

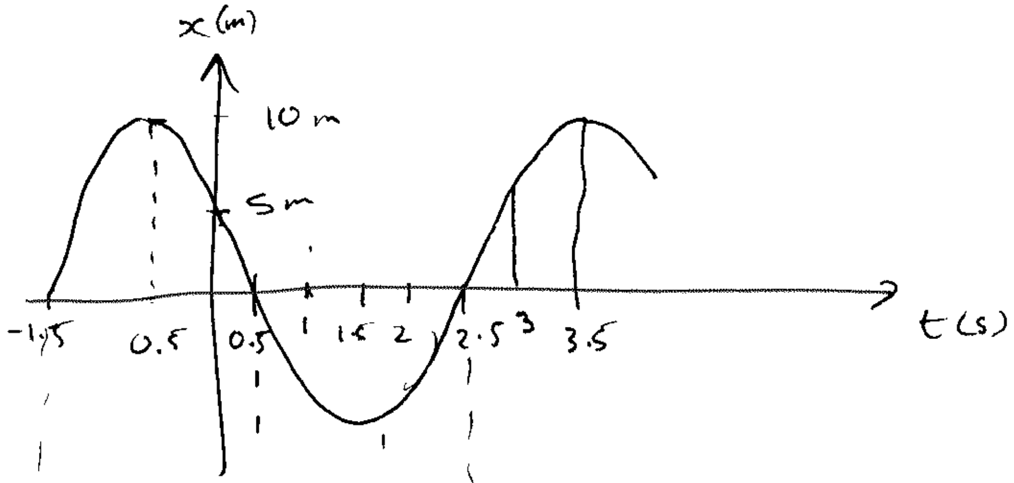
at $t = 0$ s

3. Consider a system of two blocks attached to a pivot by very light, rigid rods. ~~At $t = 0$s~~ The gun is fired and the bullet strikes the block m_1 and is embedded in the block's center. The system of blocks (which was initially stationary) spin on the table about the pivot and block m_1 will collide with block m_3 at time $t = t_1$. The coefficient of kinetic friction between each of the the blocks and the table is μ_k. The angle θ and the distance d are also known. (you may neglect frictional forces during the collisions only.)

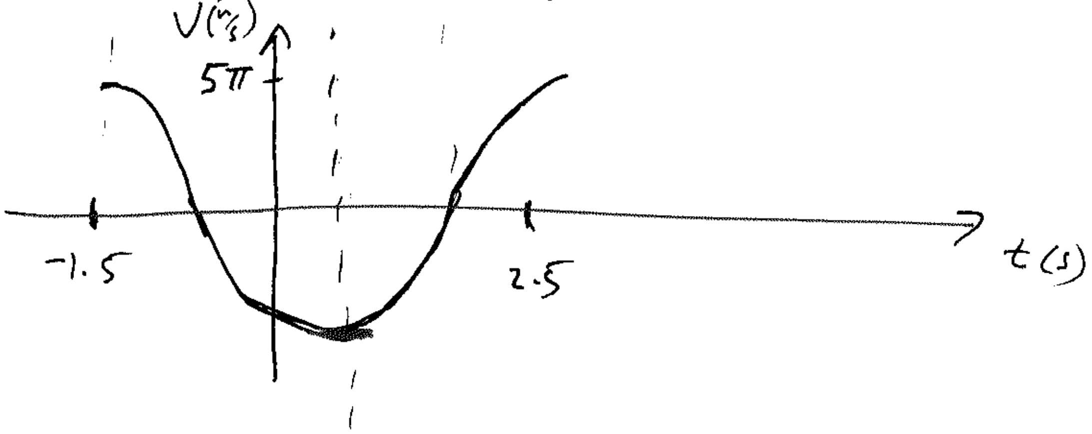


   a) Calculate the speed of the bullet before it strikes the block m_1.
   b) If m_1 collides elastically with m_3, calculate the speed of m_3 right after the collision.

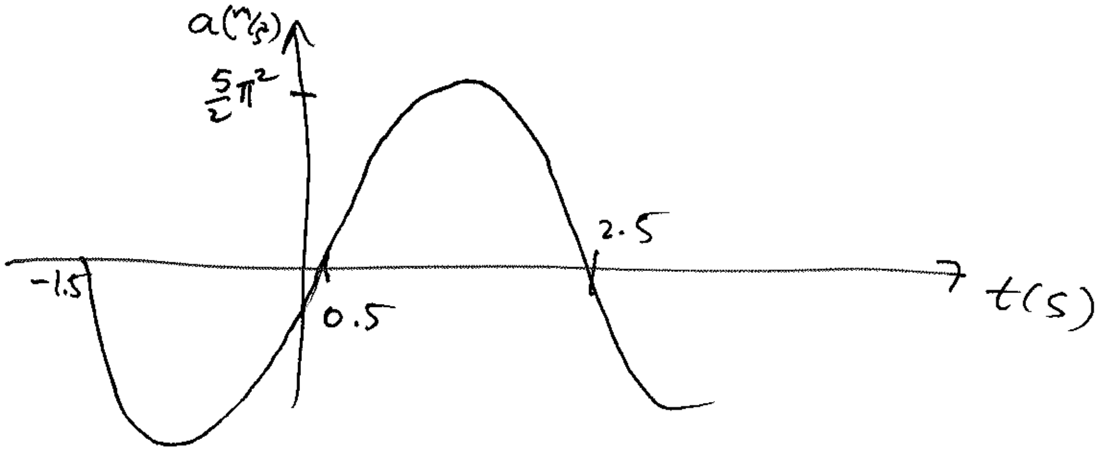
(I)



a) Velocity - time graph



acceleration - time graph



b) Amplitude = max. disp. from rest = 10m

~~Frequency~~

Period = time from crest to crest = 4.0s

Frequency = $\frac{1}{4}$ HZ $(1/T)$

by the way $\omega = 2\pi f = \frac{\pi}{2}$ $\quad (\omega^2 = \frac{\pi^2}{4}$

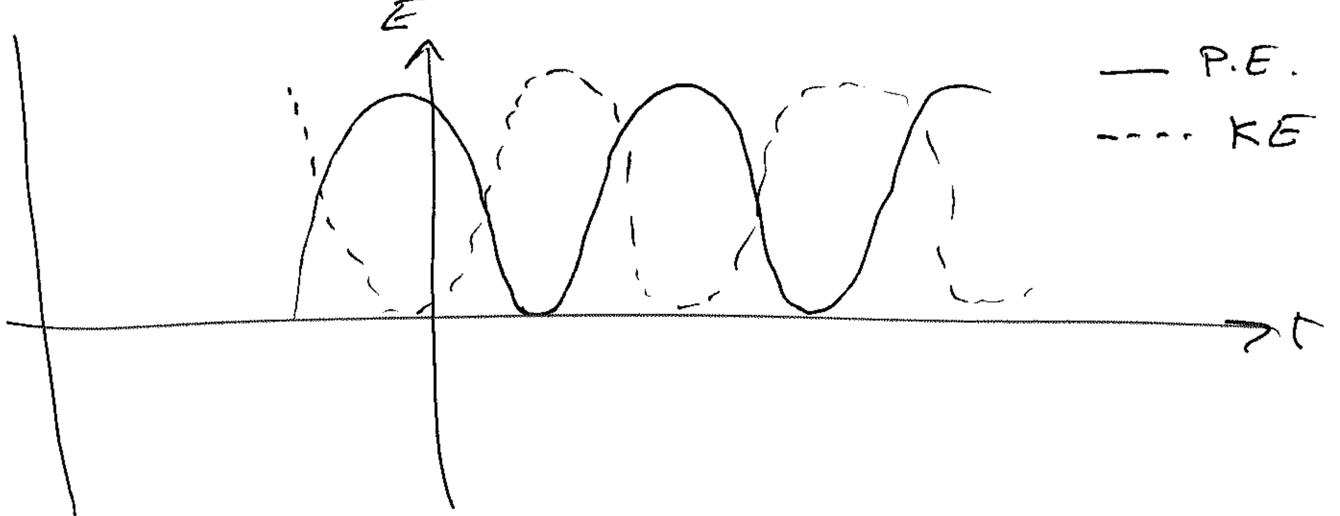
- Phase constant

@ $t = 0\,s$ $\quad x = 5\,m$

so $A\cos(\omega t + \delta)$ gives $\delta = 60°$ or $1.05\,rad$

$$(10\,m)\cos(\delta) = 5 \Rightarrow \delta = \cancel{60°}\ \cancel{\text{or } 1.05\,rad}\ \cancel{87° \text{ or } 1.52}$$

c)



2) $M = 1.98 \times 10^{30}\,kg$, $\quad r = 10\,km$ $\quad T = 1.0\,s$

a) $T = \frac{1}{f}$ so $f = 1\,Hz$ $\quad \omega = 2\pi\,s^{-1}$

$R\omega = V$ so $10\,km \times 2\pi\,s^{-1} = \underline{20\pi\,\frac{km}{s}}$

b) $|\vec{F}| = G\frac{M_{star}\,m_{object}}{r^2}$

$$g = \frac{G M_{star}}{r^2} = \frac{6.67 \times 10^{-11}(1.98 \times 10^{30}\,kg)}{(10 \times 10^3)^2}$$

$$= 1.32 \times 10^{12}\,m/s^2$$

c) radius of orbit $= 10000\,m + 1000\,m$
$= 11000\,m \quad (11\,km)$

(Next page)

$$T = \sqrt{K}\, r^{3/2}$$

with $K = \frac{4\pi^2}{GM_*} = 2.99 \times 10^{-19}$

$$T = 6.31 \times 10^{-4}\ s$$

$f = \frac{1}{T} = $ 1585 cycles/sec or rev/sec

~~$2\pi f = 9961$ rad/s~~

d) period of satellite = period of star = 1.0 s

$$T^2 = K r^3$$

$$1 = (2.99 \times 10^{-19})\, r^3$$

or $\left[\frac{1}{(2.99 \times 10^{-19})}\right]^{1/3} = r = 1.5 \times 10^6$ m

above center of star

~~but this is almost the same as~~

$\simeq 1.49 \times 10^6$ m above surface

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e) 

Force & radius vector are anti-parallel

$(\theta = 180°)$ so $\vec{\tau} = \vec{r} \times \vec{F} = 0$

$\vec{\tau} = 0 \Rightarrow \frac{d\vec{L}}{dt} = \vec{0}$ So angular momentum is conserved.

it must always point either straight up or straight down.

III b

(3) $\Delta\vec{\theta} = \vec{\theta} + \vec{\pi} \; (-\hat{k})$ ⇐ clockwise rotation

The forces acting on the blocks, which slow them down after the collision with the bullet, are the frictional forces.

$$|\vec{f}_{f_1}| = (m_1 + m_b)\mu_k g$$

$$|\vec{f}_{f_2}| = m_2 \mu_k g$$

To get $\vec{\alpha}$ use $\Sigma\vec{\tau} = I\vec{\alpha}$

$$I_1 = (m_1 + m_b)(L_1)^2$$

$$I_2 = (m_2)[L_1 + L_2]^2$$

$$\vec{\tau}_1 = L_1 |\vec{f}_{f_1}| \hat{k} = L_1 (m_1 + m_b)\mu_k g \hat{k}$$

$$\vec{\tau}_2 = (L_1 + L_2)|\vec{f}_{f_2}| \hat{k} = (L_1 + L_2) m_2 \mu_k g \hat{k}$$

$$\Sigma\vec{\tau} = \mu_k g [L_1 (m_1 + m_2 + m_b) + L_2 m_2] \hat{k}$$

(i) $I = I_1 + I_2 = (m_1 + m_b)(L_1)^2 + m_2 [L_1 + L_2]^2$

$$\vec{\alpha} = \frac{\Sigma\vec{\tau}}{I} \Big\} \text{ a known quantity}$$

what we know:
$\Delta\vec{\theta}$, $\vec{\alpha}$, $\Delta t = t_1$

Angular momentum right before collision = Angular momentum right after collision

To get angular momentum right after collision we need ω_i, the angular velocity of the system right after bullet collides.

$$\Delta\vec{\theta} = \vec{\omega}_i t_1 + \frac{1}{2}\vec{\alpha} t_1^2$$

$$-(\theta+\pi) = -|\vec{\omega}_i| t_1 + \frac{1}{2}|\vec{\alpha}| t_1^2 \quad (\text{z-component of above eqn})$$

$$|\vec{\omega}_i| = \frac{(\theta+\pi)}{t_1} + \frac{1}{2}|\vec{\alpha}| t_1$$

Angular momentum of system right after collision with bullet

$= I\omega_i \rightarrow$ I is known (Eq (i))
ω_i is known (from above)

$$|\vec{L}_b| = |\vec{L}_f|$$

$$m_b |\vec{V}_b| \underbrace{|L_1|}_{\text{length of rod to block 1}} \sin\theta = I\omega_i$$

so $$\boxed{|\vec{V}_b| = \frac{I\omega_i}{m_b |L_1| \sin\theta}}$$

b) $$\vec{\omega}_f = \vec{\omega}_i + \vec{\alpha} t_1$$

$$-\omega_f = -\frac{(\theta+\pi)}{t_1} + \frac{1}{2}|\vec{\alpha}| t_1 + |\vec{\alpha}| t_1$$

$$|\vec{V}_{f_{m_1}}| = L_1 |\vec{\omega}_f| \leftarrow \text{linear } \cancel{\text{vel}} \text{ speed of } m_1 \text{ just before it hits } m_3$$

$\cancel{(m_1+m_b)|\vec{V}_{f_{m_1}}| = m_1 + m_2}$

V

Easiest to use Eqn from sheet

$$V_{3f} = \frac{2(m_1+m_n)}{\underbrace{(m_1+m_n)+m_3}_{\text{known}}} \underbrace{|\vec{V}_{fm_1}|}_{\text{known}}$$