28:

a) $^{234}_{92}U \rightarrow ^{m}_{q}X + ^{4}_{2}\alpha$

charge: $92 = q + 2 \Rightarrow q = 90 \rightarrow Z = 90 \Rightarrow X = Th.$

mass: $234 = m + 4 \Rightarrow m = 230$

$$\Rightarrow \boxed{X = ^{230}_{90}Th}$$

b) $^{32}_{15}P \rightarrow ^{m}_{q}X + ^{0}_{-1}e^{-} + ^{0}_{0}\bar{\nu}_e$

charge: $15 = q - 1 \Rightarrow q = 16 \rightarrow Z = 16 \Rightarrow X = S$

mass: $32 = m + 0 \Rightarrow m = 32$

$$\Rightarrow \boxed{X = ^{32}_{16}S}$$

c) $^{m}_{q}X \rightarrow ^{30}_{14}Si + ^{0}_{+1}e^{+} + ^{0}_{0}\nu_e$

$\Rightarrow$ charge: $q = 14 + 1 + 0 = 15 \Rightarrow Z = 15 \Rightarrow X = P$

mass: $m = 30 + 0 + 0 \Rightarrow \boxed{X = ^{30}_{15}P}$

d) $^{24}_{11}Na \rightarrow ^{24}_{12}Mg + ^{0}_{-1}e^{-} + ^{0}_{0}\bar{\nu} \longrightarrow ^{m}_{q}X + ^{0}_{0}\gamma$

mass $m = 24$

charge $q = 12 - 1 = 11$ $\Rightarrow X = ^{24}_{11}Na$

$\rightarrow$ This is simply γ-emission from excited $^{24}_{11}Na$ nucleus.

$$\boxed{X = ^{24}_{11}Na}$$

Note: It is possible to misread this equation as stating the electron and anti-neutrino fly off leaving a secondary decay of $^{24}_{12}Mg \longrightarrow ^{m}_{q}X + \gamma$

$\Rightarrow X = ^{24}_{12}Mg \rightarrow$ This is also an acceptible answer.

66: a) How many ^{222}Rn atoms are there in $1\,m^3$ of air given an activity density of $\frac{4\,pCi}{L}$

$$1\,Ci = 3.7\times10^{10}\,Bq \qquad \Rightarrow \frac{4\,pCi}{L} = \frac{(4)(10^{-12})(3.7\times10^{10}\,Bq)}{10^{-3}\,m^3}$$
$$1\,L = 10^{-3}\,m^3$$

$$= \frac{148\,Bq}{m^3}$$

$$\Rightarrow \frac{R}{V} = \frac{\text{decay rate}}{\text{Volume}} = \frac{148\,s^{-1}}{m^3}$$

Since $R = \lambda N = \frac{\ln(2)}{t_{1/2}} N$, $N = \frac{t_{1/2}}{\ln(2)} R$

$$\Rightarrow \frac{N}{V} = \frac{t_{1/2}}{\ln(2)} \cdot \left(\frac{R}{V}\right) \quad \rightarrow \quad t_{1/2} = 3.82 \text{ days} \simeq 3.3\times10^5\,s$$

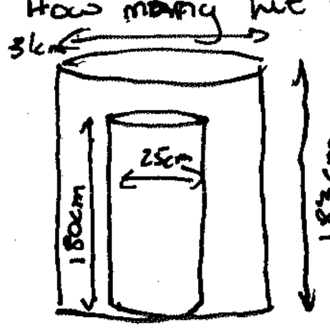
$$\Rightarrow \frac{N}{V} = \frac{(3.3\times10^5\,s)(148\,s^{-1})}{0.6931} \approx 7\times10^7 \frac{\text{atoms}}{m^3}$$

$\Rightarrow$ There are approximately 7×10^7 Radon nuclei in $1\,m^3$ of air for an activity density of $\frac{4\,pCi}{L}$.

b) This part is a little longer.

Need: $\rightarrow$ How many α's hit the person in a year
$\rightarrow$ How much energy each one has.

How many hit the subject in a year.



The air around the person is contained between the two cylinders shown (I didn't include air under his/her feet)

$$V_{Air} = \pi\cdot\left(\frac{31\,cm}{2}\right)^2 \cdot 183\,cm - \pi\left(\frac{25\,cm}{2}\right)^2 \cdot 180\,cm$$
$$= 5\times10^4\,cm^3 = 5\times10^{-2}\,m^3$$

So the volume of air we're considering is

$$V_{Air} \simeq 5\times10^{-2} m^3.$$

Given $\frac{4\,pCi}{L} = \frac{148\,Bq}{m^3}$ from part a), we know the activity in this volume of air is

$$R = \frac{148\,Bq}{m^3} \times 5\times10^{-2} m^3 = 7.4\,Bq = 7.4\,\frac{\text{decays}}{\text{sec}}$$

Each decay produces 1 α-particle

$$\Rightarrow R_\alpha = 7.4\,\frac{\alpha\text{ particles}}{\text{sec}}$$

Since only about 50% of them hit the person, then

$$R_{person} \approx 3.8\,\frac{\alpha\text{ particles}}{\text{sec}}$$ ← Rate at which the person absorbs α-particles

Over a year, (3.2×10^7 s), the person absorbs

$$N_{absorbed} = 3.8\,\frac{\alpha\text{ particles}}{s} \cdot 3.2\times10^7 s \approx 1\times10^8\ \alpha\text{-particles}$$

$\Rightarrow$ over the course of a year, the person absorbs 1×10^8 of these α-particles.

We now need to figure out how much energy each α-particle carries.

The decay of Radon

$$^{222}_{86}Rn \rightarrow {}^{m}_{q}X + {}^{4}_{2}\alpha \qquad \begin{array}{l} \text{charge: } 86 = q + 2 \Rightarrow q = 84 \\ \rightarrow \text{mass } 222 = m + 4 \Rightarrow m = 218 \end{array}$$

$$^{218}_{14}Po$$

$$\Rightarrow {}^{222}_{86}Rn \rightarrow {}^{218}_{84}Po + {}^{4}_{2}\alpha$$ ← this is the nuclear process
← add electrons to get atomic process.

$$\left({}^{222}_{86}Rn + 86e^-\right) \rightarrow {}^{218}_{84}Po + 86e^- + {}^{4}_{2}\alpha$$

$$\underbrace{\left({}^{222}_{86}Rn + 86e^-\right)}_{\simeq \text{ Mass of Radon atom}} \rightarrow \underbrace{\left({}^{218}_{84}Po + 84e^-\right)}_{\simeq \text{ Mass of Polonium atom}} + \underbrace{\left({}^{4}_{2}\alpha + 2e^-\right)}_{\simeq \text{ Mass of helium atom}}$$

→ Ignoring the electronic binding energies (they are very small and inconsequential to this calculation), we can use the data for atomic masses in Appendix C

$$\begin{array}{rl} 222.017571 \text{ Amu} & \leftarrow \text{Mass of Radon-222 atom} \\ -218.008965 \text{ Amu} & \leftarrow \text{Mass of Polonium-218 atom} \\ -\ \ \ 4.002602 \text{ Amu} & \leftarrow \text{Mass of Helium-4 atom} \\ \hline \end{array}$$

$$\Rightarrow \quad 6\times10^{-3}\text{ Amu} \times 1.67\times10^{-27}\frac{\text{kg}}{\text{Amu}} = 9.93\times10^{-30}\text{ kg}$$
$$\simeq 1\times10^{-29}\text{ kg}.$$

$\Rightarrow$ 1×10^{-29} kg is converted to energy

$$E = mc^2 = 1\times10^{-29}\text{ kg}\cdot\left(3\times10^{8}\text{ m/s}\right)^2 = 9\times10^{-12}\text{ J}.$$

$\Rightarrow$ Each α-decay of Radon-222 liberates $\approx 9\times10^{-12}$ J.

How much of this energy does the α-particle get?

1) This is a nuclear process $\Rightarrow$ the electrons are essentially spectators in this reaction.

2) The initial speed of the ^{222}Rn is small

Taking the speed of the Radon to be initially 0, and using non-relativistic momentum conservation

$$0 = m_{Po} V_{Po} + m_\alpha V_\alpha \Rightarrow V_{Po} = \frac{m_\alpha}{m_{Po}} V_\alpha$$

Non-relativistic energy:

$$\tfrac{1}{2} m_{Po} V_{Po}^2 + \tfrac{1}{2} m_\alpha V_\alpha^2 = \Delta E$$

$$\Rightarrow \frac{1}{2} m_P \left(\frac{m_\alpha}{m_{Po}} V_\alpha \right)^2 + \frac{1}{2} m_\alpha V_\alpha^2 = \Delta E$$

$$\Rightarrow \underbrace{\frac{1}{2} m_\alpha V_\alpha^2}_{E_\alpha} \left(\frac{m_\alpha}{m_{Po}} + 1 \right) = \Delta E$$

$$\Rightarrow E_\alpha = \frac{\Delta E}{1 + (m_\alpha / m_{Po})} \simeq 0.98\,\Delta E \quad \text{since } m_\alpha / m_{Po} \simeq 2 \times 10^{-2}$$

~~Since~~ So, to 1 sig. fig., the emitted α-particle has about

$$E_\alpha \simeq 1 \times 10^{-12}\ \text{J}$$

Some of this energy is lost to EM interactions with the electrons, and to ~~other~~ interactions with other particles in the air. But it's a ~~good~~ rough estimate of how much energy each α-particle deposits in the person.

~~So, over the course of a year, (1x1~~

Over the course of the year, then, the subject absorbs 1×10^8 α-particles each having an energy of $\simeq 1 \times 10^{-12}$ J

$\Rightarrow$ The total energy absorbed by α-exposure is then

$$E_{TOT} = (1 \times 10^8)(1 \times 10^{-12}) = 1 \times 10^{-4}\ \text{J}.$$

The rad Dose is found, for a 65 kg individual,

$$D_{rad} = \frac{1\times10^{-4}\,J}{65\,kg} \cdot \frac{1\,rad}{10^{-2}\,J/kg} = 1.5\times10^{-4}\,rad$$

$$= 1.5\ mrad$$

and the rem dose

$$D_{rem} = D_{rad} \times RBE$$

$$= 1.5\,mrad \times 15\,\frac{mrem}{rad} \simeq 2\ mrem.$$

The yearly dose is thus estimated at $\approx$ 2mrem

c) 2 mrem/year, when compared with an average background of about 300 mrem/year is not very significant.

If this were the whole story, the EPA guidelines would seem to be a bit overly strict.

However, we've only calculated the exposure to skin. We've neglected the exposure to the inside of the body through the lungs.

The internal dose has a much higher RBE and is the reason for the EPA guidelines.

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