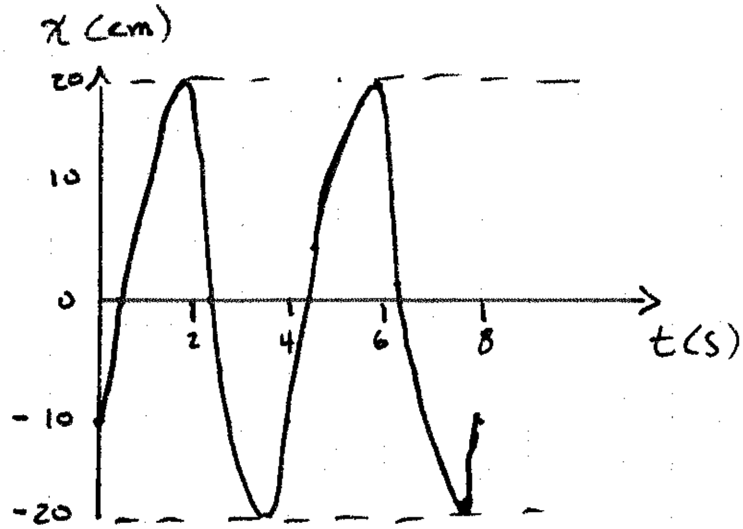
Ch. 14

#6.



Simple Harmonic motion: $x(t) = A\cos(\omega t + \phi_0)$

a) Amplitude $|x(t)|_{max} = A = 20\text{cm}$

$\Rightarrow \boxed{A = 20\text{cm}}$

b) frequency: 2 complete cycles in 8 seconds.

$\Rightarrow f = \frac{2 \text{ cycles}}{8 \text{ secs}} = 0.25 \text{ Hz}.$

angular frequency: $\omega = f \cdot \frac{2\pi \text{ rad}}{\text{cycle}}$

$\omega = 2\pi(0.25) \frac{\text{rad}}{s}$

$= 1.57 \frac{\text{rad}}{s}.$

$\boxed{f = 0.25 \text{ Hz}, \quad \omega = 1.57 \text{ rad/s}.}$

c) Phase constant.

$x(0) = -10\text{cm} = 20\text{cm} \cos(\omega \cdot 0 + \phi_0) = 20\text{cm}\cos(\phi_0)$

$\Rightarrow \cos(\phi_0) = \frac{-10\text{cm}}{20\text{cm}} = -\frac{1}{2}.$

$\phi_0 = \arccos\left(-\frac{1}{2}\right) = \pm \frac{2\pi}{3} \text{ rad}.$

which one? At $t=0$, $\frac{dx}{dt} > 0$. $\left.\frac{dx}{dt}\right|_{t=0} = -A\omega\sin(\omega \cancel{t} + \phi_0) = -A\omega\sin\phi_0$

At $t=0$, $v = \frac{dx}{dt} > 0$

$$\left.\frac{dx}{dt}\right|_{t=0} = -A\omega\sin(\cancel{\omega t}+\phi_0) = -A\omega\sin(\phi_0)$$

$$\therefore \quad -A\omega\sin(\phi_0) > 0 \quad \Rightarrow \quad \sin(\phi_0) < 0$$

$$\Rightarrow \phi_0 = -2\pi/3.$$

→ only the −ve ϕ_0 works.

In principle, we could add any number of complete cycles (2π rad) to ϕ_0 and not affect the answer

so $$\boxed{\phi_0 = \left(-\frac{2\pi}{3} + 2m\pi\right)\text{rad}}, \quad m \in \mathbb{Z}.$$

→ m is any integer.

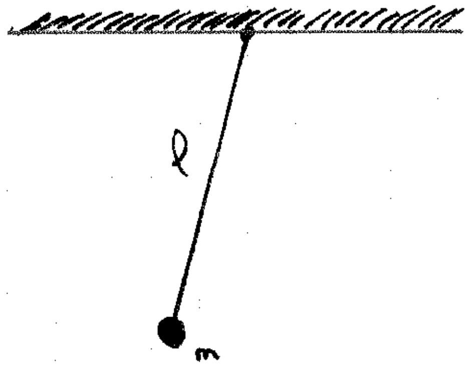
you could, for example choose

$$\phi_0 = -\frac{2\pi}{3}\text{ rad}, \ \frac{4\pi}{3}\text{ rad}, \ \frac{10\pi}{3}\text{ rad etc...}$$

your choice.

Ch. 4

#22



Simple pendulum: $\omega \approx \sqrt{g/\ell}$ for small oscillations

$$\Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{g}{\ell}} \Rightarrow T = 2\pi\sqrt{\frac{\ell}{g}}$$

a) $T = 2\pi\sqrt{\frac{\ell}{g}}$ $\Rightarrow$ no mass dependence

change m $\Rightarrow$ $\boxed{\text{period unchanged}}$

b) ℓ doubled. $T \propto \ell^{1/2}$

$$\therefore T' = 2\pi\sqrt{\frac{2\ell}{g}} = \sqrt{2}\cdot 2\pi\sqrt{\frac{\ell}{g}} = \sqrt{2}\cdot T$$

$$\Rightarrow \boxed{T' = \sqrt{2}\cdot 4s = 5.66\ s.}$$

c) length halved:

$$T' = 2\pi\sqrt{\frac{\frac{1}{2}\ell}{g}} = \sqrt{\frac{1}{2}}\cdot 2\pi\sqrt{\frac{\ell}{g}} = \frac{1}{\sqrt{2}}\cdot T$$

$$\boxed{T' = \frac{1}{\sqrt{2}}\cdot 4s = 2.83\ s}$$

d) $\boxed{\text{T independent of Amplitude, provided Amplitude is small. If amplitude is large, motion is anharmonic.}}$

Ch. 14 #28

$f = 1\,Hz.$

$\tau = 4s$

$$x(t) = A e^{-t/2\tau} \cos(\omega t + \phi_0)$$

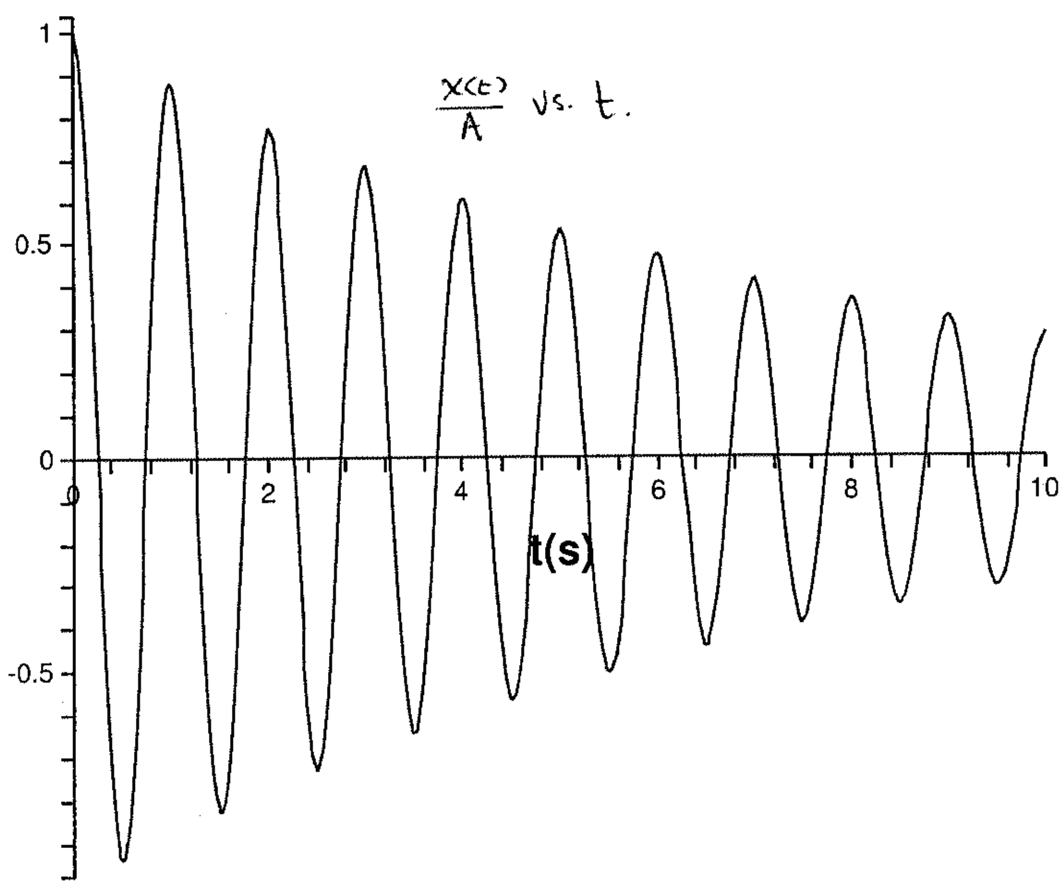
$$\omega = 2\pi \frac{rad}{cycle} \times f = 2\pi \frac{rad}{cycle} \cdot \frac{1\,cycle}{s} = 2\pi \frac{rad}{s}$$

choose $\phi_0 = 0$ rad

$$\Rightarrow \frac{x(t)}{A} = e^{-t/8s} \cdot \cos\left(2\pi \frac{rad}{s} \cdot t\right)$$

Important points: 10 complete cycles from $t=0$ to $t=10s$

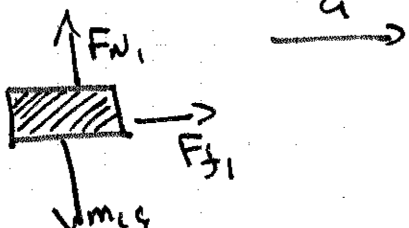
$x(8s) = \frac{1}{e} x(0) \approx 0.368\, x(0).$



Ch. 14

#54.

Forces on Top block.



$F_{N_1} - m_1 g = 0 \Rightarrow F_{N_1} = m_1 g$

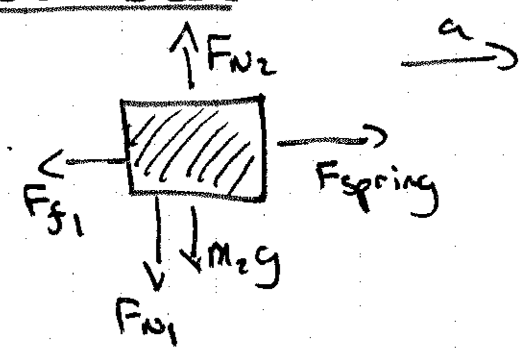
$F_f = m_1 a \Rightarrow a = \dfrac{F_f}{m_1}$

$F_f \le \mu_s \cdot F_{N_1} = \mu_s \cdot m_1 g$

$\Rightarrow |a| \le \dfrac{\mu_s \cdot m_1 g}{m_1} = \mu_s g$

$$\boxed{|a| \le \mu_s g} \quad (1)$$

Forces on Lower Block



$\Rightarrow F_{spring} - F_{f_1} = m_2 a$

$\Rightarrow F_{spring} = m_2 a + F_{f_1}$

$= m_2 a + m_1 a$

$F_{spring} = (m_1 + m_2) a$

$F_{spring} = -kx$

$\Rightarrow a = -\dfrac{k}{m_1 + m_2} x$

$\Rightarrow \dfrac{\partial^2 x}{\partial t^2} = -\dfrac{k}{m_1 + m_2} x \rightarrow$ equation for simple harmonic motion

$\omega = \sqrt{\dfrac{k}{m_1 + m_2}}$.

$\Rightarrow x(t) = A \cos(\omega t + \phi_0)$

$\Rightarrow a(t) = -\omega^2 A \cos(\omega t + \phi_0) \Rightarrow |a_{max}| = \omega^2 A \longrightarrow$

Now compare with 1.

$$|a_{max}| = \omega^2 A \le \mu_s \cdot g$$

$$\Rightarrow A \le \frac{\mu_s \cdot g}{\omega^2} \Rightarrow A_{max} = \frac{\mu_s g}{\omega^2}$$

$$\omega^2 = \frac{k}{m_1+m_2} \Rightarrow A_{max} = \frac{\mu_s \cdot g}{k/_{m_1+m_2}} = \frac{\mu_s (m_1+m_2) g}{k}$$

$$A_{max} = \frac{\mu_s \cdot (m_1+m_2) g}{k}.$$

$$= \frac{(0.5)(1kg+5kg) \cdot 9.81\,m/s^2}{50\,N/m} = \frac{0.589\,kg\,m/s^2}{N/m}$$

$$= \frac{0.589\,N}{N/m} = 0.589\,m$$

$$\boxed{A_{max} = 0.589m.}$$