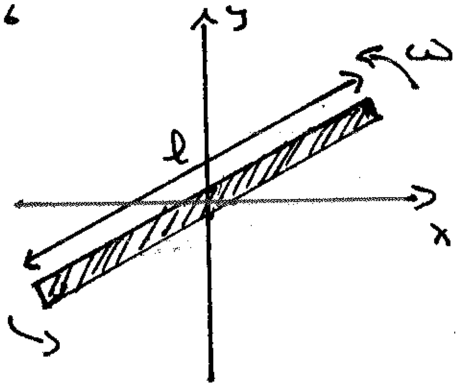
Knight 13 – 46



$l = 2m$
$m = 0.5\,kg$

$$|\vec{\omega}| = \frac{120\,rev}{min}\cdot\left(\frac{1\,min}{60\,s}\right)\cdot\frac{2\pi\,rad}{rev}$$

$$= \frac{4\pi\,rad}{s}$$

→ direction: The bar is rotating about the z-axis in a sense that it rotates counter-clockwise when viewed from the z-direction.

$$\therefore\ \vec{\omega} = \frac{4\pi\,rad}{s}\,\hat{k} = \left(0, 0, \frac{4\pi\,rad}{s}\right)$$

right-hand

→ Check: Lining thumb on the z-axis (out of page), fingers curl in the direction of rotation ✓.

Angular momentum for a rigid body

$$\vec{L} = I\vec{\omega}\ ,$$

I for a beam rotating about its centre is given on pg. 385 by

$$I = \frac{1}{12} m l^2$$

$$\Rightarrow \vec{L} = \frac{1}{12} m l^2 \vec{\omega} = \frac{1}{12}\cdot(0.5\,kg)\cdot(2m)^2\cdot\frac{4\pi\,rad}{s}\,\hat{k}$$

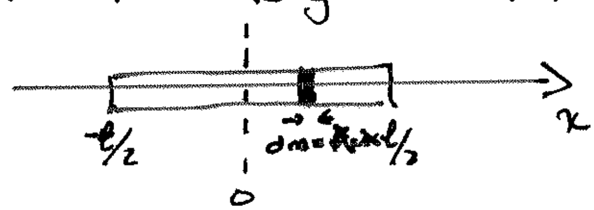
$$= \frac{2}{3}\pi\,\frac{kg\,m^2}{s}\,\hat{k} = 2.09\,\frac{kg\,m^2}{s}\,\hat{k}$$

→ If we don't have a table, how can we find I?

$$I = \int r^2\,dm\ .$$ → For a rod through its centre

$dm = \lambda dx$ → λ = linear mass density

→ $\lambda = \frac{m}{l}$



$$\Rightarrow I = \frac{m}{l}\int_{-l/2}^{l/2} x^2\,dx = \frac{m}{l}\cdot\frac{x^3}{3}\Big|_{-l/2}^{l/2} = \frac{m}{\cancel{l}}\cdot\frac{2l^3}{3\cdot 8} = \frac{ml^2}{12} = \frac{1}{12}ml^2$$

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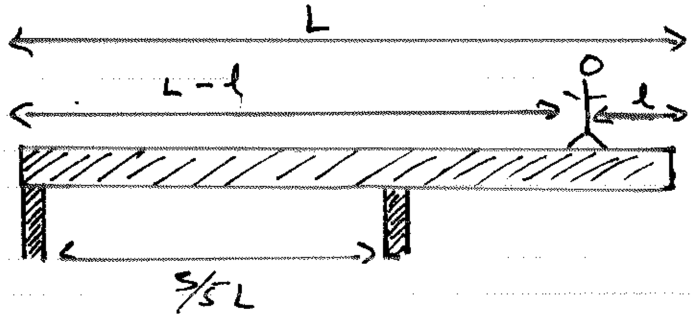
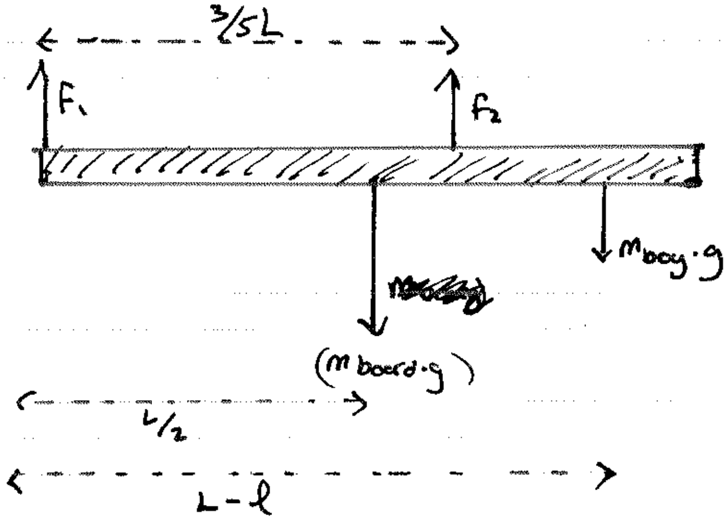
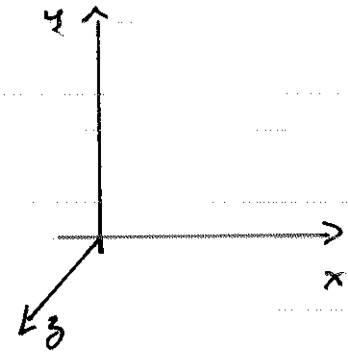


The beam is resting on the supports but not attached to them. This means that the supports can give any required upwards force to the beam, but cannot hold it down.

$\Rightarrow$ For static equilibrium to really hold, then, the support forces must point upwards (*) $\leftarrow$ will come back to this point.

Forces on Board





Static Equilibrium $\rightarrow$ $\vec{a} = 0$, $\vec{\alpha} = 0$ $\Rightarrow \sum \vec{F} = 0$, $\sum \vec{\tau} = 0$.

$$\sum \vec{F} = 0 \Rightarrow F_1 \hat{j} + F_2 \hat{j} - m_{board} g \hat{j} - m_{boy} g \hat{j} = 0$$

$$\Rightarrow F_1 + F_2 = (m_{board} + m_{boy}) g . \quad (1)$$

$\sum \vec{\tau} = 0 \Rightarrow$ $\rightarrow$ Choose left support as axis.

$$\vec{r_1} \times F_1 \hat{j} + \vec{r_2} \times F_2 \hat{j} + \vec{r}_{board} \times (-m_{board} g \hat{j}) + \vec{r}_{boy} \times (-m_{boy} g \hat{j}) = 0$$

$$\Rightarrow 0 F_1 \hat{k} + \frac{3}{5} L F_2 \hat{k} - \frac{L}{2} m_{board} g \hat{k} - (L - \ell) m_{boy} g \hat{k}$$

$$\Rightarrow \frac{3}{5} L F_2 = \left(\frac{L}{2} m_{board} + (L - \ell) m_{boy} \right) g . \quad (2)$$

Simplify (2).

$$F_2 = \frac{5}{3L} \cdot L \left\{ \frac{m_{board}}{2} + \left(1-\frac{l}{L}\right) m_{boy} \right\} g$$

$$F_2 = \frac{5}{6} m_{board}\, g + \frac{5}{3}\left(1-\frac{l}{L}\right) m_{boy}\, g \quad (2a)$$

plugging (2a) into (1)

$$F_1 = (m_{board} + m_{boy})g - F_2$$

$$= m_{board}\, g + m_{boy}\, g - \frac{5}{6} m_{board}\, g - \frac{5}{3}\left(1-\frac{l}{L}\right) m_{boy}\, g$$

$$F_1 = \frac{1}{6} M_{board}\, g - \frac{2-5\,l/L}{3} m_{boy}\, g.$$

$$\Rightarrow \quad F_1 = \left(\frac{1}{6} m_{board} - \frac{2-5\,l/L}{3} m_{boy}\right) g$$

$$F_2 = \left(\frac{5}{6} m_{board} + \frac{5}{3}\left(1-\frac{l}{L}\right) m_{boy}\right) g.$$

for $0 < l < L$, $F_2 > 0$ $\Rightarrow$ The board is always in contact with ~~supp~~ the rightmost support

However, for F_1, we find that the board can lose contact

$$F_1 \le 0$$

$$\Rightarrow \left(\frac{1}{6} m_{board} - \frac{2-5\,l/L}{3} m_{boy}\right) g \le 0$$

$$\Rightarrow \quad \frac{1}{6} M_{board} \le \frac{2-5\,l/L}{3} m_{boy}$$

$$\Rightarrow \quad \frac{m_{board}}{2\, m_{boy}} \le 2 - 5\,l/L \quad \Rightarrow \quad \frac{1}{2}\frac{m_{board}}{m_{boy}} - 2 \le -\frac{5l}{L}$$

$$\Rightarrow \quad \frac{5l}{L} \le 2 - \frac{1}{2}\frac{m_{board}}{m_{boy}}$$

$\longrightarrow$ over

$$\Rightarrow \ell \le \frac{2}{5}L - \frac{1}{10}L\frac{m_{board}}{m_{boy}}.$$

$\Rightarrow$ The board will topple for $\ell \le \frac{2}{5}L - \frac{1}{10}L\frac{m_{board}}{m_{boy}}$

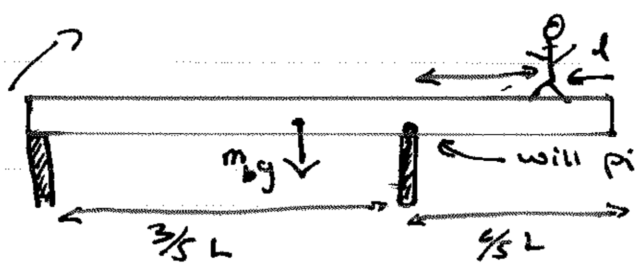
evaluating: $L = 5m$, $m_{board} = 40kg$, $m_{boy} = 20kg$.

$$\ell \le \left(\frac{2}{5} - \frac{1}{10}\cdot\frac{40kg}{20kg}\right)L$$

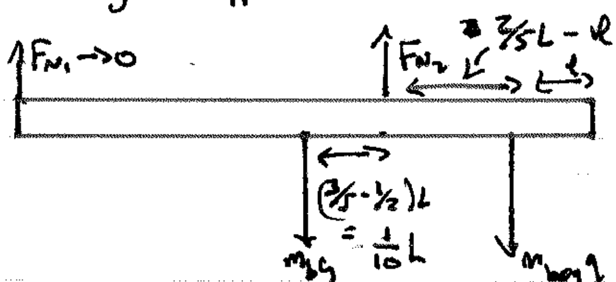
$$\ell \le \left(\frac{2}{5} - \frac{1}{5}\right)L = \frac{1}{5}\cdot 5m = 1m.$$

$\Rightarrow$ If he gets closer than 1m to the end, the board will topple.

If you have a clear picture in your mind of what's going on, you can save yourself some work.



Choosing the right support as the axis...



$$\Rightarrow \sum \tau = \frac{1}{10} m_b g L - \left(\frac{2}{5}L - \ell\right) m_{boy} g = 0$$

$$\Rightarrow \frac{1}{10} m_{beam} g L = \left(\frac{2}{5}L - \ell\right) m_{boy} g$$

$$\Rightarrow \frac{1}{10}\frac{m_{beam}}{m_{boy}} = \left(\frac{2}{5} - \frac{\ell}{L}\right)$$

$$\Rightarrow \ell = \left(\frac{2}{5} - \frac{1}{10}\frac{m_{beam}}{m_{boy}}\right)L = \frac{1}{5}L = 1m \text{ as before}$$

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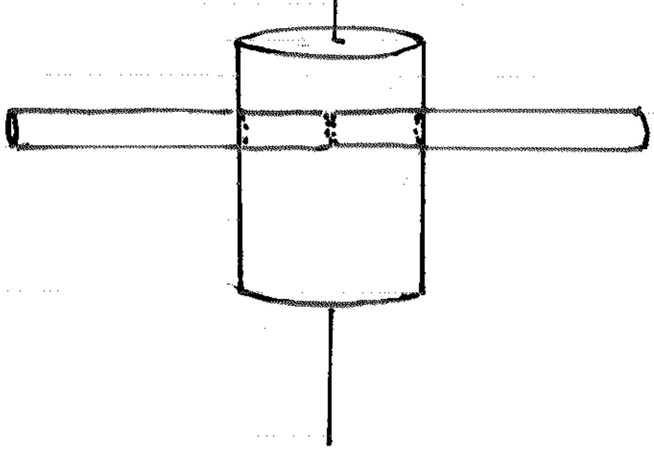
$\omega_0 = 1.5\,\text{rpm} = \frac{\pi}{20}\,\frac{\text{rad}}{\text{s}}$



$M_{arm} = 2.5\,kg$

$L_{arm} = 0.78\,m$

$M_{Torso} = 40\,kg$

$R_{Torso} = 0.2\,m$

"Arms" → rods about an end. → $I_{arm} = \frac{1}{12} M_{arm} L_{arm}^2$

"Torso" → cylinder about its centre → $I_{Torso} = \frac{1}{2} M_{Torso} R_{Torso}^2$

$$\Rightarrow \quad I_{Total} = \frac{1}{2} M_{Torso} R_{Torso}^2 + 2 \times \left(\frac{1}{12} M_{arm} L_{arm}^2 \right)$$

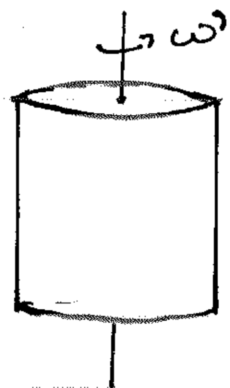
$$I_{Total} = \frac{M_{Torso} R_{Torso}^2}{2} + \frac{M_{arm} L_{arm}^2}{6}.$$

$$\Rightarrow |\vec{L}| = \left(\frac{M_{Torso} R_{Torso}^2}{2} + \frac{M_{arm} L_{arm}^2}{6} \right) |\vec{\omega_0}|$$

Since no external torques are acting, the angular momentum is conserved.

→ Now she raises her arms

ω'



$M_{TOT} = 45\,kg$

$R = 0.2\,m = R_{Torso}$

$$\Rightarrow I' = \frac{1}{2} M_{TOT} R_{Torso}^2$$

Since L is fixed can find $|\vec{\omega}'|$

$$|\vec{L}| = I' |\vec{\omega}'| \quad \Rightarrow \quad |\vec{\omega}'| = \frac{|\vec{L}|}{I'} = \frac{|\vec{L}|}{\frac{1}{2} M_{TOT} R_{Torso}^2}$$

since $|\vec{L}| = \left(\frac{M_{TORSO} R_{TORSO}^2}{2} + \frac{M_{arm} L_{arm}^2}{6} \right) |\vec{\omega_0}|$

$$|\vec{\omega'}| = \frac{\frac{1}{2} M_{TORSO} R_{TORSO}^2 + \frac{1}{6} M_{arm} L_{arm}^2}{\frac{1}{2} M_{TOT} R_{TORSO}^2} |\vec{\omega_0}|$$

$$|\vec{\omega'}| = \left\{ \frac{M_{TORSO}}{M_{TOT}} + \frac{1}{3} \frac{M_{arm}}{M_{TOT}} \left(\frac{L_{arm}}{R_{TORSO}} \right)^2 \right\} |\vec{\omega_0}|$$

To simplify, note $M_{TORSO} = M_{TOT} - 2M_{arm}$

$$|\omega'| = \left\{ 1 - \frac{2M_{arm}}{M_{TOT}} + \frac{1}{3} \frac{M_{arm}}{M_{TOT}} \left(\frac{L_{arm}}{R_{TORSO}} \right)^2 \right\} |\vec{\omega_0}|$$

$$= \left\{ 1 + \frac{M_{arm}}{M_{TOT}} \left[\frac{1}{3} \left(\frac{L_{arm}}{R_{TORSO}} \right)^2 - 2 \right] \right\} |\vec{\omega_0}|$$

$M_{arm} = 2.5\,kg$

$M_{TOT} = 45\,kg$ $\Rightarrow \frac{M_{arm}}{M_{TOT}} = \frac{2.5}{45} = \frac{1}{18}$

$L_{arm} = 0.78\,m$

$R_{TORSO} = 0.2\,m$ $\Rightarrow \frac{L_{arm}}{R_{TORSO}} = \frac{0.78}{0.2} = 3.6$

$\vec{\omega_0} = \frac{\pi}{20} \frac{rad}{s}$

$$\Rightarrow |\omega'| = \left\{ 1 + \frac{1}{18} \left[\frac{1}{3} (3.6)^2 - 2 \right] \right\} \cdot \frac{\pi}{20} \frac{rad}{s}$$

$$= 1.13 \times \frac{\pi}{20} \frac{rad}{s} = 0.177 \frac{rad}{s}$$

or $|\omega'| = 1.69 \frac{rev}{min}$