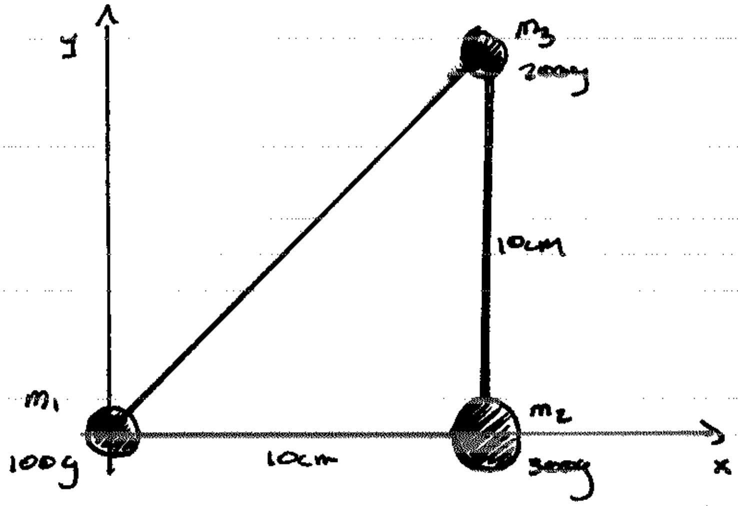
Knight ch. 13

#10.



$\vec{r}_1 = (0,0,0)$

$\vec{r}_2 = (10cm, 0, 0)$

$\vec{r}_3 = (10cm, 10cm, 0)$

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3}{m_1 + m_2 + m_3}$$

$$= \frac{100g\,(0,0,0) + 300g\,(10cm,0,0) + 200g\,(10cm,10cm,0)}{100g + 200g + 300g}$$

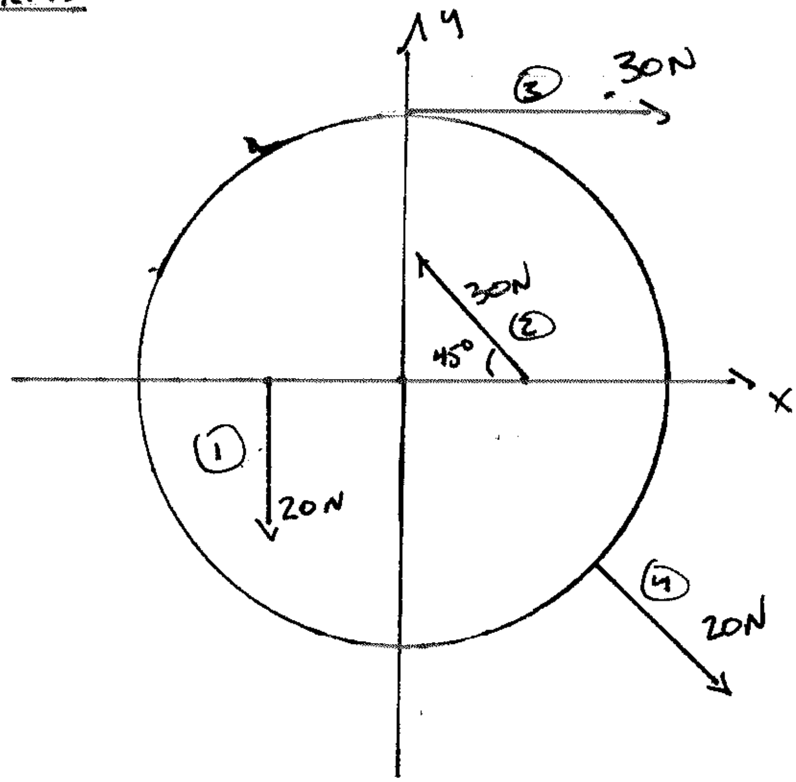
$$= \frac{(0,0,0) + (3000g\cdot cm, 0, 0) + (2000g\cdot cm, 2000g\cdot cm, 0)}{600g}$$

$$= \left(\frac{5000}{600}cm, \frac{2000}{600}cm, 0\right)$$

$$\boxed{\vec{r}_{cm} = (8.33cm, 3.33cm, 0)}$$

Knight Ch. 13

14



A note on cross products for those who know how to evaluate determinants.

$\vec{A} \times \vec{B}$ can be evaluated by the determinant equation

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

where $\vec{A} = (A_x, A_y, A_z)$

$\vec{B} = (B_x, B_y, B_z)$

Now on to the torques:

Method 1: Cross products. $\rightarrow \vec{\tau} = \vec{r} \times \vec{F}$

① $\rightarrow$ $\vec{r_1} = (-5cm, 0, 0)$, $\vec{F_1} = (0, -20N, 0)$

$\vec{r_2} = (5cm, 0, 0)$, $\vec{F_2} = (-30N\cos 45°, 30N\sin 45°, 0)$

$\vec{r_3} = (0, 10cm, 0)$, $\vec{F_3} = (30N, 0, 0)$

$\vec{r_4} = (10cm\cos 45°, -10cm\sin 45°, 0)$

$\vec{F_4} = (20N\cos 45°, -20N\sin 45°, 0)$

$$\therefore \quad \vec{\tau}_1 = \vec{r}_1 \times \vec{F}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -5\,\text{cm} & 0 & 0 \\ 0 & -20\,\text{N} & 0 \end{vmatrix} = 100\,\text{cm}\cdot\text{N}\,\hat{k} = 1\,\text{Nm}\,\hat{k}$$

$$\vec{\tau}_1 = 1\,\text{Nm}\,\hat{k}$$

$$\left[\vec{F}_2 = \frac{30\text{N}}{\sqrt{2}}(-1,1)\right]$$

$$\vec{\tau}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5\,\text{cm} & 0 & 0 \\ -\frac{30\text{N}}{\sqrt{2}} & \frac{30\text{N}}{\sqrt{2}} & 0 \end{vmatrix} = \frac{150\,\text{cm}\cdot\text{N}}{\sqrt{2}}\,\hat{k} = \frac{1.5}{\sqrt{2}}\,\text{Nm}\,\hat{k}$$

$$\vec{\tau}_3 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 10\,\text{cm} & 0 \\ 30\text{N} & 0 & 0 \end{vmatrix} = -300\,\text{cm}\cdot\text{N}\,\hat{k} = -3\,\text{Nm}\,\hat{k}$$

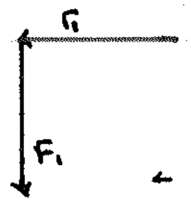
$$\vec{\tau}_4 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{10\,\text{cm}}{\sqrt{2}} & -\frac{10\,\text{cm}}{\sqrt{2}} & 0 \\ \frac{20\text{N}}{\sqrt{2}} & -\frac{20\text{N}}{\sqrt{2}} & 0 \end{vmatrix} = 0$$

$$\Rightarrow \vec{\tau}_{net} = \left(1\,\text{Nm} + \frac{1.5}{\sqrt{2}}\,\text{Nm} - 3\,\text{Nm}\right)\hat{k} = -0.939\,\text{Nm}\,\hat{k}$$

$$\boxed{\vec{\tau}_{net} = -0.939\,\text{Nm}}$$

Method 2 $\quad \vec{\tau} = |\vec{r}||\vec{F_\perp}|$ in a direction given by right hand rule.

$\vec{\tau_1}$:



$\Rightarrow \vec{\tau_1} = |5\,cm| * |20N| = 100\,cm \cdot N$

$= 1\,Nm$

$\leftarrow F_\perp = F_1$

direction $\rightarrow$ "out of page" $= \hat{k}$

$\vec{\tau_1} = 1Nm\hat{k}$

τ_2:

$\rightarrow F_\perp = F_2 \sin(45°)$

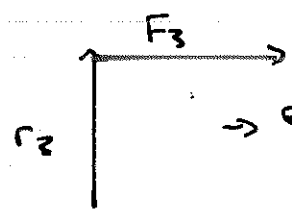
$\Rightarrow \vec{\tau_2} = |5cm| \cdot 30N\sin 45°$

$= \dfrac{150\,Ncm}{\sqrt{2}} = \dfrac{1.5}{\sqrt{2}}\,Nm$

$\rightarrow$ direction : $\rightarrow$ "out of page"

$\rightarrow \vec{\tau_2} = \dfrac{1.5}{\sqrt{2}}\,Nm\,\hat{k}$

$\vec{\tau_3}$



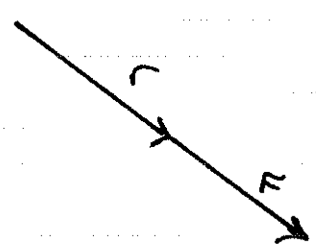
$\rightarrow F_\perp = F_3$

$\rightarrow \vec{\tau_3} = 10cm \cdot 30N = 300Ncm = 3\,N \cdot m$

direction $\rightarrow$ "into page" $= -\hat{k}$

$\Rightarrow \vec{\tau_3} = -3Nm\hat{k}$

$\vec{\tau_4}$



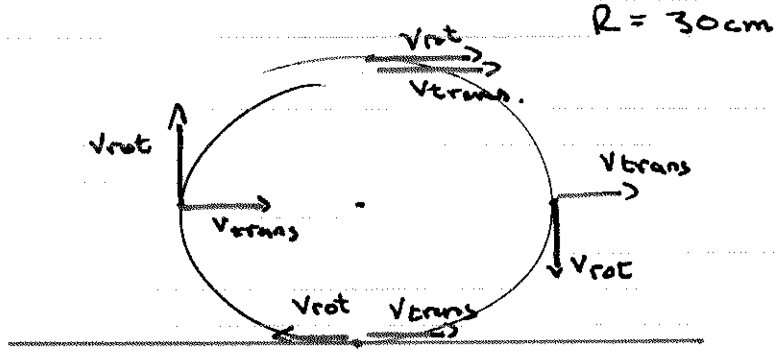
$\rightarrow F_\perp = 0 \Rightarrow \vec{\tau_4} = 0$,

$$\vec{\tau_{net}} = \vec{\tau_1} + \vec{\tau_2} + \vec{\tau_3} + \cancel{\vec{\tau_4}}^{\,0} = -0.939\,N \cdot m \cdot \hat{k}$$

$\rightarrow$ $-\hat{k}$ direction implies net torque acts to turn wheel clockwise

Ch.13 # 3

$\underrightarrow{v} = 20\,m/s$



Rolling without slipping:

To roll without slipping, the point of contact between the tire and the road must not be moving.

The velocity of a point on the edge of the tire has two parts → v_{trans} → The translational velocity of the centre of mass of the tire → same as velocity of the car

v_{rot} → The velocity due to the rotation of the tire

Referring to the above diagram, to roll without slipping, $|\vec{v}_{rot}| = |\vec{v}_{trans}| = |\vec{v}| = 20\,m/s$.

a) $|\vec{v}_{rot}| = 20\,m/s$

→ Frequency: The wheel rotates a distance $2\pi R$ in a time τ = period

$$\Rightarrow |\vec{v}_{rot}| = \frac{2\pi R}{\tau} = 2\pi R f \quad \text{as freq} = \frac{1}{\text{period}}$$

$$\Rightarrow f = \left(\frac{|\vec{v}_{rot}|}{2\pi R}\right) = \frac{20\,m/s}{2\pi \cdot 0.3\,m} = 10.6\ \frac{\text{cycles}}{\text{sec.}}$$

$$\Rightarrow f_{rpm} = 10.6\ \frac{\text{cycles}}{s} \cdot \left(\frac{60\,s}{min}\right)\left(\frac{1\ \text{rotation}}{\text{cycle}}\right)$$

$$= 637\,rpm.$$

b) a point on the top of the tire is moving with speed

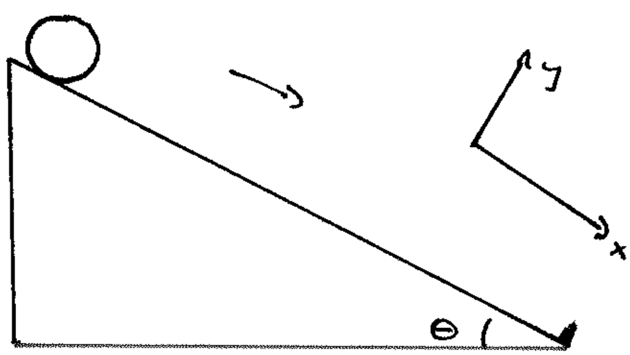
$$|\vec{v}_{top}| = |\vec{v}_{rot}| + |\vec{v}_{trans}| = 2\vec{v} = 40\,m/s.$$

$$|\vec{v}_{top}| = 40\,m/s.$$

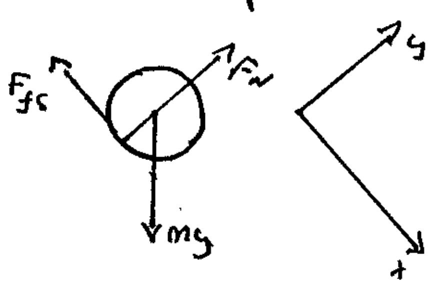
c) A point at the bottom of the tire is, as already discussed, zero

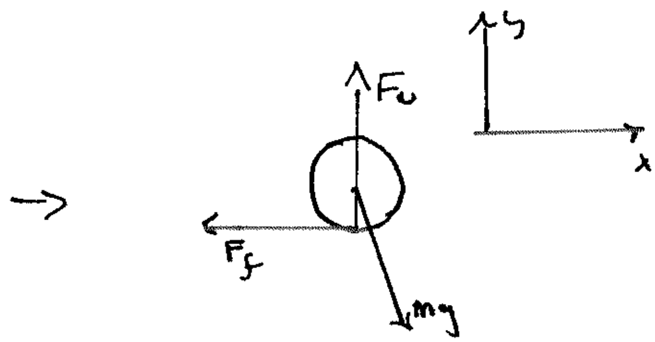
$$|\vec{v}_{bottom}| = 0.$$

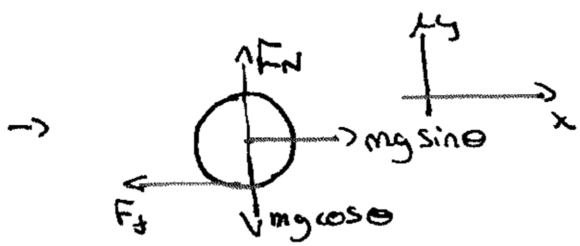
#34

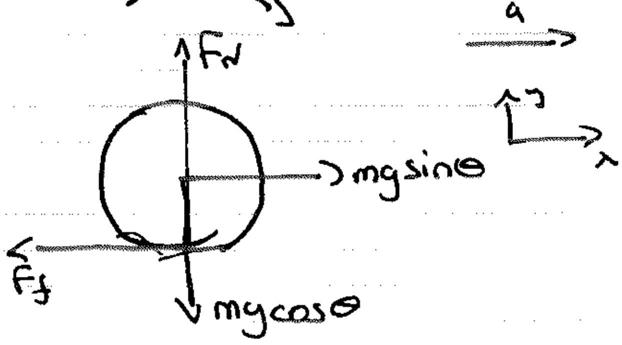


Forces on Sphere









$$\sum \vec{F} = m\vec{a}$$

$$\sum F_y = ma_y = 0 \Rightarrow |\vec{F_N}| - |mg\cos\theta| = 0 \Rightarrow \boxed{F_N = mg\cos\theta} \quad (1)$$

$$\sum F_x = ma_x = ma \Rightarrow \boxed{mg\sin\theta - F_f = ma} \quad (2)$$

And the new one ...

$$\sum \vec{\tau} = I\vec{\alpha}$$ $\Rightarrow$ Choosing clockwise rotations as +ve direction.

$$\rightarrow \boxed{R F_f = I\alpha} \quad (3)$$

No other force produces a torque about the centre (why? → Make sure you know the answer).

We need also the rolling-without-slipping constraint

$$\rightarrow \quad \omega = \frac{v}{R}$$

$$\Rightarrow \boxed{\alpha = \frac{a}{R}} \quad 4$$

4 equations for 4 unknowns → a, α, F_f, F_N

~~solving this system gives~~

$F_N = mg\cos\theta$ ①

$R^2 F_f = I R \cdot \alpha$ ②

$R^2 mg\sin\theta - R^2 F_f = R^2 ma$ ③

$R\alpha = a$ ④

$\rightarrow$ ④ into ② $\rightarrow$ $R^2 F_f = I \cdot a$ (2a)

(2a) into ③ gives $R^2 mg\sin\theta - Ia = R^2 ma$

$\Rightarrow R^2 mg\sin\theta = (mR^2 + I)a$

$\Rightarrow a = \dfrac{mg\sin\theta}{(m + I/R^2)} = \dfrac{g\sin\theta}{1 + I/mR^2}$

$$a = \frac{g\sin\theta}{1 + \frac{I}{MR^2}}$$

For a sphere, $I = 2/5 MR^2 \Rightarrow a = \dfrac{g\sin\theta}{1 + 2/5}$

$$a = \frac{5}{7} g\sin\theta$$

a) $V_f^2 = \cancel{V_i^2}^{\,0} + 2a \cdot \Delta x$

$\Rightarrow V_f = \sqrt{2a \cdot \Delta x}$ $\quad a = \frac{5}{7} g\sin\theta$, $\Delta x = 2.1\,m$

$\rightarrow V_f = \sqrt{2 \cdot \frac{5}{7}(9.81\,m/s^2)\sin(25^\circ) \cdot 2.1\,m}$

$V_f = 2.49\,m/s^2$ $\rightarrow \omega_f = \dfrac{V_f}{R} = \dfrac{2.49\,m/s^2}{0.04\,m}$

$\omega_f = 62.3$ rad/s.

$\rightarrow$ a) $\omega_f = 62.3 \ \frac{rad}{s}$

b) <u>Kinetic Energy</u>

$$T = \tfrac{1}{2} m v^2 + \tfrac{1}{2} I \omega^2$$

$$\omega = \frac{v}{R} \quad \rightarrow \quad \frac{1}{2} m v^2 + \frac{1}{2} \frac{I}{R^2} v^2$$

$$= \frac{1}{2} m v^2 \left(1 + \frac{I}{mR^2}\right)$$

Rotational ~~KE~~ $T_{rot} = \tfrac{1}{2} I \omega^2 = \tfrac{1}{2} m v^2 \left(\frac{I}{mR^2}\right)$

$\Rightarrow$ Fraction of Rotational Energy.

$$\frac{T_{rot}}{T} = \frac{\tfrac{1}{2} m v^2 \left(I/mR^2\right)}{\tfrac{1}{2} m v^2 \left(1 + \frac{I}{mR^2}\right)}$$

$$= \frac{\frac{I}{MR^2}}{1 + \frac{I}{MR^2}} = \frac{\frac{2}{5}}{1 + \frac{2}{5}}$$

$$= \frac{2/5}{7/5} = \frac{2}{7}$$

$$\boxed{\frac{T_{rot}}{T} = \cancel{\tfrac{2}{7}} \ \frac{2}{7}.}$$