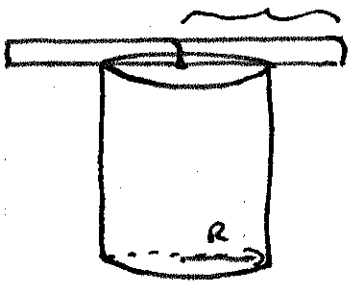
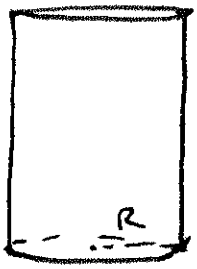
Ch. 13

#82. Before



After



$l = 0.78\,m$

$M_{arm} = 2.5\,kg$

$M_{torso} = 40\,kg$

$R = 0.2\,m$

$I_{before} = I_{cylinder} + 2I_{rod}$

$I_{cylinder} = \frac{1}{2} M_{TORSO} R^2$

$I_{rod} = \frac{1}{3} M_{arm} l^2$

$$\Rightarrow \boxed{I_{before} = \frac{1}{2} M_{TORSO} R^2 + \frac{2}{3} M_{arm} l^2}$$

$I_{after} = I_{cylinder}$

$= \frac{1}{2} M_{TOT} R^2$

$$\boxed{I_{after} = \frac{1}{2} M_{TOT} R^2}$$

Since no external torques are acting, angular momentum is conserved.

$$\vec{L}_{before} = \vec{L}_{after}$$

$$\Rightarrow |\vec{L}_{before}| = |\vec{L}_{after}|$$

$$\Rightarrow I_{before} |\vec{\omega}_{before}| = I_{after} |\vec{\omega}_{after}|$$

$$\Rightarrow |\vec{\omega}_{after}| = \frac{I_{before}}{I_{after}} |\vec{\omega}_{before}|$$

$$|\vec{\omega}_{after}| = \frac{\frac{1}{2} M_{TORSO} R^2 + \frac{2}{5} M_{arm} l^2}{\frac{1}{2} M_{TOT} R^2} |\vec{\omega}_{before}|$$

Since $M_{TOT} = M_{TORSO} + 2M_{arm}$, $M_{TORSO} = M_{TOT} - 2M_{arm}$

$$|\vec{\omega}_{after}| = \frac{\frac{1}{2} M_{TOT} R^2 + 2M_{arm}\left(\frac{1}{3} l^2 - \frac{1}{2} R^2\right)}{\frac{1}{2} M_{TOT} R^2} |\vec{\omega}_{before}|$$

$$= \left(1 + \frac{4 M_{arm}}{M_{TOT}} \frac{\left(\frac{1}{3} l^2 - \frac{1}{2} R^2\right)}{R^2}\right) |\vec{\omega}_{before}|$$

$$|\vec{\omega}_{after}| = \left(1 + \frac{4}{6}\frac{M_{arm}}{M_{TOT}}\left(\frac{2\ell^2 - 3R^2}{R^2}\right)\right)|\vec{\omega}_{before}|$$

$$|\vec{\omega}_{after}| = \left\{1 + \frac{2}{3}\frac{M_{arm}}{M_{TOT}}\left(2\left(\frac{\ell}{R}\right)^2 - 3\right)\right\}|\vec{\omega}_{before}|.$$

for $M_{arm} = 2.5\,kg$
$M_{TOT} = 45\,kg$

$\ell = 0.78m$
$R = 0.2m$

$$|\vec{\omega}_{after}| = \left\{1 + \frac{2}{3}\frac{2.5kg}{45kg}\left(2\left(\frac{0.78}{0.2}\right)^2 - 3\right)\right\}|\vec{\omega}_{before}|$$

$$= \left\{1 + \frac{2}{3}\frac{1}{18}\left(2\cdot(3.9)^2 - 3\right)\right\}|\vec{\omega}_{before}|$$

$$= \left\{1 + \frac{1}{27}\left(2\cdot(15.21) - 3\right)\right\}|\vec{\omega}_{before}|$$

$$= \left\{1 + \frac{1}{27}\left(30.42 - 3\right)\right\}|\vec{\omega}_{before}|$$

$$= \left\{1 + \frac{27.42}{27}\right\}|\vec{\omega}_{before}|$$

$$|\vec{\omega}_{after}| = 2.02\,|\vec{\omega}_{before}|$$

for $|\vec{\omega}_{before}| = 1.5$ rpm ← Note, this is ridiculously slow they probably meant rev per sec.

$$|\vec{\omega}_{after}| = 2.02 \cdot 1.85\,rpm = 3.02\,rpm$$

$$|\vec{\omega}_{after}| = 3.02\,rpm = 0.0504\,\frac{rev}{sec}.$$