



Comparing the advantages and disadvantages of physics-based and neural network-based modelling for predicting cycling power

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ARTICLE INFO

Keywords:

Model comparison
Physics-based
Neural network
Dynamic systems
Cycling power

ABSTRACT

Models of physical phenomena can be developed using two distinct approaches: using expert knowledge of the underlying physical principles or using experimental data to train a neural network. Here, our aim was to better understand the advantages and disadvantages of these two approaches. We chose to model cycling power because the physical principles are already well understood. Nine participants followed changes in cycling cadence transmitted through a metronome via earphones and we measured their cadence and power. We then developed and trained a physics-based model and a simple neural network model, where both models had cadence, derivative of cadence, and gear ratio as input, and power as output. We found no significant differences in the prediction performance between the models. Both models had good prediction accuracy despite using less input variables than traditional models and using more challenging prediction conditions by enforcing rapid speed changes during cycling. The advantages of the neural network model were that, for similar performance, it did not require an understanding of the underlying principles of cycling nor did it require measurements of fixed parameters such as system weight or wheel size. These same features also give the physics-based model the advantage of interpretability, which can be important when scientists want to better understand the process being modelled.

1. Introduction

Modelling efforts in biomechanics have traditionally focused on physics-based models (Alexander, 2003, 1995; Klika, 2011; Xiang et al., 2010). In cycling, such physics-based models have been used to predict mechanical power (Fitton and Symons, 2018; González-Haro et al., 2007; Maier et al., 2017; Martin et al., 1998). Developing these models required first identifying the input variables that affect power—such as speed or drag forces—and then identifying model parameters and how the parameters are combined with the input variables to predict the output variable. These parameters can be identified through measurements, or from data (Dahmen et al., 2012). Using this physics-based approach to develop models has at least two major challenges. First, this process can require a detailed understanding of the principles underlying a process, which may be unknown or complex. Second, real-world measurements of parameters introduce inaccuracies, which stem from both equipment and human errors. Inaccurately measured parameters will reduce the performance of the overall model. For example, to predict cycling speed from cadence a scientist would have to

both understand the underlying principles of how the angular velocity of the pedal translates to the linear velocity of the wheel, and accurately measure bike parameters such as the wheel radius.

An alternative approach to physics-based modelling is data-driven modelling. This includes neural networks, named as such because they loosely imitate how biological nervous systems learn to predict the world from experience. Mathematically, they consist of a set of functions that can be trained with data to recognize patterns in complex data sets. Because neural networks can learn from data, there are many applications for which they can be used. Modelling the input–output relationship of processes with neural networks requires an understanding of the important variables that contribute to the output (Çolak, 2021). And there needs to be a proper amount and quality of data to train the model (Klein and Rossin, 1999). But, given enough high-quality data, neural networks can approximate a wide variety of input–output relationships without explicitly having to measure many relevant fixed parameters, or understand the principles underlying a process, ameliorating the two challenges identified above (Hornik et al., 1989). For example, to predict cycling speed from cadence, a neural network could learn the

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relationship between the cadence and the cycling speed without understanding the principles that relate speed and drag to power, and without requiring measurements of the bike parameters such as the wheel radius.

While both physics-based and neural network approaches are potentially useful in biomechanics, there is a lack of research that directly compares their strengths. To the best of our knowledge, the only such example in biomechanics compares neuromusculoskeletal models with neural networks in the estimation of joint torques, finding better performance from neural network models in this use case (Zhang et al., 2023). To better understand the advantages and disadvantages of developing models using a first principles or data-driven approach, here we compare a physics-based model for predicting mechanical power in cycling with a predictive model developed using neural networks. We chose cycling for two reasons. First, the underlying principles upon which to build a physics-based model are well-understood (Debraux et al., 2011; Fitton & Symons, 2018; Maier et al., 2017; Martin et al., 1998). Second, there is both scientific and commercial interest in accurately predicting mechanical power during cycling. Scientifically, it can help to better simulate racing strategies (Dahmen et al., 2012; Fitton & Symons, 2018; Gordon, 2005; Wolf et al., 2016). Commercially, this knowledge can lead to products that help athletes to indirectly measure their power without the necessity of expensive power metres. To accomplish our goal, we first built a microcontroller-operated system that provides the cyclists with metronome-indicated changes in cadence and measures the power output. We used this system to measure cyclists' power output during two trials of cycling on a flat running track with two different gear ratios while following step changes in commanded cadence. Using this data, we developed and parameterized a physics-based model and a neural network model that best fit the simulated power to the measured power. We then compared how accurately these two models predicted the measured mechanical power and evaluated the advantages and disadvantages of each approach.

2. Methods

2.1. Data collection

We tested nine participants in this experiment (3 females and 6 males; body mass: 72.2 ± 9.4 kg; height: 177.2 ± 10.5 cm; age: 28 ± 5 years; mean \pm std). *Information about ethics approval can be found on the title page and the letter of transmittal.* All participants provided written and verbal informed consent before participating in our study.

During the experiment, participants cycled on a 400 m running track. All participants used the same bike (Specialized Tricross Comp Size 52, Specialized Bicycle Components, Inc.) and adjusted the seat height to their own preference. During cycling, they carried a backpack with a microcontroller (Teensy 3.1, Pjrc.com Llc.). The microcontroller measured torque in the pedal crank arm T_p continuously from an SRM power metre (Dura-Ace, SRM GmbH). Twice per crank arm revolution (every half pedal stroke), the microcontroller measured the crank arm angular velocity ω_p using a reed switch, and calculated the mechanical power P_p by multiplying the time-averaged torque with the angular velocity:

$$P_p = \tau_p \cdot \omega_p \quad (1)$$

As a warm-up and to familiarise participants with the bike, we first instructed them to cycle at a comfortable speed for 5–10 min. During this familiarisation period, participants chose their preferred rear gear (16.5 ± 0.5 teeth), while we kept the front gear fixed (39 teeth). We measured their preferred cadence (67 ± 12 rpm) as the average cadence during a 30 s period towards the end of the familiarisation period. Next, participants completed an 18 min trial with the rear gear being one gear over their preferred rear gear. This was followed by a second 18 min trial with the rear gear one gear under their preferred rear gear. We

instructed participants to keep their body position (i.e.: high vs. low handlebar position) the same throughout the experiment to keep their frontal area, which affects the drag, relatively constant. A metronome, controlled by the microcontroller and communicated to the participant through earphones, commanded step changes of $\pm 5\%$, $\pm 10\%$, and $\pm 20\%$ of the participant's preferred cadence, centred about the preferred cadence (Fig. 1). We instructed participants to match the metronome beat as accurately as possible with their cadence. Step changes occurred every 60 s and participants could rest for ~ 10 mins between the two trials.

2.2. Development of the physics-based model

We describe the mechanical power output of the cyclist (P) as the sum of the inertial power and the drag power. The inertial power is the kinetic energy change of the system and the drag power is the drag force multiplied by the speed (v).

$$P = mv\dot{v} + cv^3 \quad (2)$$

where m is the combined mass of the cyclist and the bike and c is the drag coefficient. Expressing speed as a function of the measured gear ratio (GR) and the measured cadence (f), with r_{rw} being the radius of the rear wheel, yields:

$$v = \frac{2\pi r_{rw} \cdot GR \cdot f}{60} \quad (3)$$

Substituting this expression for v into equation (2) and simplifying yields:

$$P = \frac{m\pi^2 r_{rw}^2}{90} GR^2 f \dot{f} + \frac{2c\pi^3 r_{rw}^3}{15} GR^3 f^3 \quad (4)$$

where f is the rate of change of the cadence. This equation expresses the output mechanical power of a cyclist as a function of the measured time-varying cadence and the experimentally-manipulated gear ratio. The only unknown and optimizable parameter is the drag number c — all other parameters in the equation can be measured (Table 1). To optimise for a drag number that best fit the predicted power to the measured power, we used a Levenberg-Marquardt optimization algorithm, implemented in Matlab's nlinfit function (R2020a, The MathWorks, Inc.) (Seber & Wild, 1989). This is a simplified physics-based model of the cycling power output. We neglected some parameters, as their significance on the model's accuracy is small. This includes the wheels' rolling resistance and their effective mass. These simplifications did not influence the comparison with the neural network model, as the neural network model also did not have any knowledge about these additional parameters.

2.3. Development of a neural network model

When developing a neural network, there are many choices to make about the architecture of the network. These choices include the structure of the data that is input into the network, types of network layers, the number of layers, the number of nodes per layer, and the type of activation function applied to each layer's output. While there are no clear rules to specify network architecture to maximise model performance on a given problem (Fiszelew and Ochoa, 2007; Hunter et al., 2012), there are certain architectures that have historically performed better on some problems than others. We used historical performance as well as pilot analyses to guide the following choices:

Data structure: To predict power for each half pedal stroke, we used input data from that half pedal stroke as well as the seven previous half pedal strokes. For all models, the input data at each half pedal stroke included the cadence and the derivative of cadence. For some models, the input data also included gear ratio (Fig. 2). We chose time windows of eight half pedal strokes because longer windows required greater

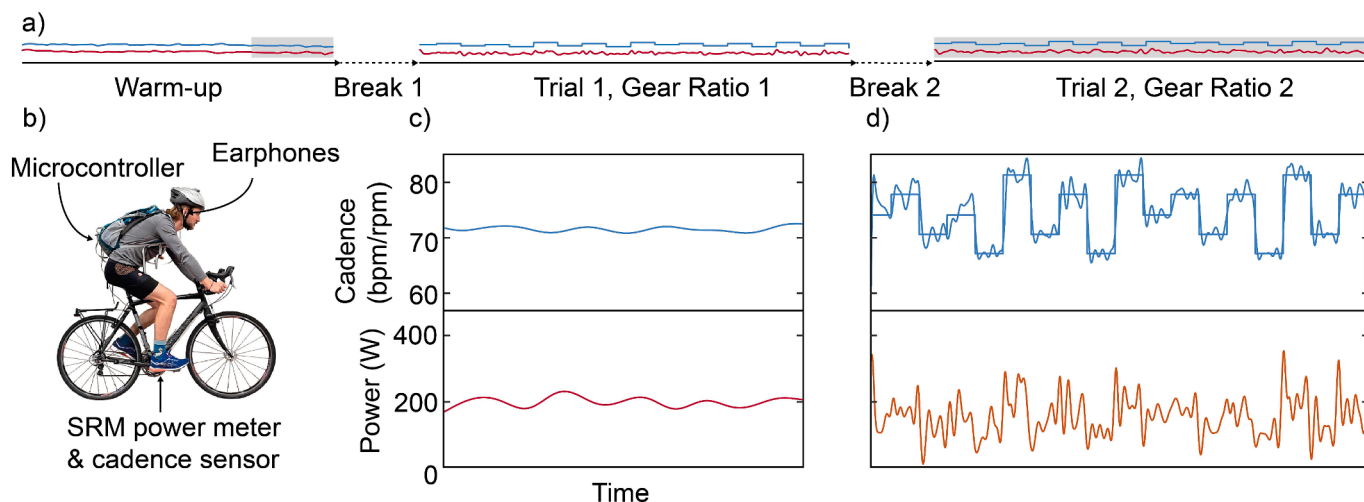


Fig. 1. The experimental setup. a) illustrates the timeline of the experiment with the Warm-up, during which we evaluated the participant’s preferred cadence (grey box), Break 1, Trial 1 with Gear Ratio 1, Break 2, and Trial 2, with Gear Ratio 2. b) illustrates the participant with the equipment. c) magnifies the data in the grey box of the warm-up. d) magnifies the data in the grey box of trial 2. Metronome cadence is measured in beats per minute (bpm), cycling cadence is measured in revolutions per minute (rpm), and power is measured in Watts.

Table 1
Values of all measurable parameters of the physics-based model.

Parameters	
r_{rw}	0.30 m
l	0.17 m
m	72.2 ± 9.4 kg (participant weight) + 13.0 kg (bike and equipment weight)
GR	2.4 ± 0.2

computational power and pilot analyses revealed good performance with our chosen window length.

Types of layers. We chose to use recurrent layers which are comprised of nodes whose output can affect the next input to nodes of the same layer. They often perform better with temporal tasks—tasks where the data changes over time—because they can store information from past data. More specifically, we used long-short-term-memory layers, which can further improve the performance over other types of recurrent layers, by prioritising which information from past data to store (LeCun et al., 2015).

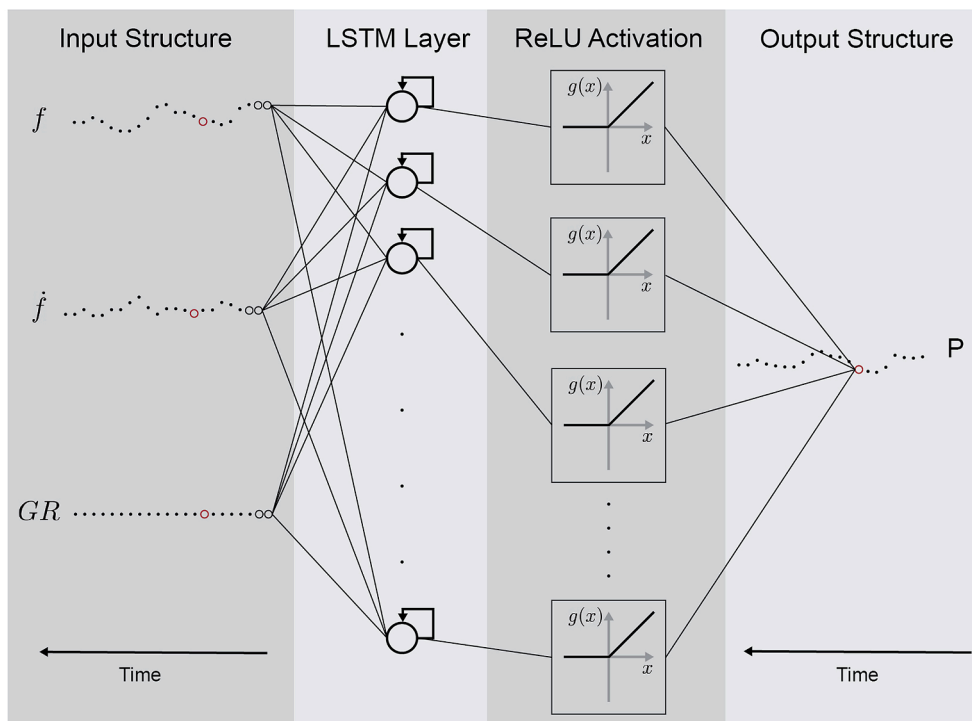


Fig. 2. Illustration of a conceptual model of the neural network from input structure to output structure. Notice that for illustration purposes the data structure of the input and output are illustrated with the time evolving from right to left. To predict one output datapoint (power P) on the right (output structure), eight input time steps per input (cadence f , cadence derivative \dot{f} , and gear ratio GR) are required. The red dots in the input structure illustrate the eighth and last input datapoint, and the red dot in the output structure illustrates the eighth output datapoint, which is the datapoint the neural network is predicting.

Number of layers. There are different advantages and disadvantages for both shallow neural networks (one hidden layer only) and deep neural networks (two or more hidden layers) (Bianchini & Scarselli, 2014; Kim & Gofman, 2018; Mhaskar et al., 2017). For simplicity, and because pilot analyses showed good performance, we chose to use only one hidden layer.

Number of nodes per layer. In pilot experiments, we found similar performance for a small number of layer nodes (8) when compared with greater numbers of nodes (16, 24, ..., and 1024). For simplicity, we chose to use 8 nodes.

Activation function. Typically, each layer in a neural network is followed by an activation function, which transforms the output of each node in the layer and provides the network with non-linear modelling capabilities. Due to their widespread success in deep neural networks, we used a rectified linear unit (ReLU) as activation functions for the long-short-term-memory layer (Ramachandran et al., 2017; Sharma et al., 2017):

$$f(x) = \max(0, x) \tag{5}$$

A ReLU activation function deactivates nodes with an output of smaller than 0, giving them the advantage of turning individual nodes on and off. We did not include additional activation functions for the

output layer, because pilot analyses revealed better performance without output layer activation functions when compared with a ReLU activation function.

Training parameters. To ensure successful training, the algorithm requires some additional parameters. Based on pilot tests, we chose an Adam optimization algorithm with a learning rate of 10^{-4} , a batch size of 32, and we did 5,000 epochs during training.

2.4. Data analysis

To better determine the advantages and disadvantages of the physics-based and neural network models, we performed two analyses: a within-trial analysis and a within-participant analysis. First, we tested the prediction performance within each trial. Here the models' aim was to learn from parts of the data within a trial and predict the rest of the data within the same trial. The neural network's input was the cadence and the derivative of the cadence and did not require knowledge of the gear ratio, as it was a fixed parameter. Second, we tested the prediction performance within each participant. Here, the models' aim was to learn from parts of both trials and predict the rest of the data within the same participant. Here, the neural network required knowledge of the gear ratio as an additional input, as it was a variable that was different

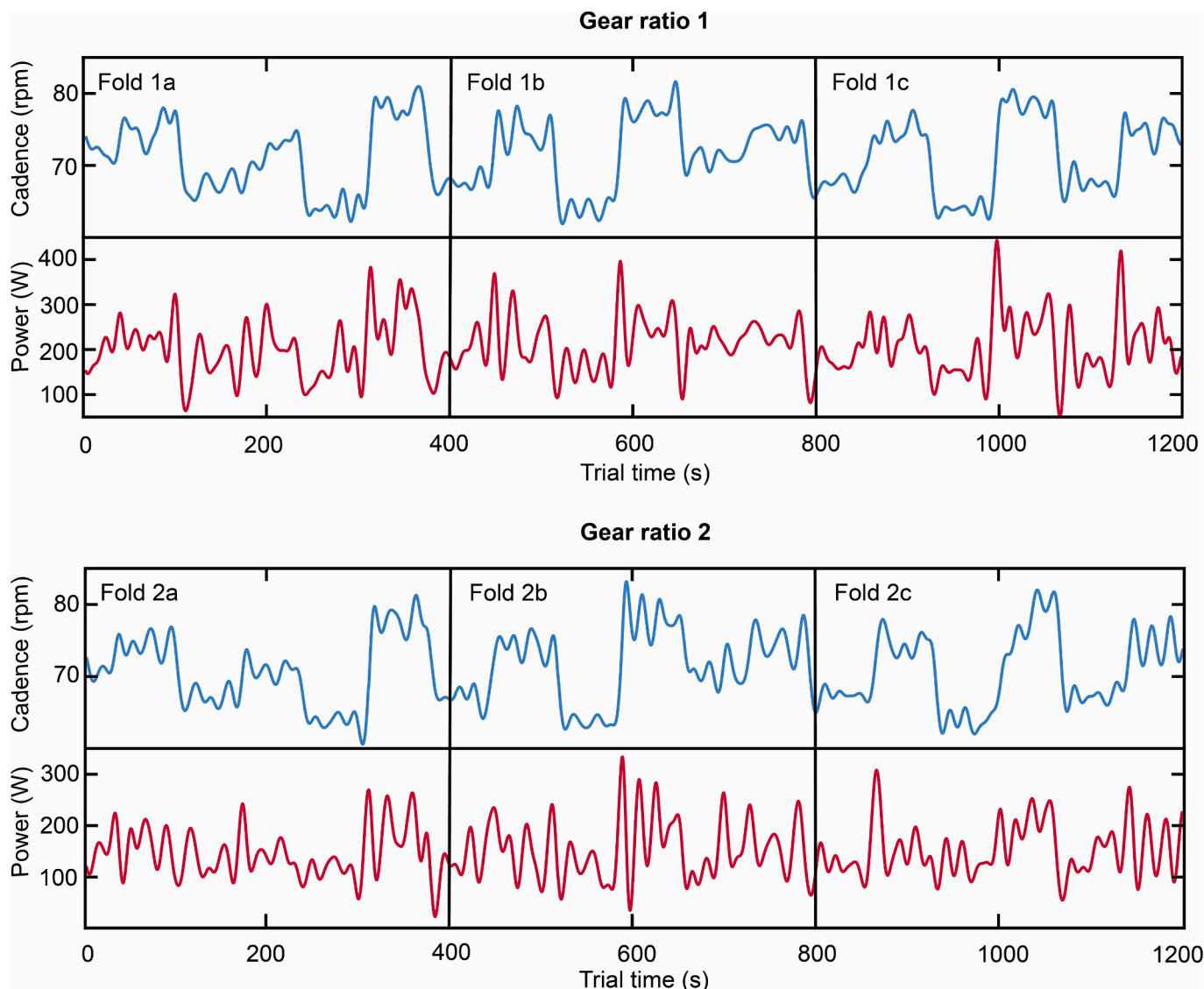


Fig. 3. Illustrates the two trials with different gear ratios, and the subsets within each trial for the within-trial and within-participant analysis.

between the two trials.

We used normalised root mean square error, normalised mean error, k-fold cross validation, and paired t-tests to compare model performance. To test the prediction performance, we calculated the normalised root mean square error, where we normalised the root mean square error by the mean of the measured data. Additionally, we also calculated the normalised mean error, where we normalised the mean error by the mean of the measured data. The normalised root mean square error emphasizes larger errors with increased sensitivity to outliers. In contrast, the normalised mean error cancels positive and negative errors which indicates the presence of a steady state error. We split up each participant's trial into three subsets, also called folds (Fig. 3). To test the performance of the physics-based model and the neural network model in the within-trial experiment we trained the models with two of the subsets within a trial and tested the accuracy of predicting the power with the third, using both the normalised root mean square error and normalised mean error. For example, we would use fold 1a and fold 1b to train the model and fold 1c to test the prediction accuracy. Here, we did 3-fold cross validation: We used each of the three subsets as a test set once to get the prediction accuracy three

times. To test the performance in the within-participant experiment, we trained the models with five of a participant's subsets and tested the accuracy of predicting the power on the sixth. For example, we would use fold 1a, fold 1b, fold 1c, fold 2a, and fold 2b to train the model and fold 2c to test the prediction accuracy. Here we did 6-fold cross validation: We used each of the six subsets as a test set once to calculate the prediction accuracy six times. For both the within-trial and the within-participant experiments we used the training phase of the analysis to determine the drag parameter of the physics-based model, and the weights and biases of the neural network model. To compare overall performance, we averaged the normalised root mean square errors of each participant and compared the mean normalised root mean square error between the physics-based model and the neural network model with a paired t-test using a significance level of $p < 0.05$.

3. Results

The physics-based model and the neural network model had similar predictive performance. The normalised mean error and normalised root mean square error for the within-trial analysis — in which the different

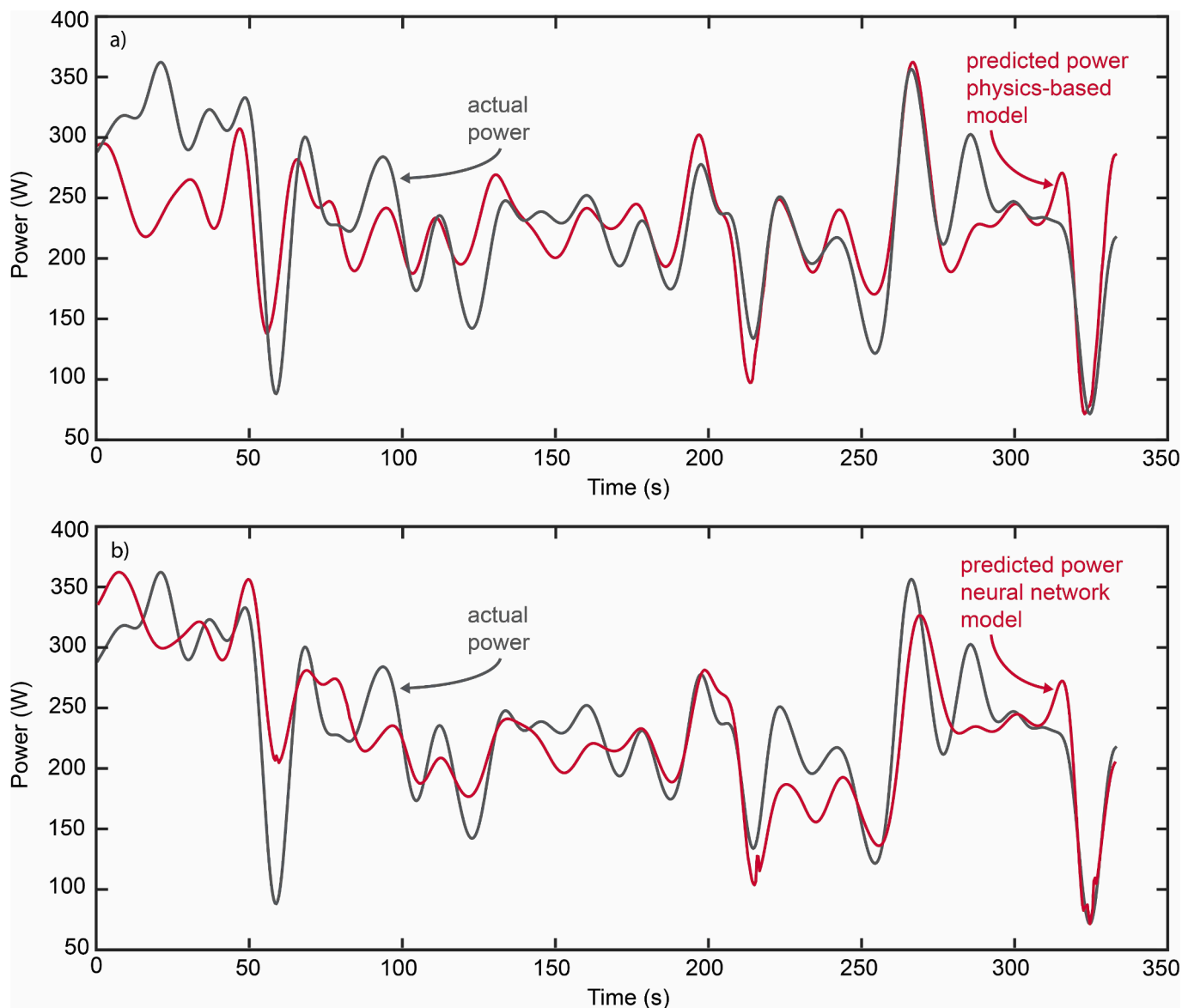


Fig. 4. Representative prediction data for the within-trial analysis. These representative trials had similar RMSEs and normalised mean errors with the overall average.

gear ratio trials were kept separate when we trained and tested the model — of the physics-based model were $1.6 \pm 1.1 \%$ and $19.6 \pm 5.1 \%$, respectively (mean between participants \pm standard deviation between participants; Fig. 4). With this predictive performance, we expect a new participant with a measured average power output of 300 W to have a predicted average power that is ~ 5 Watts (1.6 %) above the actual average power. And for 95 % of the half pedal strokes at 300 W, we expect the predicted power for this new participant to be within ~ 116 Watts ($19.6 \% \cdot 1.96$). On average, the optimised drag number was 1.2 ± 0.2 . The normalised mean error and normalised root mean square error for the within-trial experiment of the neural network model were $1.2 \pm 4.2 \%$ and $18.2 \pm 6.0 \%$, respectively (Fig. 4). The predictive performance between the two models was not significantly different ($p = 0.34$).

The normalised mean error and normalised root mean square error for the within-participant analysis — in which we combined the different gear ratio trials when we trained and tested the model — of the physics-based model were $3.2 \pm 1.8 \%$ and $20.9 \pm 5.1 \%$, respectively (Fig. 5). On average, the optimised drag coefficient was again 1.2 ± 0.2 . The normalised mean error and normalised root mean square error for the within-participant experiment of the neural network model were $4.1 \pm 10.9 \%$ and $25.4 \pm 5.6 \%$, respectively (Fig. 5). Again, the predictive performance between the two models was not significantly different ($p = 0.12$).

4. Discussion and Implications

Here we developed and compared a physics-based model and a neural network model in predicting cycling power from changes in cadence and gear ratio. In our physics-based model, we optimised for the only unknown parameter—the drag number. For the neural network model, we used a simple recurrent neural network with one long-short-term-memory layer consisting of eight nodes. We found that the neural

network model had a similar performance to that of the physics-based model.

Despite limited input variables and rapid speed changes during cycling, both models achieved good prediction accuracies. Unlike most models that incorporate measured variables like wind speed, direction, and drag coefficient, our models rely only on cadence and gear ratio (Martin et al., 1998; Fitton and Symons, 2018; Gonzales-Haro, 2007). For comparison, Martin et al. used eight input variables in developing the most comprehensive mathematical model of road cycling. Gonzales-Haro et al. (2007) evaluated the prediction accuracy of nine prominent cycling models in constant speed velodrome conditions. On average, the nine models resulted in a normalized mean squared error of $22.0 \pm 13.6 \%$ (best: 8.4 %, worst: 44.6 %). Our models performed as well as the average, achieving a mean squared error of $\sim 21 \%$ despite using fewer input variables and cycling under conditions that were more challenging to predict.

This work has at least two major limitations. First, we only used one task to compare the physics-based model and the neural network model. More experiments in the future should show how neural networks compare to physics-based models in other biomechanics applications. Second, we excluded many variables that could have positively influenced the cycling power prediction accuracy (Martin et al., 1998). For example, we did not measure headwinds or tailwinds which increase and decrease the drag force, respectively. Our participants cycled on an oval running track creating situations where in the presence of a prevailing wind, participants alternatively experienced headwinds and tailwinds with neither of these forces represented by changes in the inputs of the model. We chose to not include these variables due to the complexity of their measurement. Incorporating wind speed and direction—or any other variables for that matter—into the physics-based model would require experts in the area to understand how to incorporate these variables in the model. In comparison, incorporating variables to the neural network model requires retraining it with the new

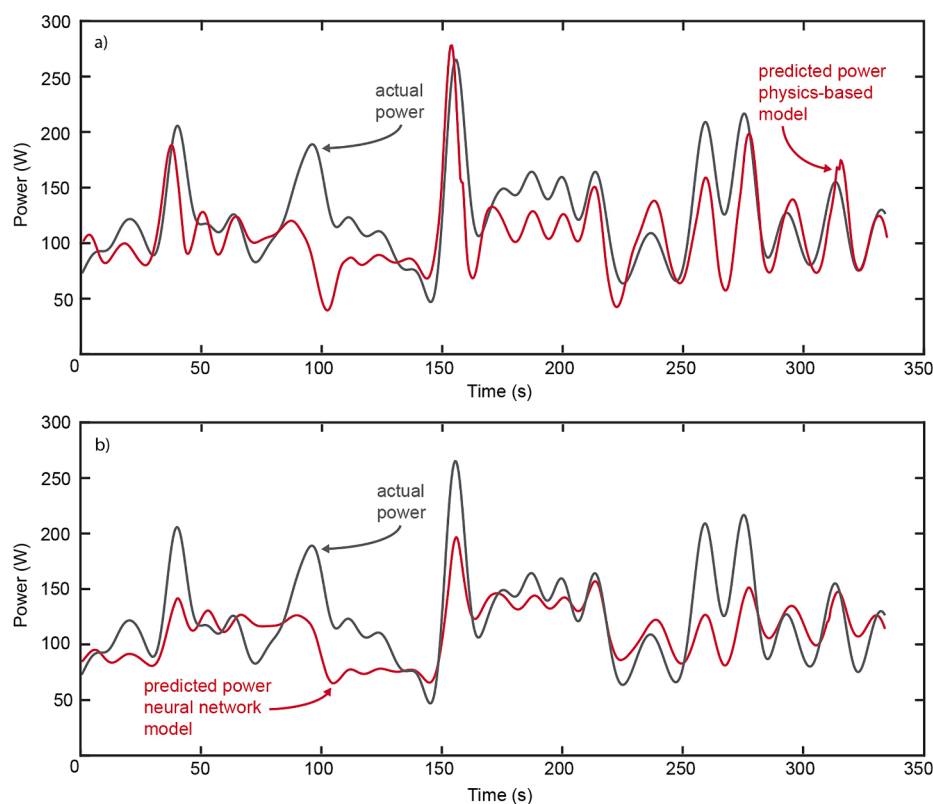


Fig. 5. Representative prediction data for the within-participant analysis. These representative trials had similar RMSEs and normalised mean errors with the overall average.

input data.

The physics-based model had more physically meaningful parameters, introducing some advantages over the neural network model. For the physics-based model we used measured parameters, such as the rear wheel radius and the participant's weight. While this approach requires more expert knowledge to develop the model, having fixed parameters makes it easier for future changes in the experimental environment. For example, if we decided to change the bike, the physics-based model would only need the new rear wheel radius and weight of the new bike, but the neural network would need additional training with the new information. Having fixed physical parameters in the physics-based model also makes it more interpretable, which can help better understand the performance of a model, and subsequently increase the fundamental understanding of the problem itself. A neural network creates its own representation of a problem, which makes it harder to interpret.

Generalizing to new cyclists would require changes and retraining for both models. In this study, we optimized both models for individual cyclists, considering no gear ratio change (within-trial) and gear ratio changes (within-participant). A next step would be to train both models to generalize to other individuals (participant-independent). To do so in the physics-based model, we could optimize for an average drag parameter. Since size and position of the cyclist also affect the drag parameter, adding these parameters into the model could also positively affect the accuracy of the model. To generalize the neural network model, we could add new input variables that would have an impact on the individuals' power output, such as the weight (already included in the physics-based model), height, and the position, and retrain a new model.

In our study, both models converged to similar, but not perfect, prediction accuracies. We expected this imperfect performance as there are many variables in cycling that affect the power output that we did not include in our models, and we challenged our models with rapid speed changes rather than the more conventional constant speeds. To improve prediction accuracy, we recommend measuring and incorporating continuous wind speed and direction into a model. We suspect that incorporating these new measurement variables into a physics-based model would prove more challenging than incorporating them into a data-driven model. This is because in the first case, one would have to understand how the wind speed and direction affects the whole system, while in the second case, one would simply require retraining the model with the new input variables added. More generally, we suspect that as the complexity of the process to be modelled increases, or as the number of required measured inputs increases, a data-driven modelling approach will prove simpler than the physics-based alternative for the problem of accurately predicting cycling power in real-world conditions.

5. Ethics Information

The Office of Research Ethics at Simon Fraser University approved the study (#20180650). All participants provided written and verbal informed consent before participating in our study.

6. Funding details

This information can be found on the title page and in the letter of transmittal.

CRedit authorship contribution statement

Patrick Mayerhofer: Writing – review & editing, Writing – original

draft, Visualization, Validation, Software, Resources, Project administration, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Ivan Bajić:** Writing – review & editing, Validation, Supervision, Conceptualization. **J. Maxwell Donelan:** Writing – review & editing, Validation, Supervision, Methodology, Investigation, Funding acquisition, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Supplementary data

Supplementary data to this article can be found online at <https://doi.org/10.1016/j.jbiomech.2024.112121>.

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