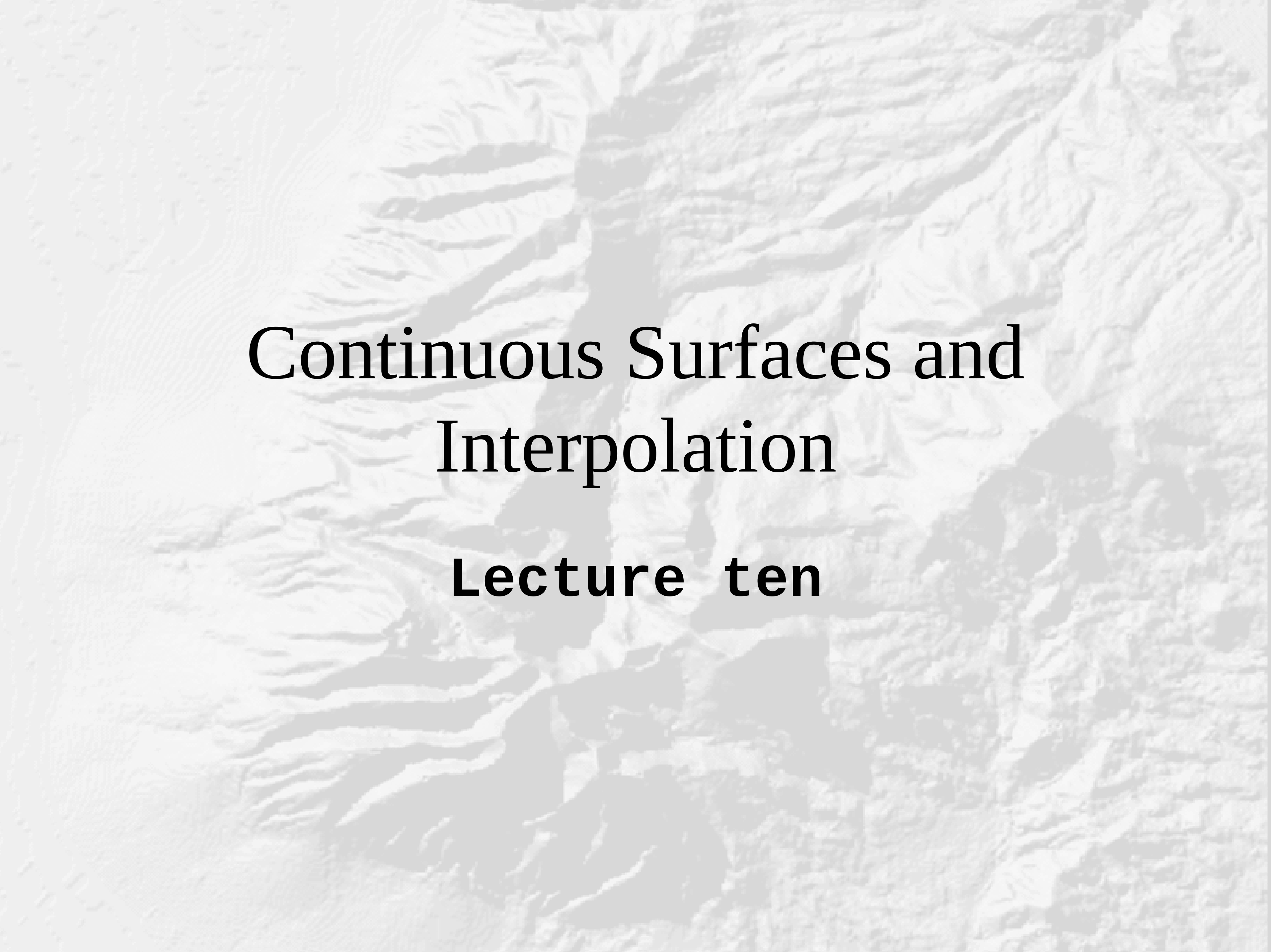


Continuous Surfaces and Interpolation

Lecture ten

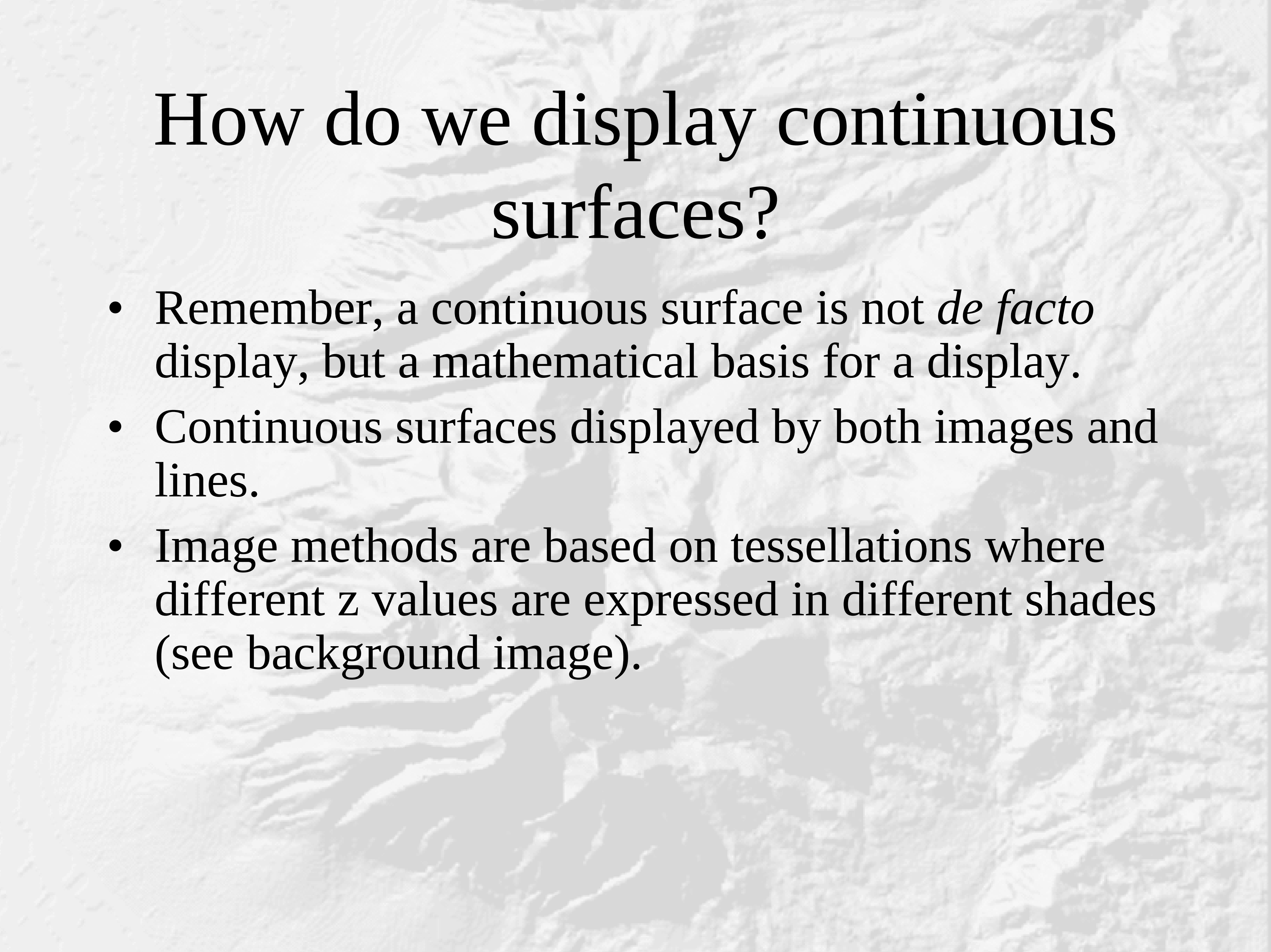
What is a continuous surface

- Continuous surfaces are models of the earth’s surface.
- Digital elevation models (DEM) or digital terrain models (DTM) are special cases of a continuous surface.

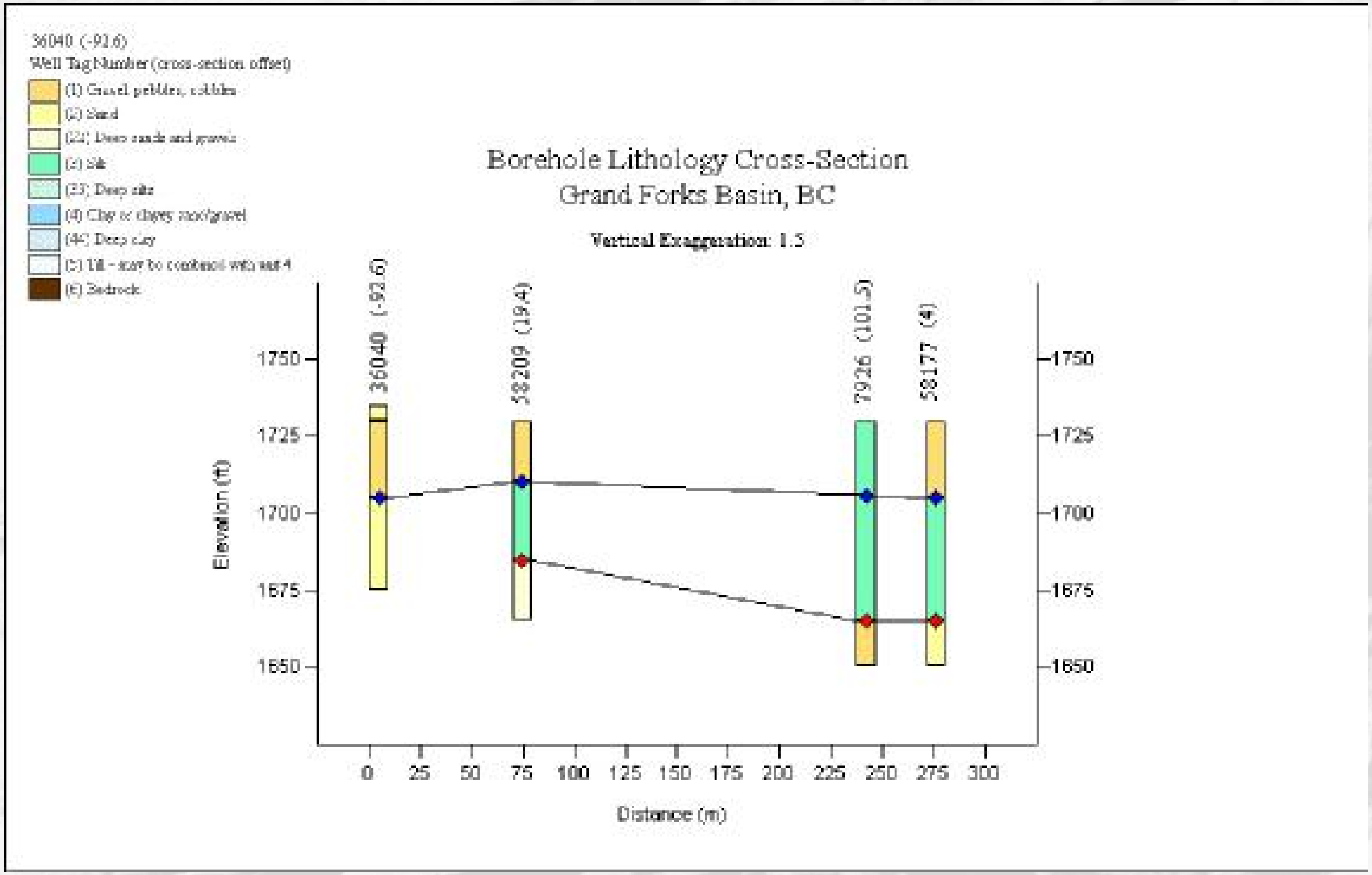
How is interpolation related to continuous surfaces?

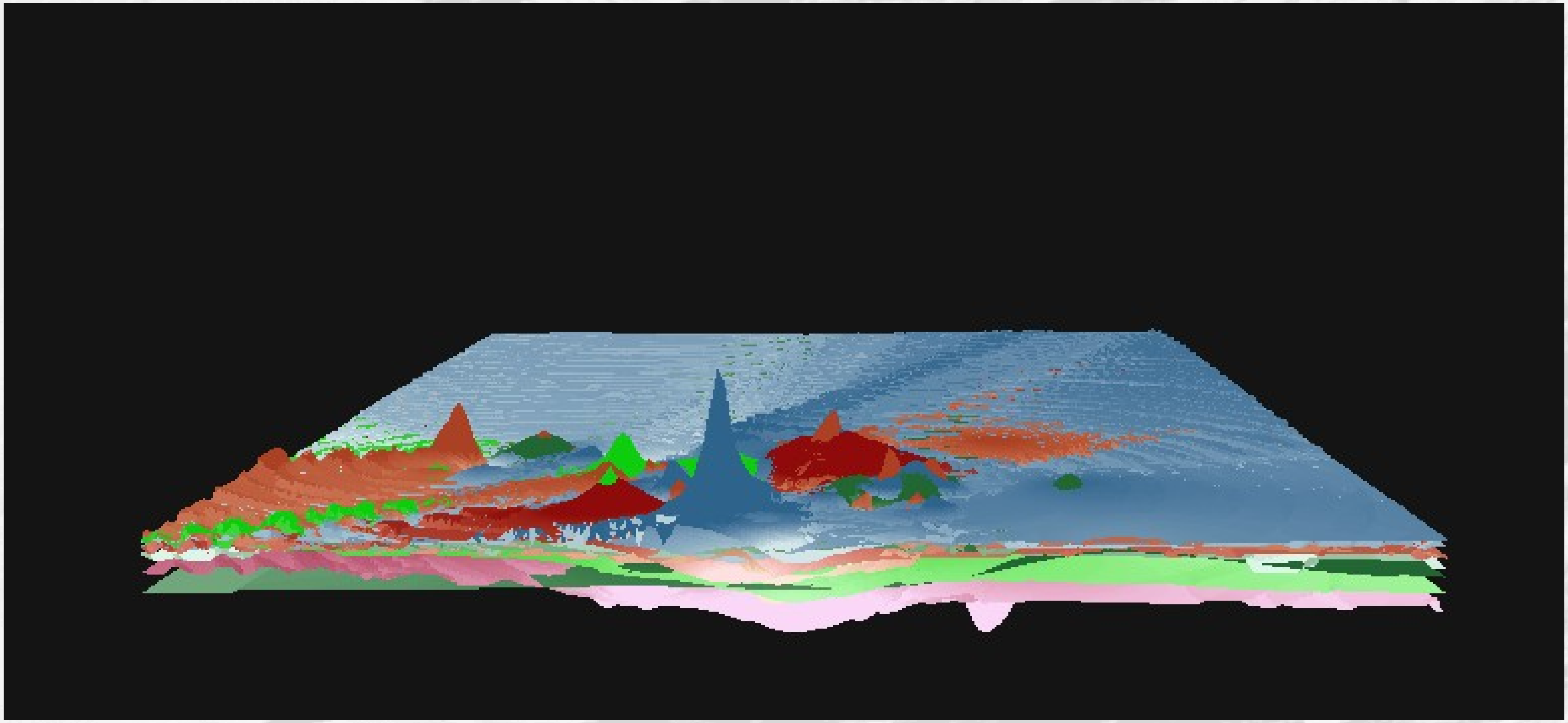
- Interpolation allow us to assign a value for attributes.
- This allows us to convert field data points to a continuous field.

How do we display continuous surfaces?



- Remember, a continuous surface is not *de facto* display, but a mathematical basis for a display.
- Continuous surfaces displayed by both images and lines.
- Image methods are based on tessellations where different z values are expressed in different shades (see background image).


36040 (-92.6)
Well Tag Number (cross-section offset)
(1) Gravel, pebbles, cobbles
(2) Sand
(22) Deep sands and gravels
(3) Silt
(33) Deep silts
(4) Clay or clayey sand/gravel
(44) Deep clay
(5) Till - may be combined with unit 4
(6) Bedrock
Borehole Lithology Cross-Section
Grand Forks Basin, BC
Vertical Exaggeration: 1.5
36040 (-92.6)
58209 (19.4)
7926 (101.5)
58177 (4)
Elevation (ft)
1750
1725
1700
1675
1650
Distance (m)
0
25
50
75
100
125
150
175
200
225
250
275
300



Spikes are missing data.

Data sources for interpolation

- Stereo aerial photos or overlapping orbital imagery
- Point samples of attribute data measured directly or indirectly in the field using regular, random, transect, progressive, etc. sampling methods.
- Digitized polygon/choropleth maps

Complexities of sampling

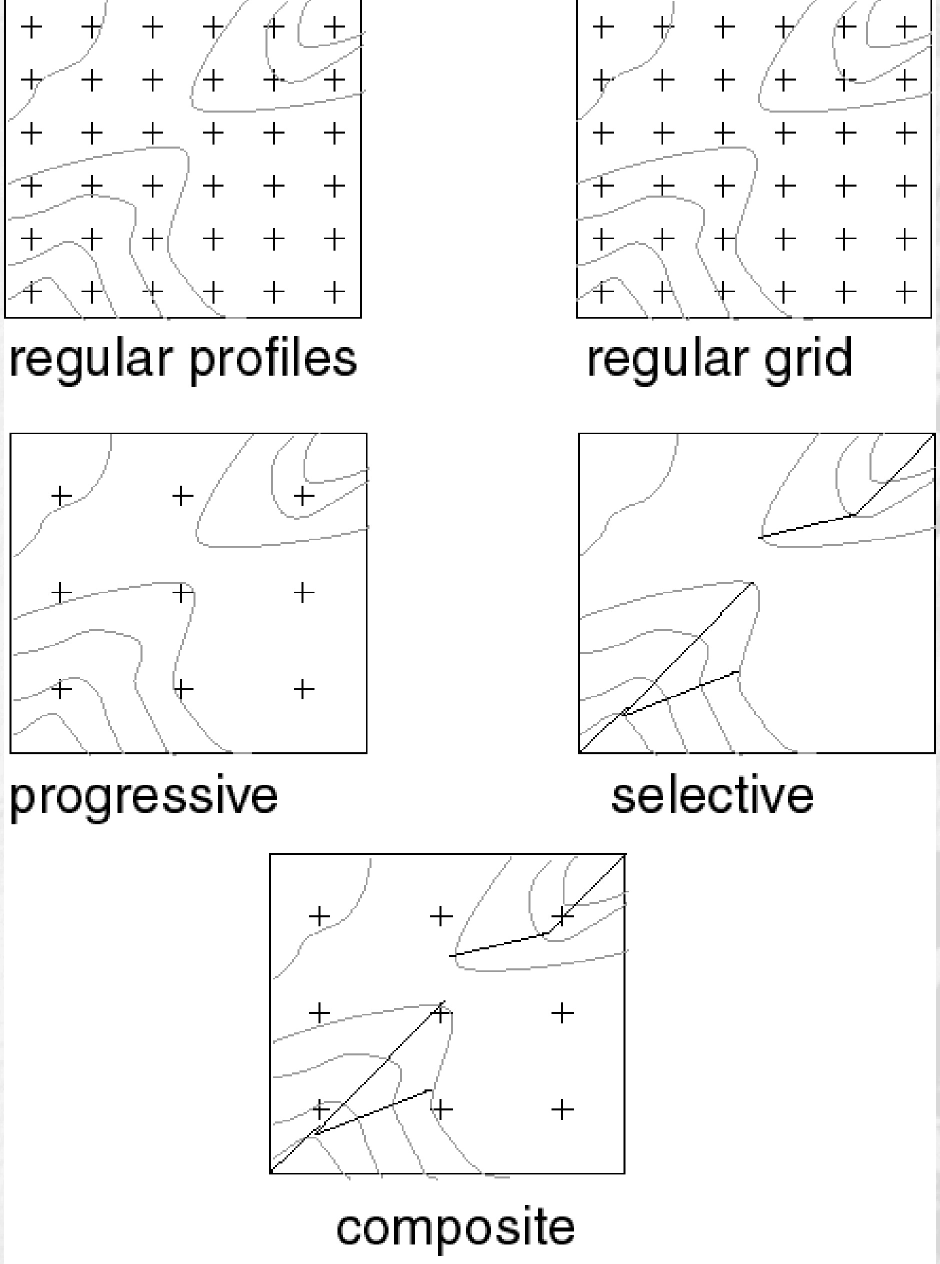
- Often samples are hard to obtain, and points of complex variation are focused upon. These points are the “hard data.”
- Knowledge of physical or social processes that underlie a phenomena (such geological processes or urban density patterns) can assist in interpolation – “soft data.”

Spatial Sampling

- Regular sampling patterns can be arranged as profiles or grids.
- Profiles are derived from information about the variability of the data whereas a regular grid uses the same grid for all areas.
- Regular sampling allows automatic extraction from RS data. The machine samples.

Progressive Sampling

- Allows the density of the sampling to fit the terrain.
- Uses a recursive process in which a low density grid is applied to start with. Then the accuracy of the sampled data is analyzed.



Sampling techniques for a DTM

Diagram should show more points in the regular grid box as profiles or “soft data” allow use of terrain information to supplement understanding, and reduce data points.

QUESTION: when would regular profiles use more data points?

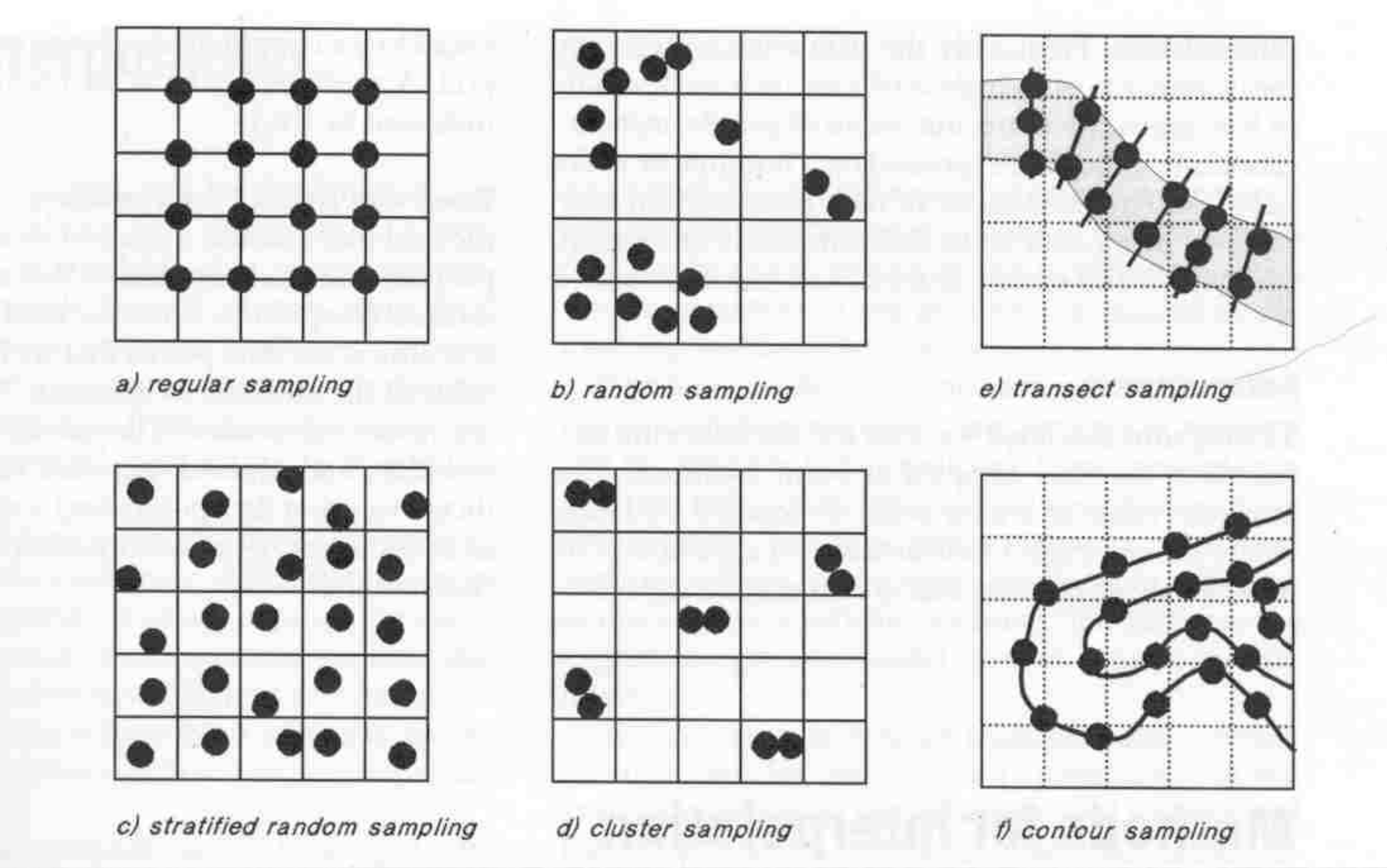
Selective Sampling

- Surface-specific point elevation data.
- A “skeleton” is created for the data that includes high and low points, saddle points; and points on streams and ridges that designate high and low edges.

Composite Sampling

- Surface-specific sampling can be combined with progressive sampling, and then the technique is called composite sampling.

Other classification of sampling techniques.



a) regular sampling

b) random sampling

e) transect sampling

c) stratified random sampling

d) cluster sampling

f) contour sampling

Structuring terrain data for representation

- Two main methods of tessellation (both based on a grid):
- (i) rectangular grid – also referred to as an elevation matrix
- (ii) TIN (triangulated regular network)
- Can also use lines.

The role of TIN and Grids

- The original data must be **structured** in order to enable it to handle the types of requests that are generated using DTMs (i.e. GIS queries)

Tessellation

- To create a continuous surface, need to divide the area into a regular or irregular grid.
- This is a covering of the surface with an non-overlapping polygons.

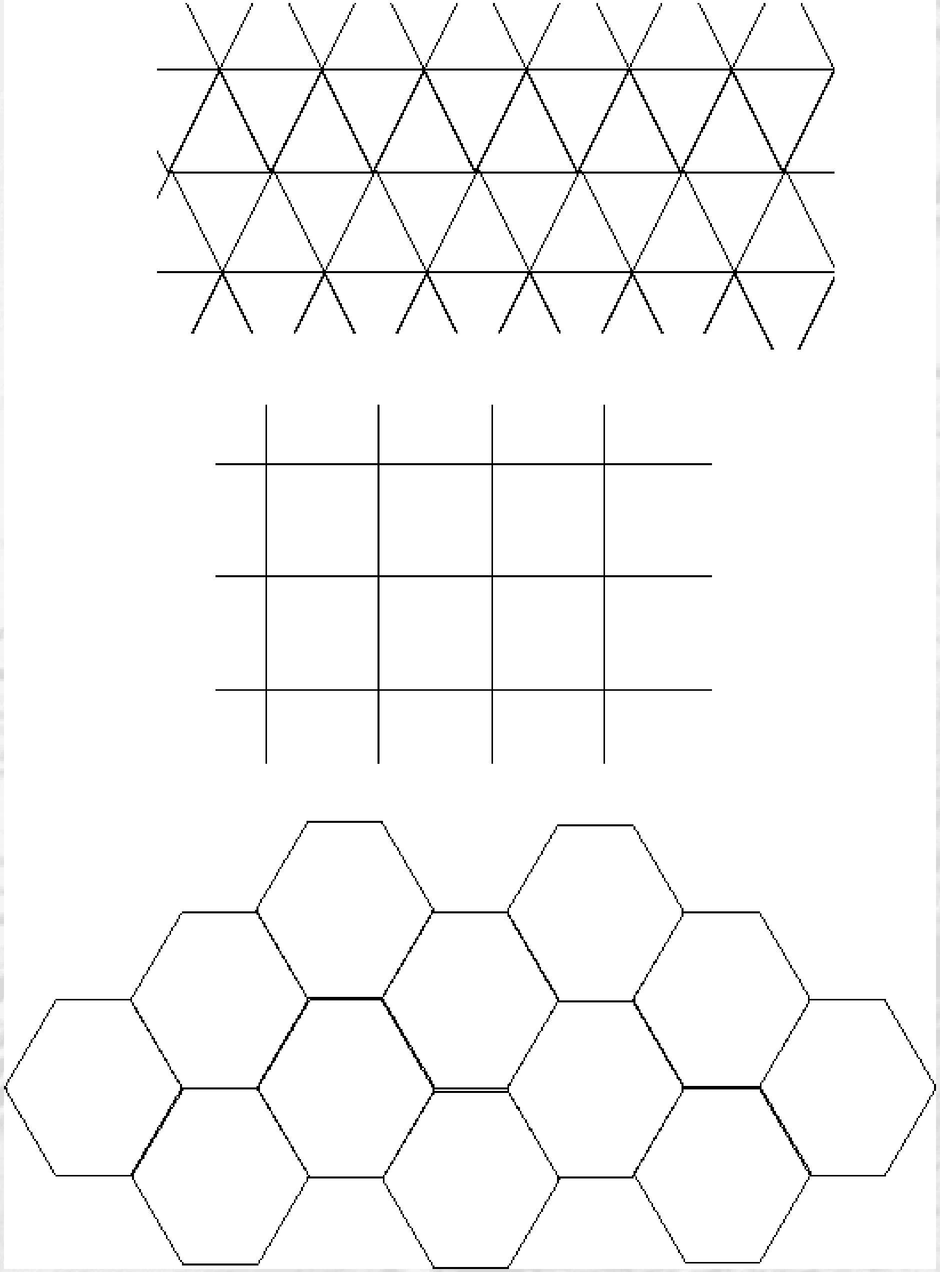
Definition

- **Proper definition of a tessellation**:
  A partition of the plane or portion of the the plane as the union of a set of disjoint areal objects.
- A regular tessellation is one in which all the polygons are regular and equal.
- The most common regular tessellation is the square (raster) grid.
- Irregular tessellations are permited (TIN).

3 permitted tessellations

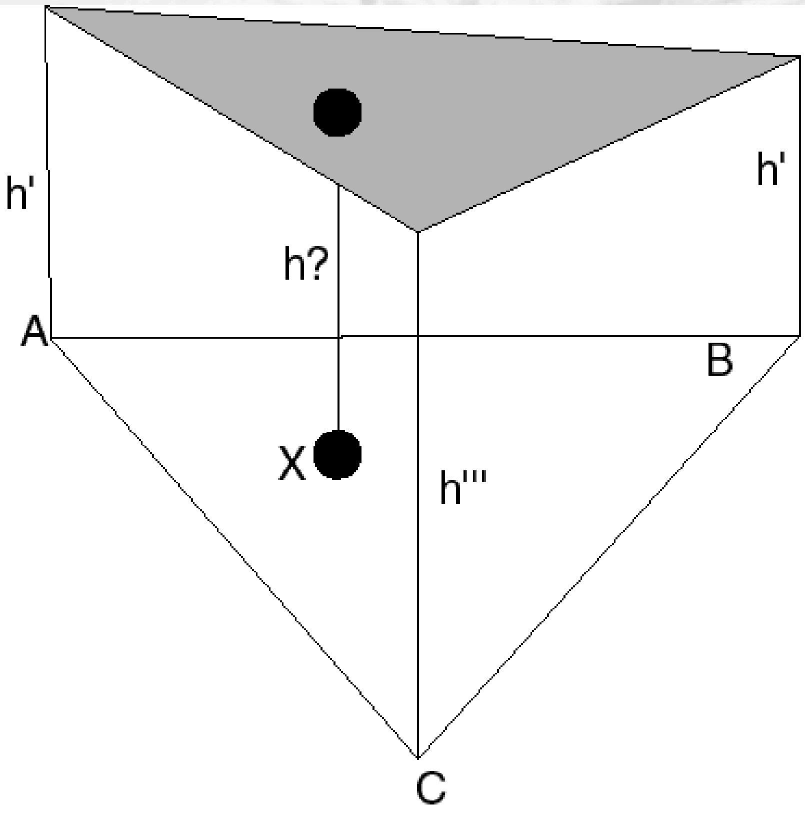
- Regular, triangular,and hexagonal are the only tessellations permitted in the Euclidean plane.

Regular, triangular and hexagonal tesselations



Using TIN to estimate elevation

- Basis for 3-D analyst
- Assume triangle *ABC* is part of a TIN. We know the coordinates of *a*, *b*, and *c*. We also know each of the heights, h' h'' *and* h''' at *A*, *B* and *C*.
- If *X* is inside the boundary of *ABC*, then $x = \alpha \text{ x } \boldsymbol{a}$ $+ \beta \text{ x } \boldsymbol{b} + \gamma \text{ x } \boldsymbol{c}$
  where α, β and γ are scalar coefficients that *can* be uniquely determined, such that
  $\alpha + \beta + \gamma <= 1$.
- The height *h* at the point *X* can now be found using this equation:
  $h = \alpha \text{ x } \boldsymbol{h'} + \beta \text{ x } \boldsymbol{h''} + \gamma \text{ x } \boldsymbol{h'''}$



Linear interpolation is used to find the height at x.

$x = \alpha \times a + \beta \times b + y \times c$

where α, β and y are scalar coefficients that can be uniquely determined, such that their sum ≤ 1.

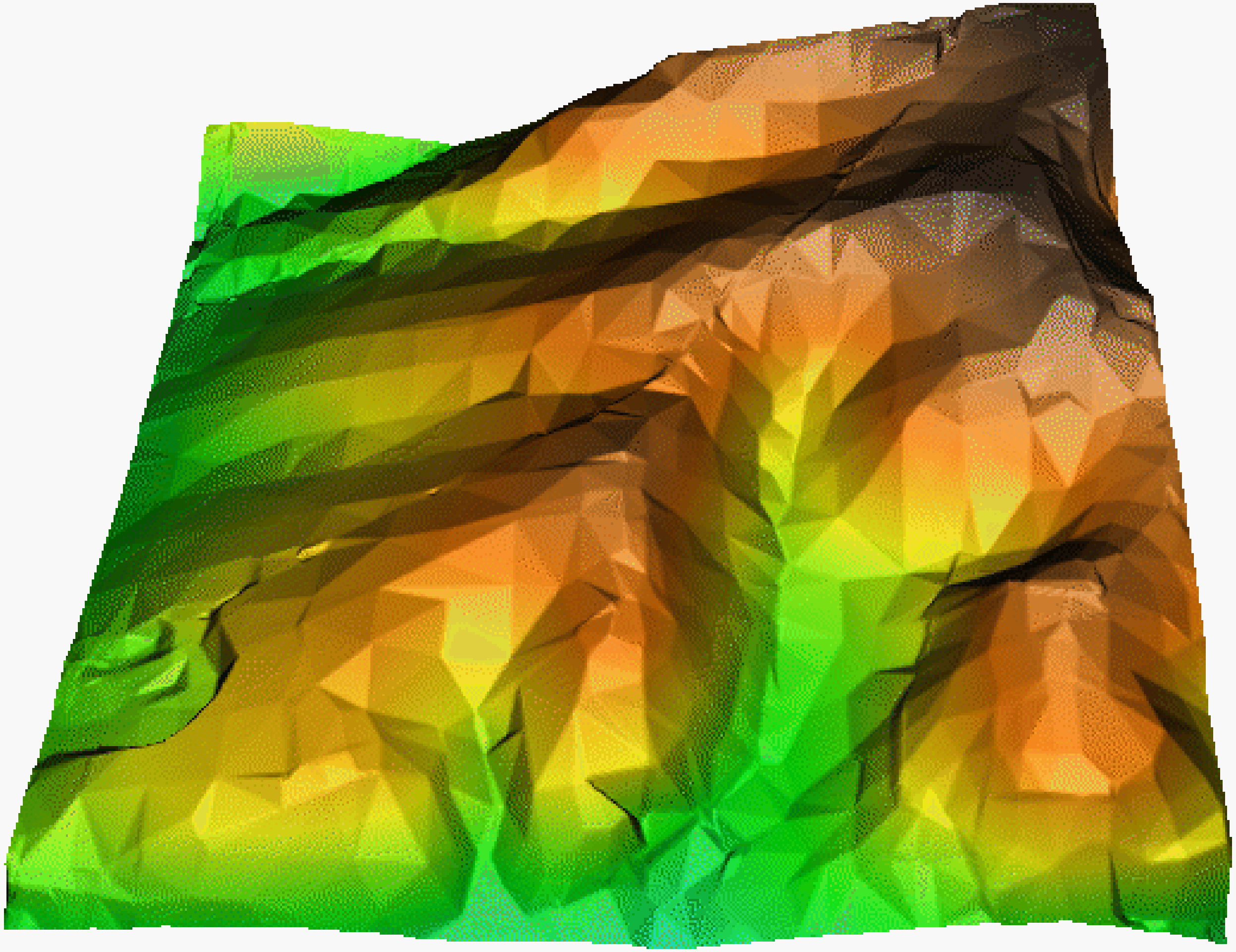
The height h at the point X can now be found using this equation:

$h = \alpha \times h' + \beta \times h'' + y \times h'''$

diagram adopted from Worboys, M. 1995. *GIS: A Computing Perspective.* London: Taylor & Francis.

Assumptions of this equation

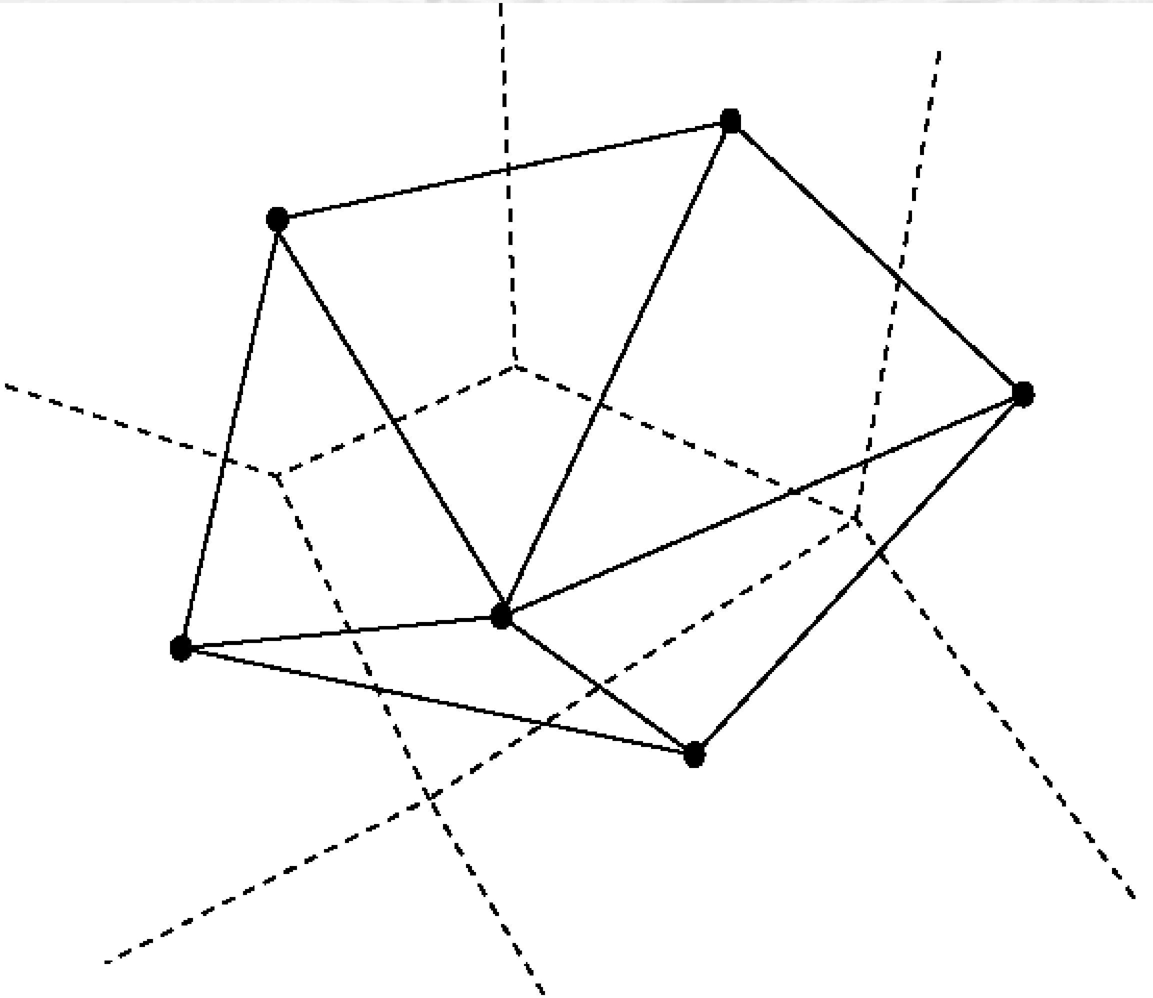
- The TIN is a *first order discretization*.
- This means that the values that the TIN uses are actual measurements, not derived values.
- It is quite legitimate to base a TIN on derived values but that particular *linear interpolation* won’t work.
- Different equations are used for *higher order* interpolation surfaces. These are usually more complex equations.



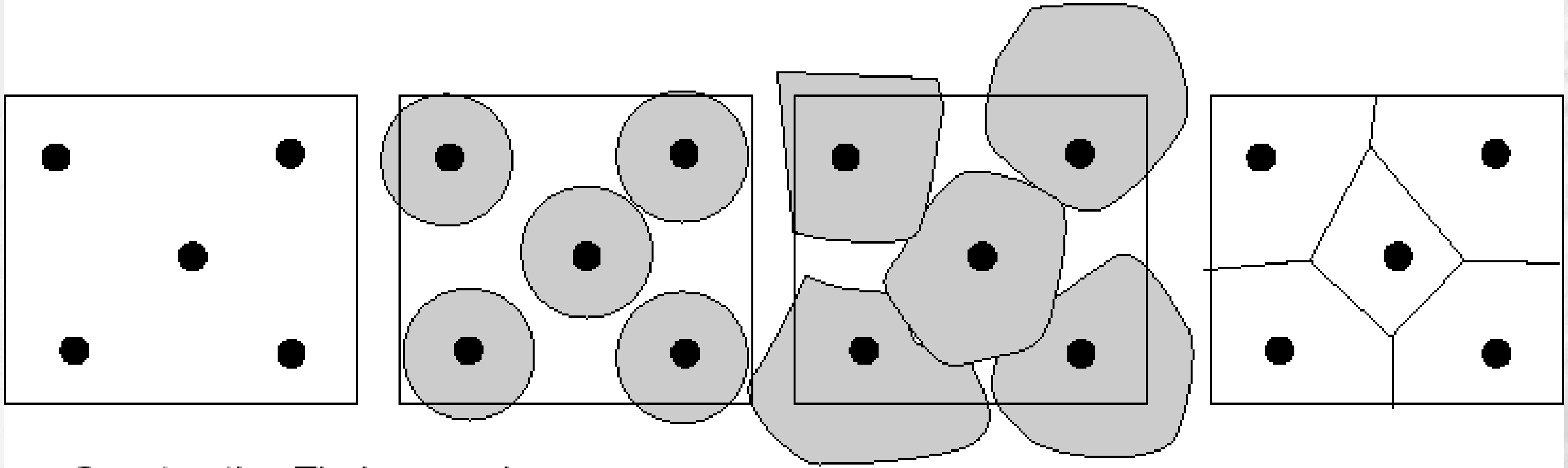
DTM based on TIN

Delauney Triangulation

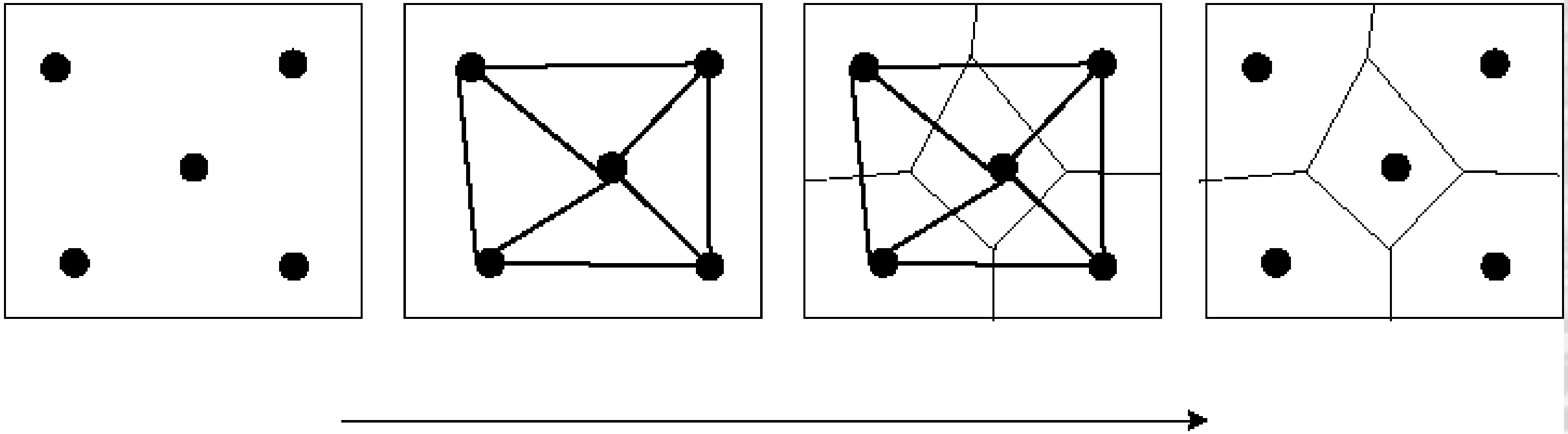
- Constituent triangles are as *equilateral as possible*.
- The dual of a Delaunay triangulation (but not all TIN) is called a **Voronoi Diagram** or **Theissen polygon**.



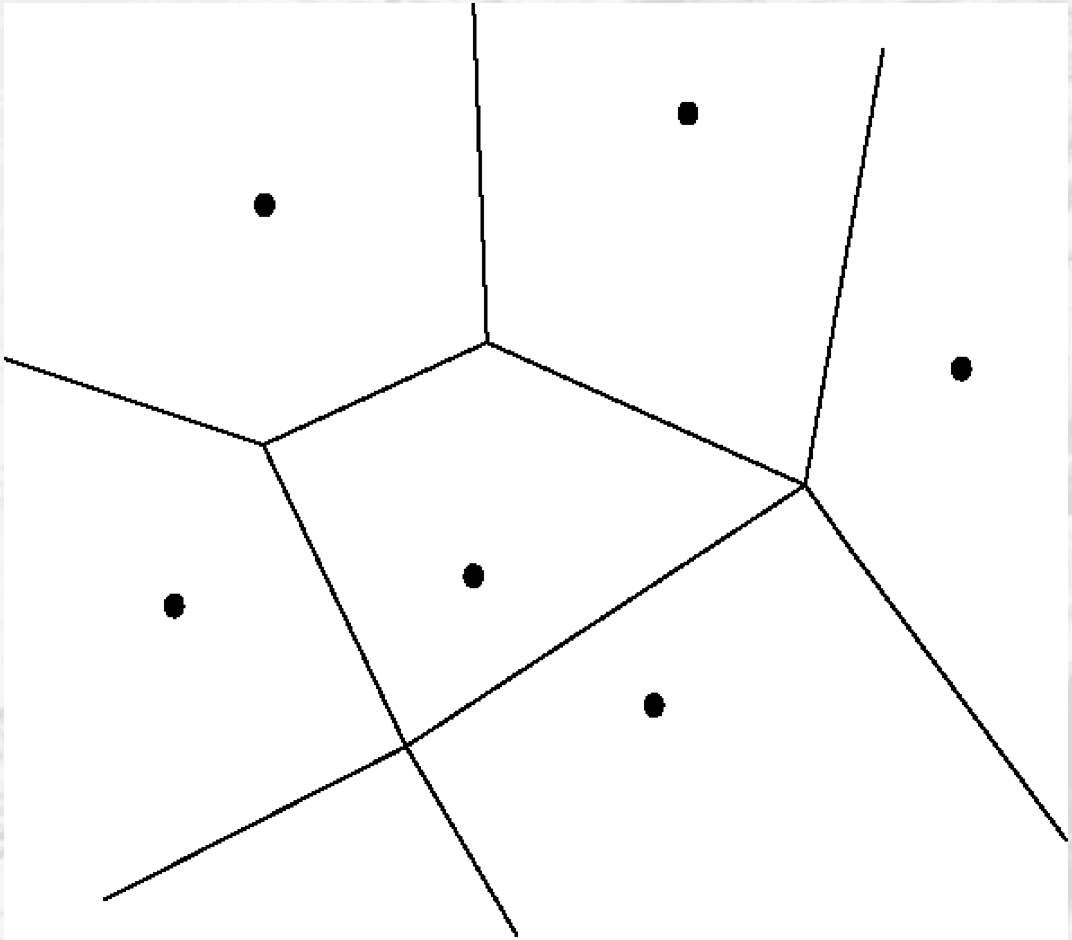
Delaunay triangulation of Voronoi diagram. This is a version of TIN with the most equilateral triangles possible.



Constructing Theissen polygons



Constructing Theissen polygons from an initial triangulation



Voronoi diagram of proximal regions surrounding city hospitals.

diagram adopted from Worboys, M. 1995. GIS: A Computing Perspective. London: Taylor & Francis.

Constraints to Theissen Polygons

- Hospital catchment example.
- Must assume that none of the points are collinear. We measure distance “as the crow flies.” in this example
- BUT travel distance (by road) could also be used to determine the regions.

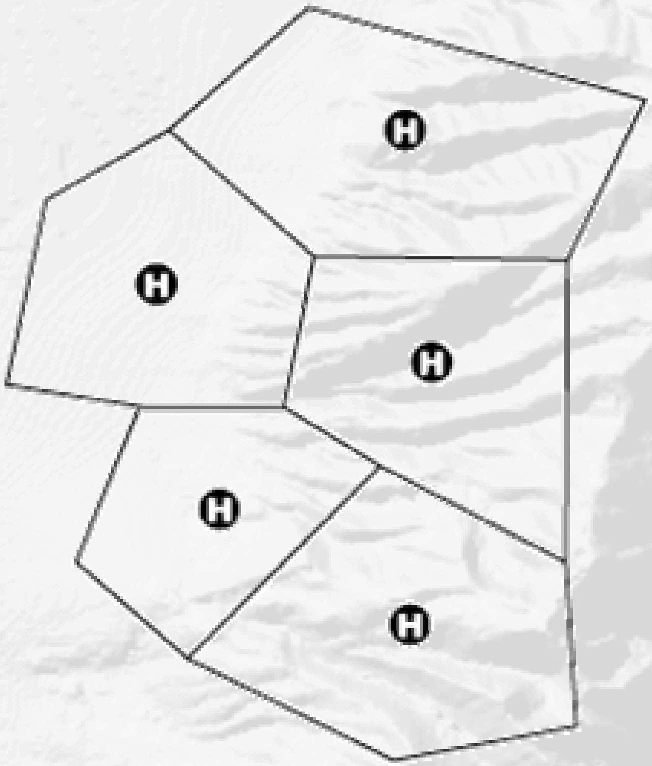


Figure 3a: Hospitals with simple catchments

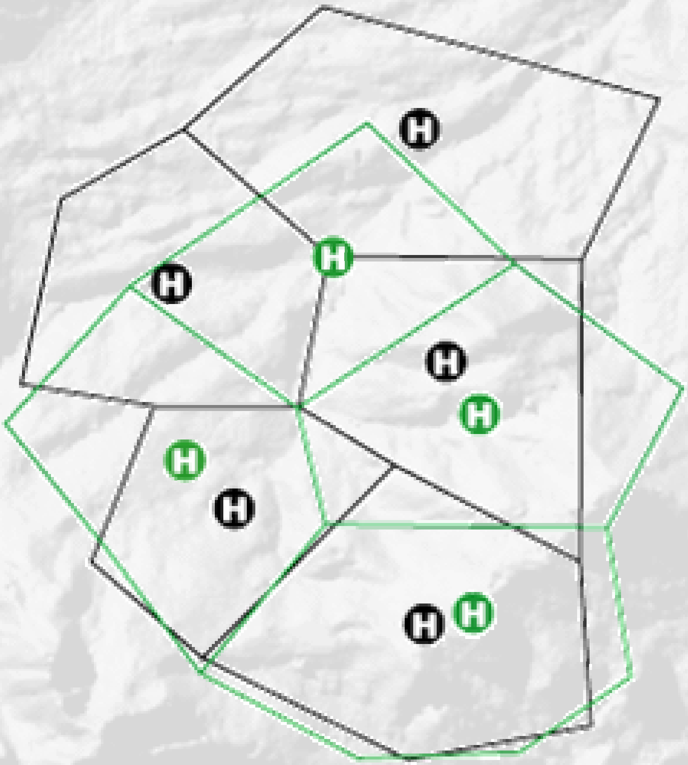


Figure 3b: Multiple proximate competing hospitals with overlapping catchments

Gridded data

- **M**uch elevation data is now acquired from gridded coverages. These are used to create *gridded DTMs*.
- Traditionally, gridded DTMs were created from aerial photography which, where there is limited vegetation, can deliver sub-meter accuracy DTMs.

Grids

- **GRIDS** provide a matrix structure that records topological relations implicitly (just like raster—topology is based on location) rather than explicitly encoded or calculated.

Gridded data sources

- Main source of gridded data is imagery.
- RS methods can provide world-wide coverage but have a number of generic limitations:
  1. No sensor can measure ground elevation beneath forest cover;
  2. Even without surface cover, there is significant random error that is caused by limitations of the sensors and surface roughness.
  3. Usually ground control points are used but often these are hard to locate in remote regions.
  4. Air-borne RS delivers better elevation data but is more expensive to fly.

RS data for grids

- **Remotely sensed data** must be filtered to remove “surface noise” which can be both random and systematic.

Using archival data

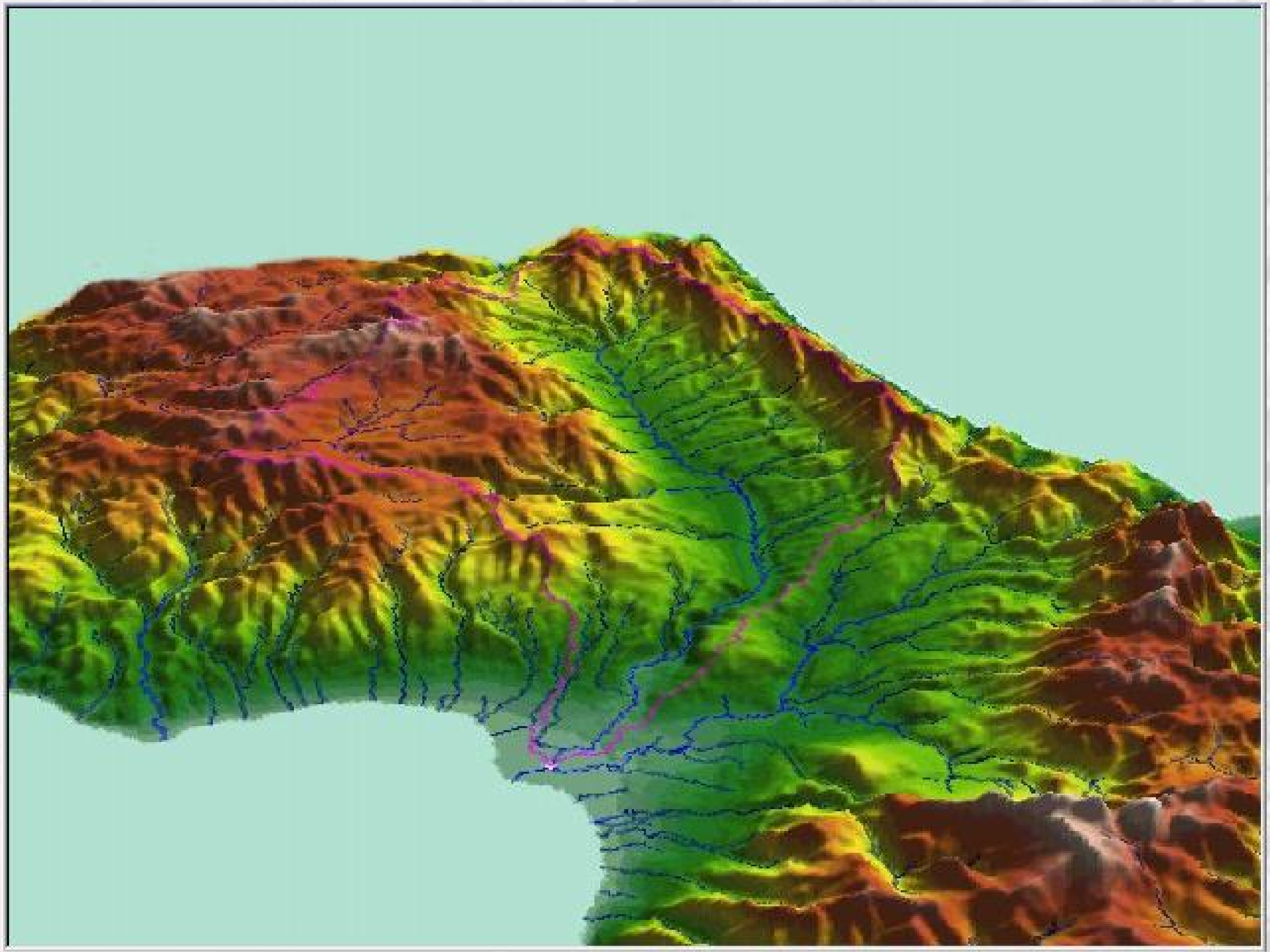
- **Contour and stream line data:** Contour maps are still used extensively as a data source for DTMs
- NMAs worldwide have been working on the conversion of paper topo maps to digital data, but cartographic sources remain in use.
- Cartographic data sources tend to be used when large areas must be covered and money is short (i.e. most government agencies).

Advantages of Contour Data

- In areas of low-relief, contours can give superior data because they pick up elevation changes that are occluded in the RS coverage by error in classification.
- QUESTION: Why do DTMs often refer to hydrological data?

Digital Terrain or Elevation Models

- Terrain data capture results in a series of fairly uneven data elements.
- These are the “original measurements”and topological relations between the data elements must be established.
- Topological definition depends on TIN or Grid.
- An interpolation model is then used to approximate the surface behaviour.
- DTMs are not built directly from the terrain data, but undergo a series of transformations.



Interpolation for DTMs

- Interpolation: the process of estimating elevations in regions where no data exist. Interpolation is used for the following operations:
  (i) computing elevations (z) at a given location (point data)
  (ii) computing an elevation (z) for a specified grid cell:
  (iii) computing the locations (x,y) of points along contours (this is used for contour interpolation); i.e. inverse interpolation.
  (iv) “resampling” or either densifying or coarsening DTMs based on grid data. That is, with scale changes or accuracy change.

Ways to classify interpolation

- For point-based interpolation, models can be separated between exact and approximate methods. Exact methods preserve values at the appointed data points while approximate methods smooth out the data.
- Another differentation is between global and local methods. Global methods use all the point data while local methods work with surface patches. Local is area by area, whereas global is more gestalt.

General Principles of Interpolation

- (i) the biggest indicator of DTM quality is the quality of original data points:
- (ii) when selecting an interpolation method, you need to take into account : the degree to which structural features (ex. faults and elevation changes) are represented and how easily the interpolation can be adapted to varying terrain.

Examples of interpolation techniques

- 1. Triangulation: interpolation is based on fitting polynomial functions to triangles.
- 2. Interpolation from contour data: Most methods use interpolation along straight lines (either pre-defined lines or in the direction of steepest slope).
- 3. Local surface patches: In this method, the area is divided into regions that are rectangular and overlap. Each region is interpolated using a global method and then the regions are fitted back together and the edges are smoothing. (sort of a global/local technique).

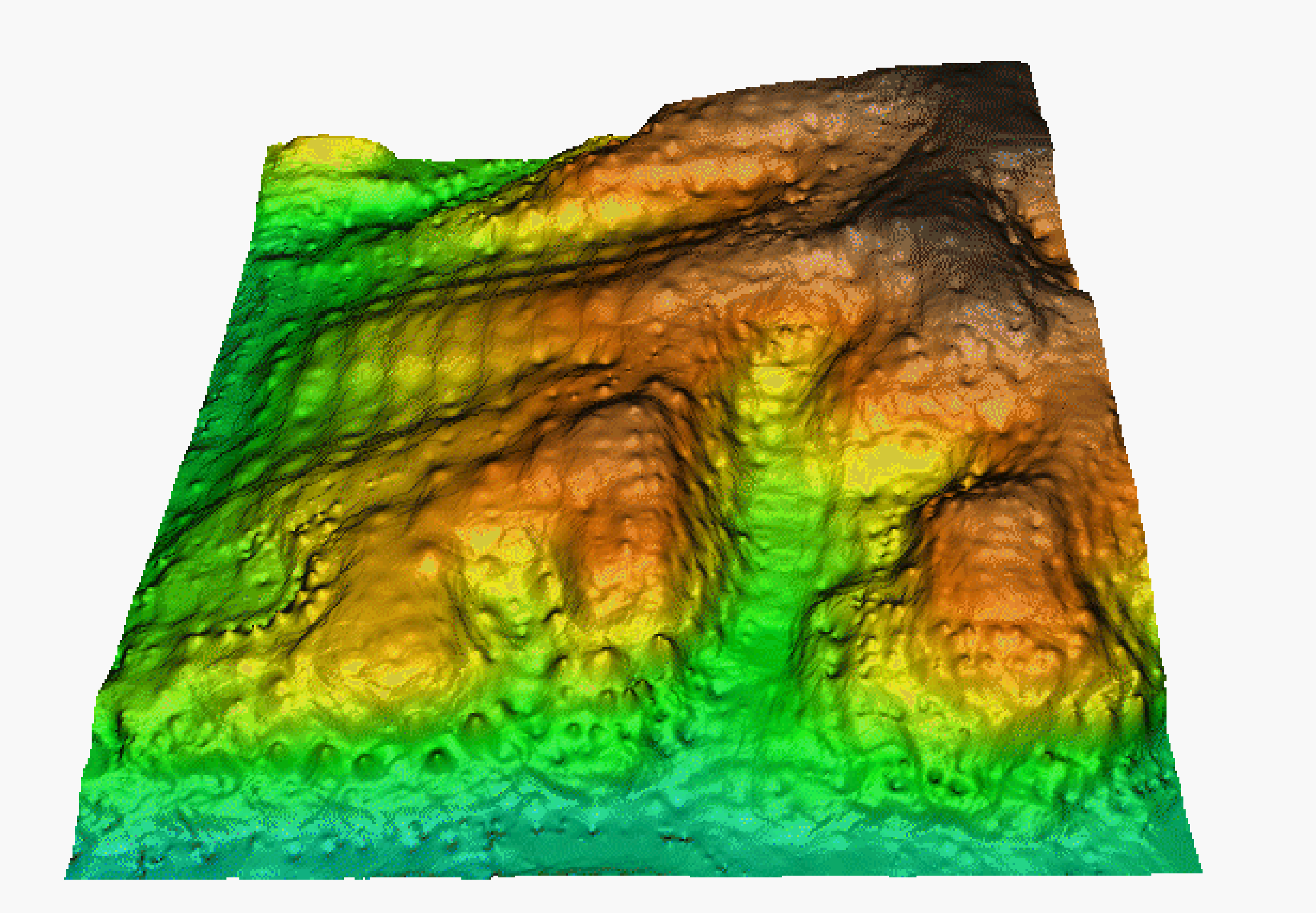
Inverse Distance Weighting

- IDW assigns higher weights to estimate points which are close to the data points.
- Based on SAC, IDW is simple and intuitive.
- A geographical law, as such, states that zones of influence decay over distance. IDW decay rates may be adjusted by changing the power or by a stratified distance approach, with respective power changes.

The general formula for IDW is:

$$\hat{z}_0 = \frac{\sum_{i=1}^{n} w_i z_i}{\sum_{i=1}^{n} w_i}$$

Where the hat over the *z* indicates an estimated value of surface height. The subscript *0* refers to the estimation point and the subscript *i* refers to the sample points falling within the zone of influence. The weights are related to distance by $w_i = 1/{d_{i0}}^2$ where d_{i0} is the distance from point *i* to point *0"*,



DEM produced by IDW

Kriging and geostatistics

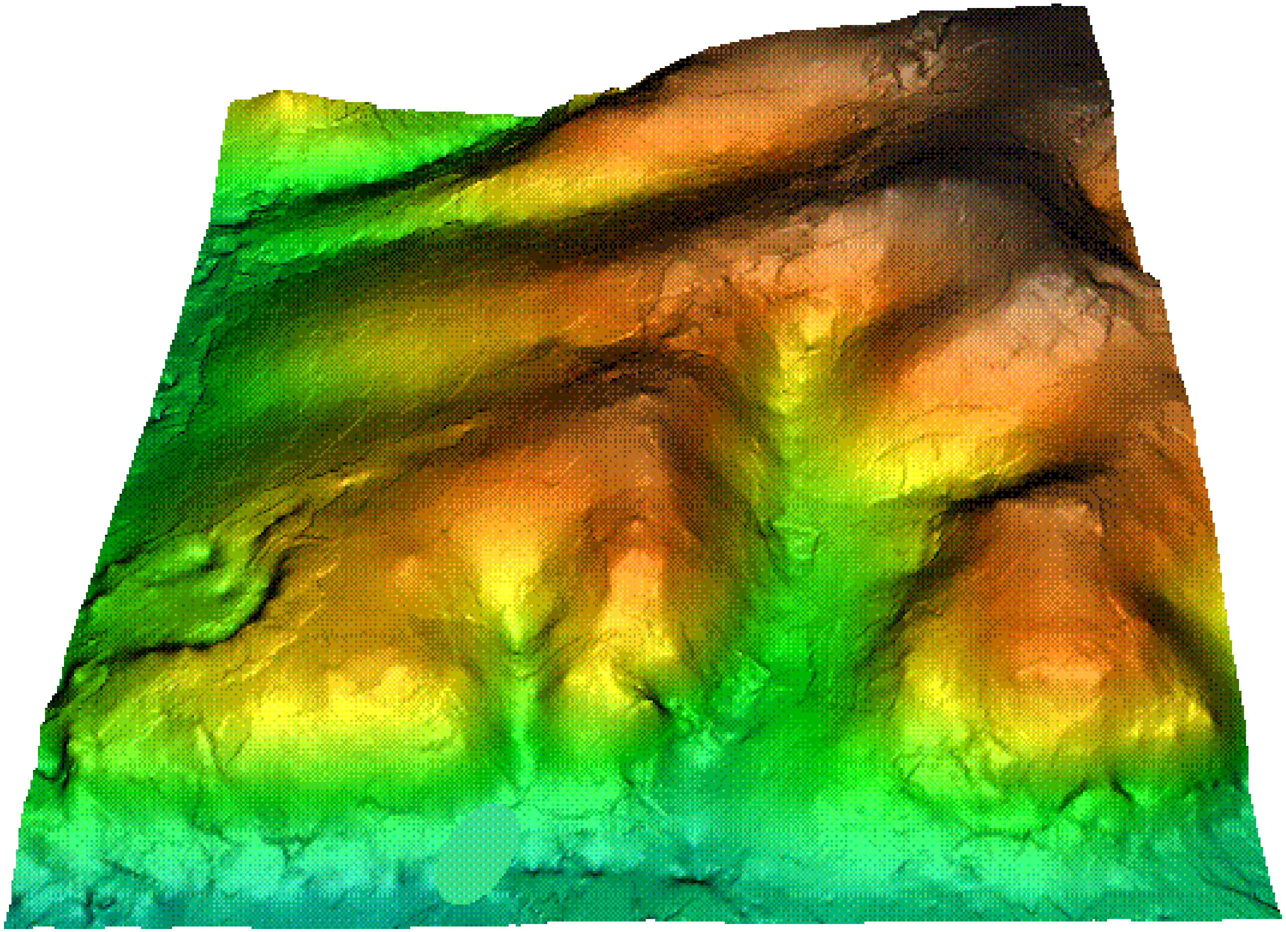
- Kriging is a statistical approach to interpolate data based upon spatial variance.
- Similar to IDW whereby proximity and influence are assumed to be related, kriging recognises that spatial variance is a function of distance.

Background to kriging

- *"The method rests on the recognition that the spatial variation of any geological, soil, or hydrological property, known as a 'regionalised variable', is too irregular to be modelled by a smooth mathematical function but can be described better by a stochastic surface. The interpolation proceeds by first exploring and then modelling the stochastic aspects of the regionalised variable. The resulting information is then used to estimate the* λ_i *weights for interpolation." Burrough, (1994).*

Basic Kriging

- **U**sed in geostatistics to estimate surfaces using known values and a semivariogram to determine unknown values.
- The procedures involved in kriging incorporate measures of error and uncertainty when determining estimations.
- Based on a semi-variogram, optimal weights are assigned to unknown values.



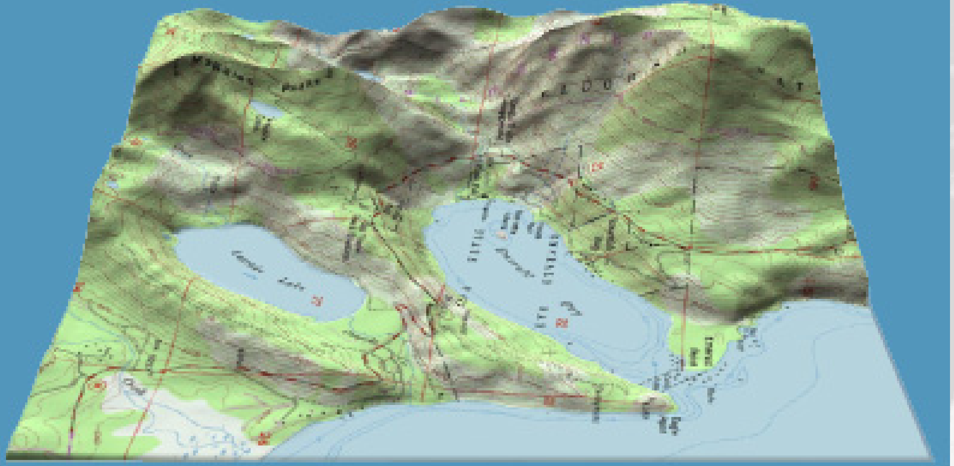
DTM produced by Kriging. Same area as IDW example.

Uses of DTMs

- **Volumes.** Accurate surveys of excavation volumes are critical for planning road work, building industrial complexes,etc. **Visualization techniques**. Creation of DTMs allow the use of “fly-throughs”.

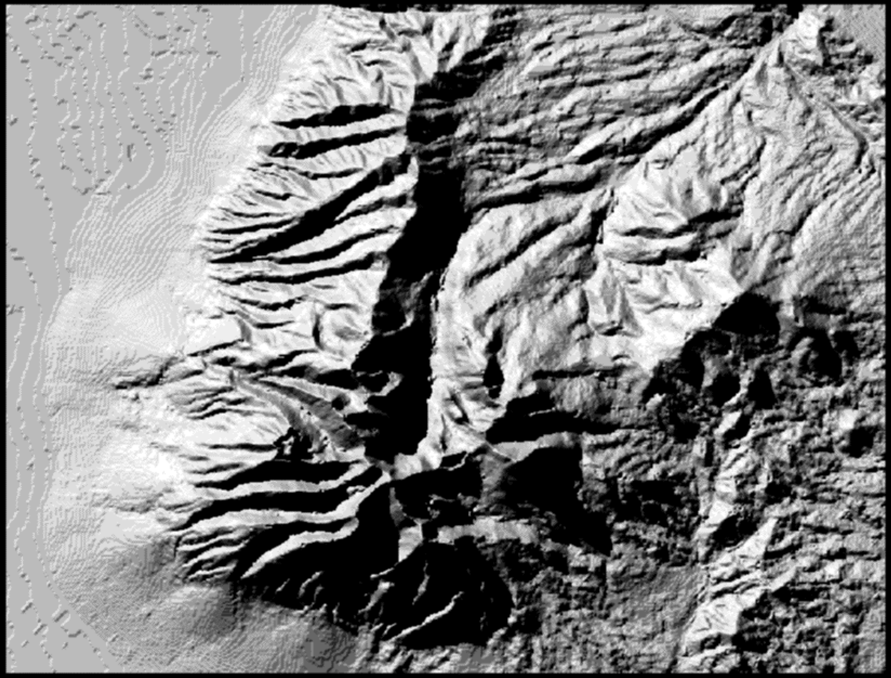
More uses of DTMs

- **Shading and draping**.
- **Hydrological modelling.**
- **Other uses.** Engineers, geomorphologists, geologists etc. all use DTMs.

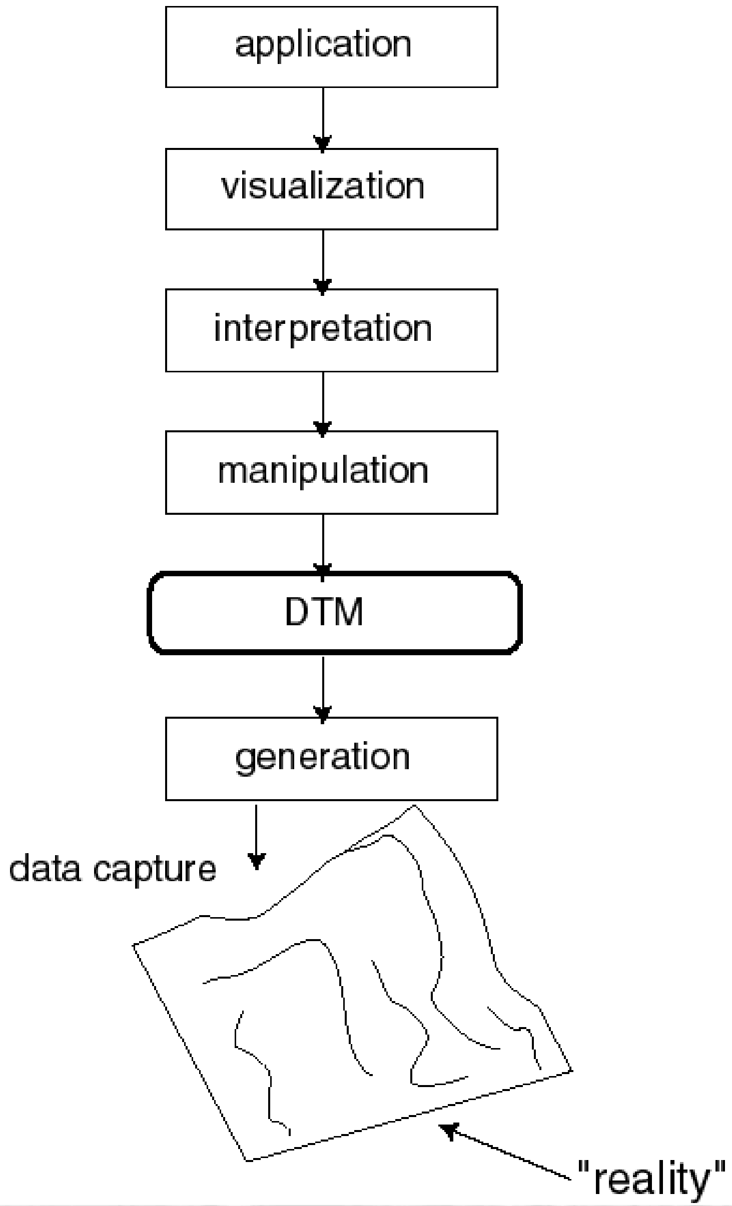


Shaded relief map

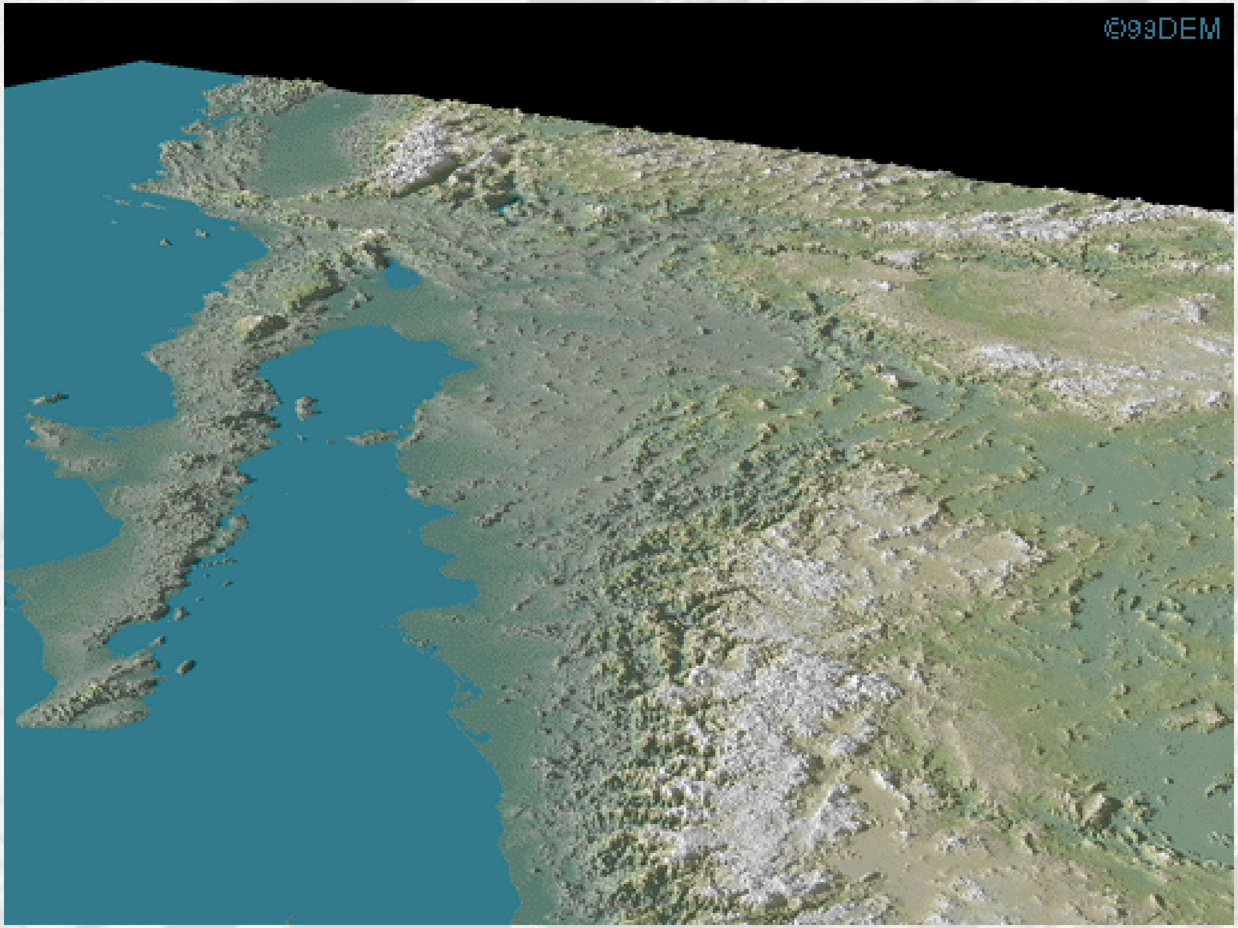
Draping



Tasks involved in creating a DTM modelling system

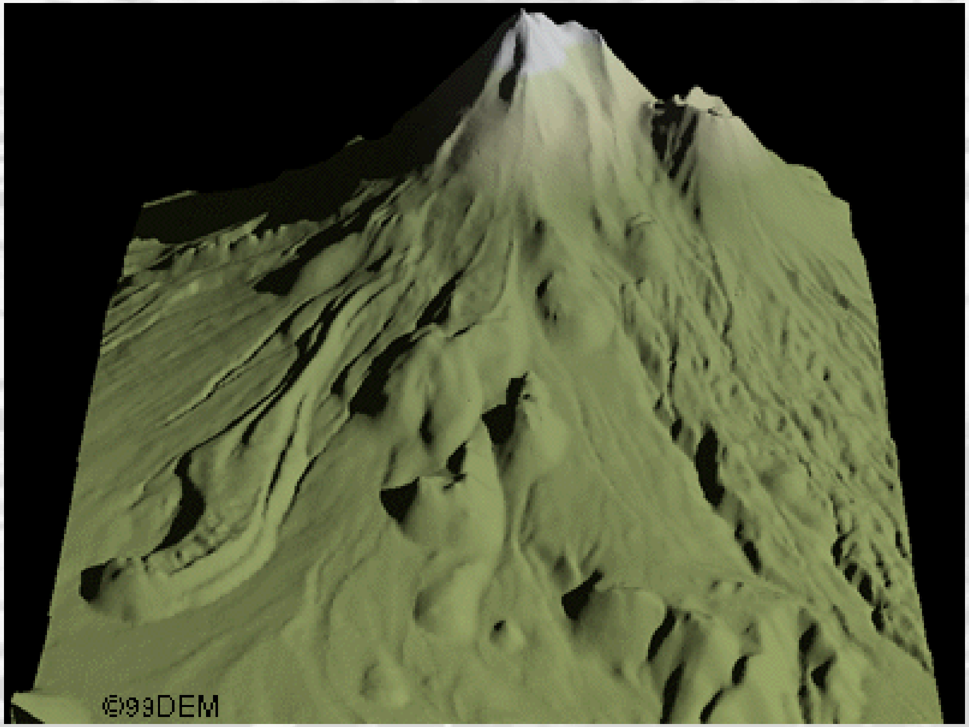


3-D visualization of geological data using DEM (1)

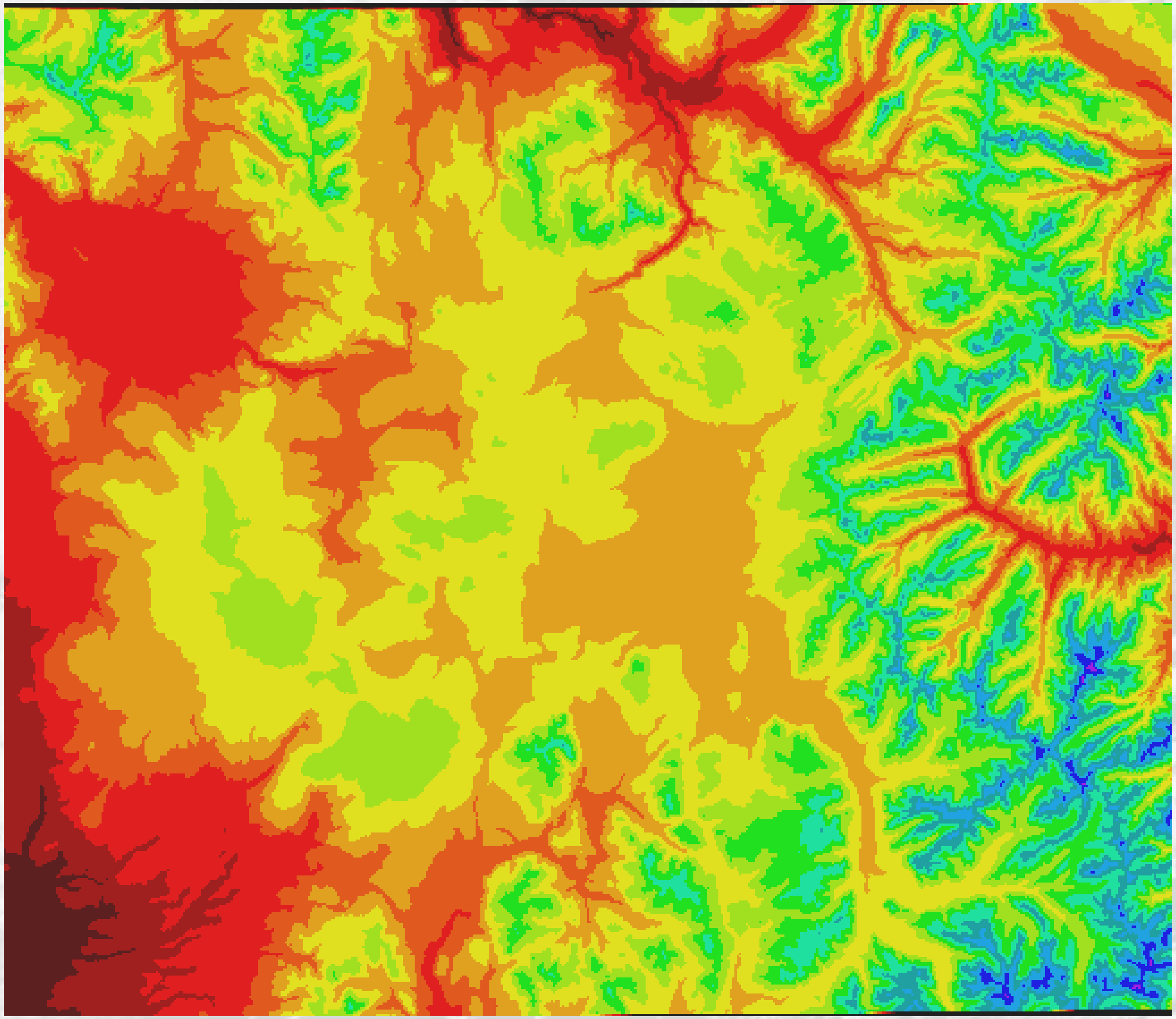


Sea of Cortez and southern basin and range, North America

3-D visualization of geological data using DEM (2)



Mount Shasta, California



Yellowstone DEM