JOURNAL OF ENVIRONMENTAL ECONOMICS AND MANAGEMENT 12, 144–154 (1985)

Pigovian Taxes Which Work in the Small-Number Case

DONALD WITTMAN[1]

Merrill College, University of California, Santa Cruz, California 95064

Received May 17, 1982; revised April 1984

An appropriately conceived pollution tax can achieve a Pareto optimal equilibrium which is (1) stable in the presence of myopia, (2) not subject to strategic manipulation even in the small-number case, and (3) resistant to inefficient cost shifting by the participants when transaction costs are low. A considerable amount of confusion in the literature exists because different authors use different tax formulas (often implicitly) and different assumptions regarding conjectural behavior. Some of this confusion is cleared up by formally presenting various Pigovian tax formulas, explicitly considering whether there is Cournot or Stackleberg behavior, and comparing the properties of the various configurations. © 1985 Academic Press, Inc.

Pigovian taxes have been under attack. Even Baumol [1, 2], an ardent supporter, has abandoned their use in the small-number case (where one or both sides is composed of a small number of participants). Brown and Holahan [5] have shown that Pigovian taxes are subject to strategic manipulation, while Coase [8], Buchanan and Stubblebine [7], and others have shown that Pigovian taxes do not work when there are zero transaction costs.[2] In this paper I show that an alternative formulation of Pigovian taxes (in the generic sense) is not subject to such strategic manipulation and that Pigovian taxes can be devised so that they work even when transaction costs are low. Furthermore, the feasibility of such a tax is demonstrated by the widespread existence of a common law liability rule with similar measurement requirements.

As an important by-product a considerable amount of confusion in the literature regarding Pigovian taxes is cleared up. Different authors use different formulas (often implicitly) for Pigovian taxes and different assumptions regarding conjectural behavior. In this paper the different tax formulas are formally presented and the different conjectures are explicitly considered. A comparison of the properties of the various combinations is then made.

I. A MODEL OF THE EXTERNALITY PROBLEM

As our primary example, we use the factory smoke–commercial laundry case. The factory smoke blackens the laundry hung out to dry (no other damage takes place).[3]

[1] I thank William Baumol for many useful suggestions. This research was supported by The National Science Foundation.

[2] Strategic manipulation can arise only when a particular party has a significant effect on the welfare of others, and low transaction costs between polluters and pollutees exist only when very few people are involved.

[3] There are only production externalities and no direct consumption externalities. This assumption is made for simplicity of exposition only. Except for Section VI it will be assumed that transaction between the factory and the laundry is impossible. We will not consider the administrative costs associated with Pigovian taxes.

0095-0696/85 $3.00

The laundry and the factory can undertake costly measures to reduce damage from pollution. It is assumed that the government chooses a tax rule which encourages the most efficient combination of prevention measures.

The basic issues can be captured in the following simple model. Let x be the dollar cost to factory X of damage prevention. For a single polluting firm in a perfectly competitive industry these costs are identical to reduced profits (due, for example, to increased smoke abatement equipment or to decreased production). Let y be the dollar cost to the laundry Y of damage prevention (e.g., hanging clothes indoors, reducing the amount of clothes cleaned). Thus x and y are inputs into the reduction of damage. The total cost to society is the sum of the damage plus the cost of damage reduction.

$D(x, y)$ is the damage (actual or expected) in dollar amounts.

$$D_x < 0, \quad D_y < 0. \tag{1}$$

We assume that D is a convex function of x and y and that all damage falls on Y (the laundry). In particular, we assume that $D_{xx}, D_{yy}, D_{xy} > 0$. This last inequality says that the inputs are substitutes and implies that the optimal response to an increase in smoke is an increase in self-protection by the laundry.[4]

$$C^S(x, y) = x + y + D(x, y) \tag{2}$$

is the total cost to society.

It is assumed that the government wants to minimize total cost. First-order conditions for an interior solution are (3) and (4).

$$C_x^S = 1 + D_x = 0. \tag{3}$$

The last dollar spent on prevention reduces damage by 1 dollar.

$$C_y^S = 1 + D_y = 0. \tag{4}$$

Given our assumptions the second-order conditions are satisfied.

Let that pair x, y which minimizes C^S be denoted by x^*, y^*. (5)

II. PIGOVIAN TAX FORMULAS

A variety of Pigovian tax formulas is possible. We will consider several of these possibilities with our main emphasis being on the marginal cost tax.

The most commonly discussed Pigovian tax is a tax on X for *actual damages*.

$$\text{TAD} = D(x, y). \tag{6}$$

[4] These assumptions are standard in the literature. See Brown and Holahan [5], Brown [4], and Diamond [10], who make similar assumptions regarding second-order conditions in accident law. For a proof that these conditions imply substitutability see footnote 16. While the discussion in the text will be in terms of actual damage, the proofs still hold if damage takes place probabilistically. Let $p(x, y)$ be the probability of an accident and $d(x, y)$ be the damage if an accident occurs. We can redefine $D(x, y)$ to be expected damage with $D(x, y) = p(x, y)d(x, y)$.

Another possibility is to charge X for the *actual costs* to Y.

$$\mathrm{TAC} = y + D(x, y). \tag{7}$$

The *marginal cost* Pigovian tax does not depend upon what Y *actually* does but rather upon what Y *should* do given x. X is charged for the cost effective behavior that should be undertaken by Y given x and the resulting damage that exists (or would exist) when y undertakes this cost effective behavior. In order to emphasize this point that the measure does not depend upon actual costs but rather on cost effective marginal cost, we will refer to the tax as a mitigated (or cost effective) marginal cost rule. While this approach may be unfamiliar to some, the technique of looking at what should have happened rather than at what actually did happen is common throughout tort law. In fact, as will be shown later in the paper, the liability rule counterpart of the marginal cost Pigovian tax is known as the doctrine of mitigation of damages.

$$\mathrm{TMC} = R^y(x) + D(x, R^y(x)) \tag{8}$$

In (8), $y = R^y(x)$ is the optimal amount of input y given any level of input x. $R^y(x)$ is an alternative version of (4) with y as an explicit function of x. Note that $R^Y(x)$ only equals y^* when x equals x^*.

One could also create a step function version of TMC by altering the tax so that X did not pay any tax if the marginal reduction in costs that X imposed on Y were less than the cost of the increased x. More formally

$$\mathrm{TMC}' = 0 \quad \text{if} \; -\left[\frac{dR^y(x)}{dx} + D_x(x, R^Y(x)) + D_y(x, R^Y(x))\frac{dR^Y(x)}{dx}\right] \leqslant 1 \tag{9}$$

$$\mathrm{TMC}' = \mathrm{TMC} = R^Y(x) + D(x, R^Y(x)) \text{ otherwise.}$$

The top line of (9) is equivalent to $0 \leqslant 1 + D_x + (dR^Y(x)/dx)[1 + Dy]$. This inequality is equivalent to $x \geqslant x^*$.[5] Thus TMC′ looks like TMC for $x < x^*$. Under TMC′ we would not observe an $x > x^*$ since this would not reduce X's tax from x^* (there being no tax at x^*), but it would increase X's costs beyond x^*. Note that the government need not know x^* initially.

While a step function may appear strange, many liability rules actually used in tort cases have this property. For example, negligence with contributory negligence would make X liable only if x were less than x^* and y were greater than or equal to y^*.

A step function is the way Pigovian taxes and fines work in practice. A driver is fined for speeding above the optimal amount.[6] Below the optimal amount he is neither taxed (unless he is going too slowly) nor subsidized. Similarly, charges for noise and air pollution are levied if the pollution is above a certain level; below that level there is typically neither a charge nor a subsidy.[7]

[5] This is demonstrated in Appendix 1. Note that step functions can also be created for TAD and TAC.

[6] The driver is being taxed either for the cost of defensive actions by other drivers or for the expected cost of accidents from driving over the speed limit.

[7] For example, a person may be fined for disturbing the peace if the person has created noise above certain acceptable levels. The person is not fined for all noise above zero sound.

It is illuminating to compare a standard to the marginal cost step function. Their differences can be understood best via a graph. Let the y axis stand for the marginal cost of the tax (or standard) to the polluter and the x axis stand for the dollar expenditure on damage prevention by the polluter. Reading the graph from right to left, the marginal cost of the tax to the polluter under a marginal cost step function looks like a step at x^* and then an upward sloping ramp. In contrast, the graph of the marginal cost to the polluter under a standard has a spike or very high wall at x^*. The wall is much higher than the step.[8]

Finally, we consider a tax based on global information. Taking a global look, the charge to the factory per unit of smoke is set equal to the marginal damage of a unit of smoke when the optimal amount of smoke damage is being produced.[9] Because the amount of tax is based on the amount of smoke, it is easiest to understand the analysis if we explicitly consider damage as a function of smoke. Let $D'(s(x), y)$ be the damage.[10] $s(x)$ is the amount of smoke. $ds/dx < 0$, $D'_s = \partial D'/\partial s > 0$.
The tax based on the *optimal* amount of *smoke* is then:

$$\mathrm{TOS} = D'_s(s(x^*), y^*)s(x). \tag{10}$$

Note that the tax rate is independent of the level of y that Y actually chooses since it is based on y^* and not y. This tax clearly requires considerable prior information regarding the relevant cost curves.

TOS is different from TAD in several respects. TAD is a function of y (as well as x) while TOS is only a function of x. Furthermore, TAD is a tax equivalent to the full amount of the actual damage, while TOS is a tax based on the optimal amount of damage and the actual amount of smoke. Therefore at x^*, y^* TOS is a tax greater than the amount of damage as the tax is based on marginal damage (D'_s) at x^*, y^* and marginal damage increases as smoke increases from 0 to $s(x^*)$.[11]

III. COURNOT EQUILIBRIUM

It is very easy to demonstrate that all of these formulas have a Cournot equilibrium at x^*, y^*.[12]

For any given x, the costs of any change in y on X or Y are fully internalized by Y (the laundry) as the damage falls only on Y (and x is given). Therefore, given x^* the laundry will choose the optimal expenditure on damage prevention, y^*. The

[8] If the government has underestimated the cost of smoke prevention by the polluter and has set x^* too high, under a standard the polluter is stuck at the high wall and too much expenditure on damage prevention by the polluter will take place. Under the step function marginal cost rule, the polluter will reduce his expenditures below x^* if his actual savings from not undertaking damage reduction are greater than the increased tax. At x^* the marginal tax from decreased x is the same for the step function and the regular marginal cost rule. Note that if a violation of a standard is imperfectly detected and this detection becomes more likely the greater the degree of violation, then a standard starts to look like a step function marginal cost tax. If polluters who do not violate the standard are sometimes mistakenly fined for violating the standard, then the standard begins to look more like the regular marginal cost rule.

[9] This is similar to the method presented by Maler [11], Baumol [1], and Baumol and Oates [3].

[10] Note that s in $s(x)$ stands for smoke while S in C^S stands for society.

[11] A constant can be added or subtracted from the tax without changing the marginal relationships. For example, $\mathrm{TOS}' = D''_s(s(x^*), y^*)[s(x) - s(x^*)]$ would result in a zero tax at $x = x^*$. The above set of Pigovian tax formulas is not exhaustive. For example, see White and Wittman [13] for a discussion of double Pigovian taxes.

[12] The demonstration that the Cournot equilibrium is Pareto optimal has been done for most of these taxes. See Browning [6] and the previous citations.

laundry will increase damage prevention (y) until the marginal cost of damage prevention equals the amount of decreased damage. This can be proven more formally.

$$C^{Y}(x, y) = y + D(x, y) \tag{11}$$

is the total cost to Y. Y wants to minimize this cost given $x = x^*$.

First-order conditions for an interior minimum when x^* is treated as a given:

$$C_y^Y = 1 + D_y(x^*, y) = 0. \tag{12}$$

This is the same as the first-order condition for minimum social cost (4).

For TMC, the factory (X) internalizes all costs that should be optimally undertaken by Y for any given x regardless of what Y actually chooses to do. The factory will increase x until the extra cost of preventing smoke emission is equal to the marginal cost it *imposes* on the laundry. This is the efficient amount of smoke prevention by the factory and involves the same marginal calculation that would be undertaken by a firm formed by the merger of the factory and the laundry. Thus X will choose x^*. This will now be demonstrated more formally.

$$C^{\mathrm{XMC}}(x, y) = x + \mathrm{TMC} = x + R^Y(x) + D(x, R^Y(x)) \tag{13}$$

is the total cost to X when a mitigated marginal cost Pigovian tax is used.

X wants to minimize his total cost. First-order conditions:

$$C_x^{\mathrm{XMC}} = 1 + \frac{dR^Y(x)}{dx} + D_x + D_y \frac{dR^y(x)}{dx} = 0 \tag{14}$$

$$= 1 + D_x + \frac{dR^Y(x)}{dx}\left[1 + D_y\right] = 0. \tag{15}$$

If $1 + D_x = 1 + D_y = 0$, then $C_x^{\mathrm{XMC}} = 0$. Therefore if Eqs. (3) and (4) are satisfied, Eq. (15) is satisfied. Under our assumptions concerning second-order partial derivatives, there is only one interior extremum for C^{XMC} as well as C^{S}.[13] Given x^*, we have already shown that Y's optimal response is y^*. Thus TMC produces a Cournot equilibrium at x^*, y^*.

The other tax formulas also have a Cournot equilibrium at x^*, y^*.[14]

[13] Because a fixed change in the tax will not affect the marginal relationships, a whole family of taxes based on TMC is possible. One important possibility is TMC″ = TMC $- y^*$ for all x. Then at x^*, y^* TMC = TAD (and only at x^*, y^*). This would allow implementation of the alternative tax formula TMC″ without changing the actual amount of taxes under the old formula TAD if the old formula actually produced the correct result.

[14] We will not clutter up the paper with formal proofs; instead we will use this footnote to provide the intuition. Given y^* (or any other y), the costs of any change in x on X and Y are fully internalized by X (the factory) when the factory is taxed according to TAD. When the factory pays for the full amount of damages, it increases its own damage prevention until the marginal cost of damage prevention equals the amount of decreased damage. At equilibrium *all* the marginals are equated—the same conditions required for a social optimum. A similar logic holds for TAC. The proof for TMC′ is similar to that for TMC. Turning our attention toward the tax based on the optimal amount of smoke (TOS) it is relatively easy to show that this tax too will produce an optimal result. The tax per unit of smoke is the marginal damage per unit of smoke at x^*, y^*. If the factory produces less smoke than optimal, the cost of producing less smoke is greater than the tax savings by the assumption of increasing costs of smoke prevention. For the same reason, the factory will not produce too much smoke as the tax on the extra unit of smoke ($D_S'(x^*, y^*)$) is greater than the cost of smoke prevention.

IV. STACKLEBERG EQUILIBRIUM

When there are small numbers of polluters Stackleberg conjectures are likely to result. Ideally one would like to create a tax that had a Pareto optimal Stackleberg equilibrium as well as a Cournot equilibrium. Such a tax could handle both the small-number and the large-number case. This kind of versatility is extremely important in devising pollution taxes because of all the possible numerical combinations of actors that can arise in externality problems. Only tax TMC and TOS satisfy this requirement.

With tax TAD, X is not charged for the cost of prevention by $Y, (y)$. Therefore if X is the Stackleberg leader, X will try to shift the cost of prevention onto Y. For example, the factory may billow out smoke hoping that the laundry will bring its clothes indoors so that there is no damage and no tax on X.[15]

Via a similar argument it can be shown that if the laundry (Y) is the Stackleberg leader, the equilibrium will not be Pareto optimal under tax TAD or TAC. For example, the laundry might purposely hang out pollution-sensitive laundry. If the factory's optimal response to such a move were to eliminate smoke entirely, the laundry as a Stackleberg leader would be nonoptimally shifting the cost of damage prevention onto the factory (if the social cost/benefit equation provided for some smoke to be produced at the efficient outcome).

Taxes TMC, TMC′, and TOS are not subject to Stackleberg manipulation. With regard to TMC, the tax is based on what X does and Y should do. Therefore, X does not care what Y actually does and so Stackleberg manipulation by either is not possible. If X is the "Stackleberg leader," his conjectures will be identical to those when he is a Cournot follower as the tax is based on an optimal response by Y. Y cannot be a Stackleberg leader with this formula. A similar logic holds for TMC′.

Under TOS the tax on X also does not depend on what Y actually does. So again, neither side can be a Stackleberg leader.

In this section we have shown that taxes TAD and TAC are subject to strategic manipulation but that Pigovian taxes TMC, TMC′, and TOS overcome certain problems associated with the small number case.

V. MYOPIA AND STABILITY

The fact that a Pareto optimal equilibrium exists is not at all interesting if the system will not move toward equilibrium when it is not actually at this optimal point. In this section we consider the case where the government does not initially know x^*, y^*. The object is for the government to create a tax rule which will sooner or later move the parties to x^*, y^* even though the government does not know this point when the rule is initiated. This property will be defined as stability in the presence of myopia by the government.

It is relatively easy to demonstrate that TMC is stable even though the government does not initially know x^*, y^*. Under TMC the factory must account for all marginal costs it imposes on Y. If $x < x^*$, then any increase in x will decrease the total costs to X as the marginal cost of damage prevention is less than the decrease

[15] Davis and Whinston [9] and Brown and Holahan [5] have argued that Pigovian taxes based on damage are not strategy proof. Again we will not clutter up the paper with a formal proof.

in the Pigovian tax. Therefore X will move toward x^*. As x approaches, x^*, $R^Y(x)$ will approach y^*. x and y are gross substitutes. That is, the optimal response to more damage prevention by X is less damage prevention by $Y(dR^Y(x)/dx < 0)$.[16] If $x < x^*$, then $y > y^*$. Consequently, an increase in x toward x^* will result in a decrease in y toward y^*. Thus TMC is stable.[17] A similar proof shows TMC′ to be stable.

Taxes TAD, TAC, TMC, and TMC′ have stability in the presence of myopia but TOS does not. Thus, only taxes TMC and TMC′ have both a stable Pareto optimal Cournot equilibrium and an efficient Stackleberg equilibrium.

VI. PIGOVIAN TAXES AND ZERO TRANSACTION COSTS

So far we have assumed that transaction costs between X and Y are high.[18] However, it is possible that in some cases (where there are few parties involved or there is a class action suit) transaction costs between X and Y are low, allowing the participants to negotiate with each other. It is therefore desirable that the government devise a tax system that is flexible enough to handle both high and low transaction costs cases.

It has been argued by numerous authors that with zero transaction costs, Pigovian taxes are inefficient and inferior to liability rules.[19] The argument runs along the following lines. Under a Pigovian tax, the laundry is incurring costs from the smoke for which it is not being compensated. It is willing to bribe the factory to produce less smoke at the same time the government is taxing the factory to produce the optimal amount of smoke. Thus at x^*, y^*, the marginal cost (Pigovian tax plus lost bribe) to the factory producing the last unit of smoke is greater than the social marginal cost (which is equivalent to the Pigovian tax); which in turn is equal to the marginal cost of preventing the smoke. Therefore the factory will produce less smoke than is optimal. In a nutshell, the laundry and the factory will contract to increase total profits by reducing the amount of Pigovian taxes going to the government. In contrast, under a liability rule there is no leakage to the government and consequently they will choose x^*, y^*.

However, it is very easy to devise Pigovian taxes which provide the correct incentives in low transaction cost situations.

Under TMC′, X is not taxed if he provides the optimal amount of x. Therefore he cannot be bribed to produce too much x and too little smoke. Step functions can

[16] The following is a proof that $dR^Y(x)/dx < 0$. We make use of the implicit function theorem. (a) $1 + D_y(x, y) = 0$. Taking the total differential of this expression. (b) $D_{yx}\,dx + D_{yy}\,dy = 0$ or (c) $-D_{yx}/D_{yy} = dy/dx = dR^Y(x)/dx < 0$.

[17] Brown and Holahan [5] have already shown that tax TAD has a stable Cournot equilibrium. A similar proof shows TAC to be stable also. Since TOS depends initially on a priori knowledge of x^*, y^*, it is not useful when the government does not initially have this information. An alternative version of TOS which sets charges per unit of smoke and then adjusts the charges as more information is acquired, has properties similar to TAD. It has a stable Cournot equilibrium but unfortunately it is subject to Stackleberg manipulation.

[18] We note that the major issues of industrial pollution and traffic congestion typically involve high transaction costs.

[19] This argument in favor of liability rules when there are zero transaction costs has previously been made by Coase [8], Buchanan and Stubblebine [7], and Brown and Holahan [5]. In contrast to Schulze and d'Arge [12], in this paper zero transaction costs implies zero transaction costs for the potential entrants as well.

also be created from taxes TAD, TAC, and TOS. Thus, even if there are zero transaction costs, a slight alteration of the ordinary Pigovian tax rules will allow Pigovian taxes to achieve the optimal outcome.

VII. INFORMATIONAL REQUIREMENTS

It is useful to briefly consider the informational requirements of these rules. TAD requires observation (or estimation) of the actual damage. TAC requires in addition knowledge of the damage prevention by Y. TMC and TMC′ require knowledge of the damage prevention by X and an estimate of the optimal y for the *given* level of x, but no knowledge of the actual damage and prevention costs by Y. As already noted the courts often engage in this type of estimation when assessing liability. When this type of tax (or liability rule) is implemented, actual behavior will often coincide with optimal behavior so that the amounts can be observed. Thus while TMC requires different information from TAD and TAC it may not require more (or substantially more) information than TAD and TAC even though it has a great many desirable qualities that the other two taxes do not possess.

In contrast TOS requires the government to know the optimal amounts x^*, y^* initially instead of searching and moving toward them. TOS initially requires full global information and thus its information requirements are much more severe than the other taxes. The information requirements of TMC, TAD, and TAC can be obtained by independently estimating the relevant values or providing negative sanctions for dishonest reporting. (For example, courts make use of expert testimony, two sides, and sanctions for perjury in order to obtain more accurate information.) The actual use of these methods as opposed to the hypothetical use of the "demand revelation" processes suggests that these former methods are more practical.

A comparison of the properties of the various tax rules can be found in Table I.[20]

TABLE I

Tax	Cournot efficient?	Stackleberg efficient?	Stable if myopia?	Is zero transaction cost efficient?	Information requirements
TAD	Yes	No	Yes	No	Actual damage
TAC	Yes	Yes for X No for Y	Yes	No	Actual cost to Y
TOS	Yes	Yes	No	No	Global knowledge
TMC	Yes	Yes	Yes	No	Optimal cost to Y for given x; actual x
TMC′	Yes	Yes	Yes	Yes	Optimal cost to Y for given x; actual x

[20] This list of properties is not exhaustive. See White and Wittman [14] for a discussion of tax and liability rules when the government knows the slopes of the marginal cost curves but not their intercepts. The liability rule which they consider can be translated into Pigovian tax TMC.

VIII. FEASIBILITY

In theory one could devise various schemes but are they practical? Here we demonstrate the feasibility of a marginal cost Pigovian tax rule by showing that the analogous liability rule is used throughout the common law.

A liability rule can be viewed as a Pigovian tax transferred to the victim. Thus the liability version of tax TMC is

$$\mathrm{LMC} = \mathrm{TMC} = R^Y(x) + D(x, R^Y(x)). \tag{16}$$

X's liability to Y is a transfer of TMC to Y. If there are many laundries each one is compensated for the marginal costs imposed on it. This is the idea behind a class action suit. The strict liability scheme proposed here is based on mitigated marginal cost and not on actual damage alone. Thus the laundry is compensated for marginal *costs*—the costs of cost effective self-protection and residual damage.[21]

In fact, the marginal cost liability rule, LMC, is known in tort law as the doctrine of mitigation of damages. For example, in nuisance law, if a farmer has a field known to be swept by sulfur fumes (which will kill any crop), he cannot collect for seeds and labor in planting the crop for he has not undertaken cost effective self-protection.[22] That is, he cannot collect for damages which could have been mitigated.

In contract law there is the doctrine of avoidable consequences. If X sells to Y a quantity of pork packed in barrels of brine with a warranty that the barrels will not leak and Y finds that some of the barrels are leaky but does not repack the pork in good barrels, then Y can get judgment for only the costs of repacking in good barrels (self-protection) and not the value of the pork spoiled after his discovery of the defects.[23] Again, the amount of X's liability to Y is based on what Y should do and not on what Y does. This prevents strategic manipulation by Y in this small-number situation. Because of this, Y will tend to do what he should. Thus, we have strict liability without distortion.[24]

The liability rule counterpart to TAD (LAD) is also quite prevalent. However, it is often the case that the other rules would provide the same result if they were used instead of the LAD rule. For example, in medical malpractice the doctor is typically liable for the actual damage to the patient. Typically, the patient's conscious behavior is not an input into the reduction of damage during the operation. Thus y and $R^Y(x) = 0$ so that LAD, LMC, and LAC are equivalent. Of course, the doctor is only liable if negligent so we have step function versions of these rules. Even here if the patient exacerbates the condition via postoperative carelessness the doctor is only liable for the damages that would have occurred if the patient had not been careless. Thus we are back to an LMC rule.

[21] An alternative liability rule $\mathrm{LMC}' = \mathrm{TMC} - y^*$ would compensate only for damages and not for self-protection by Y when $x = x^*$. Since y^* is a constant (not dependent on x), this liability rule has all the good marginal qualities of LMC.

[22] See United Verde Extension Mining Company vs Ralston, 296 P. 262 (1931).

[23] Restatements of the Law of Contracts 503 1932. See also Daley vs Irwin, 205 P. 76 (1922).

[24] Y is only compensated for mitigated damages. Because of this, people will not expose themselves gratuitously to external diseconomies hoping to bargain (assuming low transaction costs) with the factory for some out-of-court settlement. The threat is not viable because mitigated damages are zero in such cases. See Wittman [15] for other liability rule applications.

The pervasive use of LMC suggests that measures of optimal (rather than just actual) costs are a feasible possibility for Pigovian taxes.

IX. CONCLUSION

In this paper we have argued that charging for mitigated marginal cost rather than for actual damage avoids many pitfalls typically associated with Pigovian taxes. In particular, this pricing formula makes the Pigovian tax strategy proof, Pareto optimal, and stable even in the small number case. In contrast, a Pigovian tax based on damage is neither stable nor strategy proof (unless courts initially know the optimal outcome).

Furthermore, slight alterations in the Pigovian tax formulas allow Pigovian taxes to automatically lead to a Pareto optimal outcome even when transaction costs are zero.

APPENDIX 1

PROPOSITION. *$0 < 1 + D_x + (dR^Y(x)/dx)[1 + Dy]$ is equivalent to $x \geqslant x^*$.*

Proof. By Eq. (14) at x^*

$$C_x^{\mathrm{XMC}} = 1 + D_x + \frac{dR^Y(x)}{dx}\left[1 + D_y\right] = 0. \tag{a}$$

As x increases beyond x^*, C_x^{XMC} increases. This can be demonstrated by looking at the second derivative.

$$\begin{aligned} C_{xx}^{\mathrm{XMC}} = D_{xx} + D_{xy}\frac{dR^Y(x)}{dx} + \frac{d^2R^Y(x)}{dx^2}[1 + Dy] \\ + \frac{dR^Y(x)}{dx}D_{yx} + \frac{dR^Y(x)}{dx}D_{yy}\frac{dR^Y(x)}{dx}. \end{aligned} \tag{b}$$

The third term equals 0 at x^* as $1 + D_y(x^*, R^Y(x^*)) = 0$. We have already shown in footnote 16 that $dR^Y(x)/dx = -D_{yx}/D_{yy}$. Substituting this into (b) and combining terms, we get

$$D_{xx} - 2\frac{\left[D_{xy}^2\right]}{D_{yy}} + \frac{\left[D_{xy}\right]^2}{D_{yy}}. \tag{c}$$

Multiplying through by $D_{yy} > 0$ we get:

$$D_{xx}D_{yy} - \left[D_{xy}\right]^2. \tag{d}$$

This expression is greater than zero by assumption (Eq. (1)) and with the first-order conditions is sufficient for cost minimization. A similar proof can be used to show that as x decreases from x^*, $1 + D_x + (dR^Y(x)/dx)[1 + D_y]$ decreases from zero.

REFERENCES

1. W. J. Baumol, On taxation and the control of externalities, *Amer. Econ. Rev.* **62**(3), 307–322 (1972).
2. W. J. Baumol, On taxation and the control of externalities, *Amer. Econ. Rev.* **64**(3), 472 (1974).
3. W. J. Baumol and W. E. Oates, "The Theory of Environmental Policy: Externalities, Public Outlays, and the Quality of Life," Prentice-Hall, Englewood Cliffs, N.J. (1975).
4. J. P. Brown, Towards an economic theory of liability, *J. Legal Stud.* **2**(2), 323–349 (1973).
5. J. P. Brown and W. L. Holahan, Taxes and legal rules for the control of externalities when these are strategic responses, *J. Legal Stud.* **9**(1), 166–178 (1980).
6. E. Browning, External diseconomies, compensation, and the measure of damage, *S. Econ. J.* **43**(3), 1279–1287 (1977).
7. J. M. Buchanan and W. C. Stubblebine, Externality, *Economica* **29**, 371–384 (1962).
8. R. H. Coase, The problem of social costs, *J. Law Econ.* **3**, 1–44 (1960).
9. O. A. Davis and A. Whinston, On externalities, information and the government-assisted invisible hand, *Economica* **33**, 1–44 (1960).
10. P. Diamond, Accident law and resource allocation, *Bell J. Econ. Manag. Sci.* **5**(2), 366–416 (1974).
11. K. G. Maler, "Environmental Economics: A Theoretical Inquiry," Johns Hopkins Press, Baltimore, Md. (1974).
12. W. Schulze and R. C d'Arge, The Coase proposition, informational constraints and long-run equilibrium, *Amer. Econ. Rev.* **64**(4), 763–772 (1974).
13. D. Wittman and M. J. White, Pollution taxes and optimal spatial location, *Economica* **49**, 297–311 (1982).
14. D. Wittman and M. J. White, A comparison of taxes, regulation, and liability rules under imperfect information, *J. Legal Stud.* **12**(2), 323–349 (1983).
15. D. Wittman, Optimal pricing of sequential inputs: Last clear change, mitigation of damages, and related doctrines in the law, *J. Legal Stud.* **9**(1) 166–178 (1980).