ECON 402 Summer 2006

Assignment 2 - KEY

1. Suppose $B = 100$, $p_y = 1$ and p_x falls from 1 to 0.25. If

$$u = 2x - \frac{1}{2}x^2 + y$$

Compute CV and EV. What do you notice? Why does this happen?
Answer: First find x and y

$$\begin{aligned} MU_x &= 2 - x \\ MU_y &= 1 \\ MRS &= MU_x/MU_y = 2 - x = p_x/p_y \\ x^* &= 2 - p_x/p_y \\ y^* &= \frac{B}{p_y} - 2\frac{p_x}{p_y} + \left(\frac{p_x}{p_y}\right)^2 \end{aligned}$$

Now find the indirect utility function and expenditure function

$$\begin{aligned} v &= \frac{2p_y^2 - 2p_xp_y + 0.5p_x^2 + Bp_y}{p_y^2} \\ B^* &= \frac{p_y^2v - 2p_y^2 + 2p_xp_y - p_x^2}{p_y} \end{aligned}$$

Letting $B = 100$, $p_y = 1$.

$$\begin{aligned} v &= 2 - 2p_x + 0.5p_x^2 + B \\ B^* &= v - 2 + 2p_x - 0.5p_x^2 \end{aligned}$$

when $p_x = 1$ $v = 100.5$. When$p_x = 0.25$, $v = 101.53125$.

$$\begin{aligned} CV &= 98.96875 - 100 = -1.03125 \\ EV &= 101.03125 - 100 = 1.03125 \end{aligned}$$

The Income elasticity of demand for x is zero.

2. Skippy is a risk averse individual with \$500 income assigns a utility of 100 to \$450 and a utility of 120 to \$500. She is willing to pay at most \$50 for a lottery ticket that pays \$250 with probability of 1/2 and \$0 with probability 1/2. Then, True or False, her utility of \$700 is 140.
ANS: If she buys a ticket for \$50, her income is \$450. If she wins her income becomes \$700 (450+250). Since \$50 is the most she will pay, it is the point where she is indifferent

$$\begin{aligned} U(500) &= 1/2U(450) + 1/2U(700) \\ U(700) &= 2U(500) - U(450) \\ U(700) &= 2 \cdot 120 - 100 \\ U(700) &= 140 \end{aligned}$$

3. A ship is overdue in port and a shortage of water develops. The limited supplies available are divided amongst all those on board. Myrtle (one of the crew) receives 225 pints of water, which is her supply from today (day 1) until the ship docks. Her utility function is

$$u = 600P - 2.5P^2$$

where U is utility an P is her daily consumption of water (in pints). For simplicity, today's utility from water is assumed to be independent of yesterday's consumption. The probability of making landfall at

the end of day 1 is 0.6, at the end of day 2 is 0.3 and the end of day 3 is 0.1. How many pints of water does Myrtle allocate to consumption on each of the three days?
ANSWER

$$P_1 = 110 \quad P_2 = 95 \quad P_3 = 20$$

To solve, equate the expected marginal utilities across three days

$$\begin{aligned}
&E(MU_1) = E(MU_2) = E(MU_3) \\
&\pi_1(600 - 5P_1) = \pi_2(600 - 5P_2) \\
&\pi_2(600 - 5P_2) = \pi_3(600 - 5P_3) \\
&and \\
&P_1 + P_2 + P_3 = 225
\end{aligned}$$

Note that π_i is the probability of needing water on day i (or probability of still being at sea during day i). To find the probabilities one must do a little reasoning: First, since there it a 0.6 chance of landing at THE END OF THE FIRST DAY, we know Myrtle needs water for day one and there is a 0.4 chance she does not land, so that is the probability she will need water in day 2. With a 0.6 chance of landing at the end of day one and a 0.3 chance of landing at the end of day 2, there is a 0.9 (6+3) chance she will be ashore after two days. Therefore there is a 0.1 chance of being at sea in day 3. This makes the probabilities for the above calculations:

$$\pi_1 = 1 \quad \pi_2 = 0.4 \quad \pi_3 = 0.1$$

4. An individual has no current endowment and can gain rights to consume only by working in one of two industries, producing the same good.
In the risky industry wages = 100 units of x ($= w_R$). The probability of state a is $13/20 = \pi_a^R$ and the probability of state b is $7/20 = \pi_b^R$.
In the safe industry wages = 81 units of x ($= w_S$). The probability of state a is $1/2 = \pi_a^S$ and the probability of state b is $1/2 = \pi_b^S$.
In state a the worker contracts asbetosis and his utility would be $U(x, 0) = x^{1/2}$. In state b the worker is healthy and his utility would be $U(x, 1) = 2x^{1/2}$.
The worker is indifferent between the industries except for the increased risk of state a.

   (a) Show that the worker is indifferent between the two industries, given the risk and relative wages and absent opportunities to gamble before job choice.

$$\begin{aligned}
EU^R &= (13/20)(x^{1/2}) + (7/20)(2x^{1/2}) = (27/20)\,x^{1/2} = 27/2 = 13.5 \\
EU^S &= (1/2)\left(x^{1/2}\right) + (1/2)\left(2x^{1/2}\right) = (3/2)x^{1/2} = 27/2 = 13.5
\end{aligned}$$

   (b) Without calculating a numerical result, determine whether the worker would be willing to make a fair gamble in X prior to choice of industry, the gain or loss to be paid concurrently with his receipt of wages.

$$\begin{aligned}
EMU^R &= (27/40)\,x^{=1/2} = 0.0675 \\
EMU^S &= (3/4)x^{=1/2} = 0.0833
\end{aligned}$$

   (c) Show how, if a gamble were chosen, the outcome of the gamble would affect his choice of occupation.
   ANSWER: Since MU safe > MU Risky, If he wins, he takes the safe job; if loses,he takes the risky job.

   (d) Show that the optimal gamble, denoted $(1/2, 1/2.B, -B)$ that maximizes expected utility is $B =$ 19. (Hint: equate marginal utilities across industries)

$$\begin{aligned}
(27/40)\,(100 - B)^{=1/2} &= (3/4)(81 + B)^{=1/2} \\
9(100 - B)^{=1/2} &= 10(81 + B)^{=1/2} \\
\frac{1}{81}(100 - B) &= \frac{1}{100}(81 + B) \\
B &= 19
\end{aligned}$$