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# The Lead-Lag Puzzle of Demand and Distribution: A Graphical Method Applied to Movies

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Understanding the lead-lag relationship between distribution and demand is an important and challenging issue for all marketers. It is particularly challenging in the movie industry, where the very short lifespan and decaying revenue and exhibition patterns of motion pictures means that the associated time series are short and nonstationary, rendering existing econometric methods unreliable. We propose an alternate method that uses state-space diagrams to determine lead-lag relationships. Straightforward to apply and interpret, it takes advantage of the eye's ability to see patterns that algebra-based formulations cannot easily recognize. A number of validation tests are provided to illustrate the usefulness and limitations of the method. We study the weekly data for 231 major movies released in 2000–2001. While econometric methods do not provide consistent results, the graphical method of visually inferred causality clearly shows a pattern that demand leads distribution for most movies. In other words, the dominant industry pattern is one of movie exhibitors monitoring box office sales and then responding with screen allocation decisions. The managerial implications of these findings are discussed.

*Key words*: distribution; marketing tools; movies; state-space diagrams; time series
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## 1. Introduction

Researchers in marketing have been critically concerned with determining causal relationships among marketing variables. One of the most enduring puzzles concerns the relationship between distribution and demand. One can plausibly argue that demand drives distribution. For example, independently owned retailers would choose to allocate their scarce shelf space only to products that are selling well and withdraw shelf space when sales start to decline. That is, demand leads distribution.[1] On the other hand, one may also argue that marketing communication builds demand for a product, but that demand would only be realized if the product were readily available. This is especially true in a competitive environment where customers come to the distributor and decide what to buy based on which brands are available (Bronnenberg et al. 2000). As a result, demand follows distribution.

The movie industry, which has recently attracted much attention from researchers, constitutes a prime example of the distribution-versus-demand puzzle. In this context, distribution primarily concerns the number of theaters a studio may obtain for a particular movie. The demand is the box office sales of the movie. Besides the highly risky decisions of movie production, the studios constantly face the challenge of adopting appropriate distribution strategies (Vogel 2001). In an analysis of the movie industry, Sawhney and Eliashberg (1996) suggest that a consumer's moviegoing behavior can be divided into two phases—a time to decide and then a time to act. If a specific movie is not available in "a the-

[1] In its early years Snapple was able to receive distribution from national supermarket chains as a result of its successful sales in the convenience store sector (see Kotler 1997, pp. 536–537).

ater near you," potential moviegoers might not be able to act on their desire of seeing it. With an average Hollywood movie lasting about twelve weeks or less in wide distribution, consumers may never see a movie despite their desire to do so (at least until the video comes out—see Lehmann and Weinberg 2000). This implies that the distribution of movies directly influences their demand, and it may be beneficial for the studios (and theaters) to lead with distribution to help increase box office sales. Nevertheless, there is anecdotal evidence that the theaters may passively observe the changing patterns of box office sales. Instead of trying to influence the demand by distribution, they may opt to respond to it by adjusting the number of screens. As a result, some movies are withdrawn early because of their failure to meet box office expectations. If this effect dominates, then the movie market will show a pattern of demand leading distribution.

The existing research on movies has primarily focused on examining the factors that are correlated with box office revenue and improving the precision of its prediction (Sawhney and Eliashberg 1996, Eliashberg and Shugan 1997, Neelamegham and Chintagunta 1999). Prior studies have noted the lack of research on issues related to movie distribution (Elberse and Eliashberg 2003) and, with the notable exception of Elberse and Eliashberg (2003), who estimate the demand-distribution dynamics using simultaneous equation models, few studies have explicitly examined the dynamic correlations between box office sales and distribution intensity.

Understanding this lead-lag relationship, however, has important managerial implications. For instance, the intensity of movie promotion and the primary target of promotion (i.e., movie audience versus movie theaters) are both critical issues to the studios. Depending on whether distribution leads or demand leads, the studios' optimal strategy will differ. That is, if distribution leading demand is the dominant pattern, the studio may want to use more trade promotions and incentives to motivate theaters to carry the movie and carry it for more weeks. On the other hand, if demand leading distribution is the dominant pattern, consumer promotion becomes more important. Moreover, since movie studios typically spend the majority of their advertising funds before a movie's actual release (Vogel 2001), such findings could lead to greater effort being placed on the postlaunch period.

Furthermore, some important managerial and public policy issues in the movie industry will benefit from the identification of this lead-lag relationship. These include channel management and integration (i.e., whether movie studios will have incentive to own parts of or the entire distribution channel), antitrust (i.e., whether integration between studios and exhibitors should be allowed by law), bargaining (e.g., to what extent studios should compete or concede when negotiating with movie theaters), and contracting (e.g., how much flexibility exhibition contracts should contain in terms of withdrawing a movie from exhibition and whether exhibition contracts should be renegotiated after the movie opens).[2] Such issues are particularly timely in the United States, where a period of bankruptcy of major movie exhibitors has been followed by consolidation and emergence of larger theater companies. At the same time, most Hollywood studios are now divisions of large entertainment conglomerates. Understanding the relationship between demand and distribution will allow these new corporate entities to better structure their interactions with each other and their marketing strategies. In many non-North American markets, where there are generally fewer theaters per capita than in the United States, decisions about growth strategies and channel contracts will be influenced by enhanced understanding of the demand and distribution interactions.

Finally, the lead-lag issue between distribution and demand has broad implications beyond the movie industry. It is particularly relevant for markets such as books, broadcast and cable television, music, and fashion goods, where the product life cycle is typically short and the distributors (retailers) make frequent decisions about which products to sell and which to drop.[3]

Albeit critical, the empirical detection of lead-lag relationships between demand and distribution can be difficult.[4] Time series models have made significant contributions to the marketing literature (Dekimpe and Hanssens 2000). Methods such as Granger causality and VARX (vector autoregression with exogenous variables) are increasingly utilized to detect lead-lag relationships, make directional inferences, and examine the effects of marketing mix variables on market performance (Horvath et al. 2002, Nijs et al. 2001, Pauwels et al. 2002, Pauwels and Srinivasan 2004, Srinivasan et al. 2004). One important reason for the popularity of these methods is the growing availabil-

[2] Interested readers are referred to Vogel (2001), Ornstein (2002), and Squire (2004) for further details of these issues, some of which have been under longtime debate and are quite controversial.

[3] For recent research on product life cycle, diffusion, and distribution issues, see Golder and Tellis (2004), Van den Bulte and Stremersch (2004), Fader et al. (2004), and Naik et al. (2005).

[4] Experiments such as those reported in Lodish et al. (1995) can be used to determine whether changes in advertising expenditures result in changes in short term or long term sales. However, such experiments are relatively costly to conduct and are infrequently employed.

ity of extensive time series data.[5] However, due to the nature of the movie industry, the time series of box office sales and screens are short (typically, they have a length of twelve weeks or less) and are nonstationary. With short and nonstationary time series, the typical time series methods become unreliable and often provide conflicting results. For example, we use VARX-based Granger causality tests to determine the direction of lead-lag relationships for 231 mass-market movies released from May 2000 to December 2001 that played for at least five weeks. As we discuss formally in §3, the tests show that demand leads distribution for 12 movies, and distribution leads demand for 86. Both directions were rejected for 39 movies, and neither direction could be rejected for 94. By contrast, the graphical approach introduced in this paper takes advantage of the eye's ability to see patterns that algebra-based formulations cannot easily recognize in short time series. It finds that 90% of these movies followed a pattern of demand leading distribution.

It is thus the purpose of this note to introduce a new graphical method to help analyze the lead-lag pattern for time series data and to apply it to the movie industry. The method is based on state-space diagrams and helps analysts visualize the differences in phases between two time series. In §2, we present the details of this new method and report a set of simulation studies to demonstrate its ability to recover known lead-lag relationships in noisy data. In §3 we apply the graphical method to a set of movie data and compare the findings with those from VARX-based Granger causality tests. In §4 we discuss the substantive findings for the movie industry and conclude with limitations and future research.

## 2. Graphical Interpretation of Leads and Lags

Although the approach developed in this paper depends on the visual interpretation of trajectory curvatures in state-space diagrams, we begin by providing the analytical model underlying this approach. The basic problem that we are trying to resolve is whether there is a causal relationship between two time series, $X_t$ and $Y_t$; that is, which series leads and which lags. For example, if $X_t$ and $Y_t$ are weekly advertising expenditures and sales, respectively, and the impact of advertising on sales takes at least two weeks to appear, then we should see changes in sales lagging those in advertising by at least two weeks. In many cases (as discussed below), curvature in state-space diagrams provides a more effective way to "see" the lag pattern than currently employed econometric methods.

We will first develop the theory using periodic sine and cosine functions, and then generalize the argument utilizing the Fourier expansion of arbitrary time series to a summation of sine and cosine functions. Fourier theory also allows us to identify limitations to the method, which we explore empirically. A search of the published economics and marketing literature found no studies that have employed similar graphical approaches.

### 2.1. Fourier Series and State-Space Diagrams

To continue the above example, consider a market with stable demand that is influenced by periodic advertising, such as "pulsing," which, for expositional reasons, is presented as deviations from the mean advertising level in the simple form of $X_t = A\sin(2\pi f \cdot t)$. This series has frequency $f$ (cycles per week), cycle period $1/f$ (in weeks), and maximum advertising spending deviation of $A$. Now suppose $S$ is the mean sales and that the deviations from mean sales lag advertising by $\Delta t$ weeks, so that sales are given by:

$$Y_t = S + kX_{t-\Delta t} = S + kA\sin(2\pi f \cdot t - \theta), \qquad (1)$$

where $\theta = 2\pi f \cdot \Delta t$ is the phase difference between the two series. Figure 1 shows an example with frequency $f = 1/(2\pi)$ and phase difference $\theta = \pi/2$. In the corresponding time domain, the period of both series is 6.3 weeks, and the lag between them is 1.6 weeks. The constants $k$ and $A$ are both set to 1, and $S$ to zero in Figure 1.

Plotting the pairs $(X_t, Y_t)$ generates the state-space diagram of the above time series. Figure 2 shows this state-space diagram for $t = (1, \ldots, 6)$. With advertising leading sales and advertising plotted on the horizontal axis, the curvature of the trajectory will be counterclockwise.[6] Reversing either will reverse the curvature. If the two series are perfectly in phase, corresponding to positively correlated contemporaneous effects, the plot will be a straight line with positive slope. Thus, such state-space diagrams offer the useful feature that the direction of curvature (i.e., clockwise or counterclockwise) uniquely indicates which series is leading.[7]

[5] The twenty studies summarized in Table 2 of Dekimpe and Hanssens (2000) use time series data with an average length of 94 time periods.

[6] The analytic definition of clockwise and counterclockwise trajectory curvatures, together with an analytic example of the relation between these curvatures and the lead or lag between two periodic functions, are given in Appendix B, which can be found on the *Marketing Science* website at http://mktsci.pubs.informs.org.

[7] State-space diagrams with sinusoidal inputs are known as Lissajous figures, named after French physicist Jules Antoine Lissajous. The unique features of Lissajous figures to reflect frequency and phrase relationship of time series data are used in physics and electronics for such tasks as circuit analysis.

**Figure 1** Stylized Example of Sales Lagging Advertising Expenditure

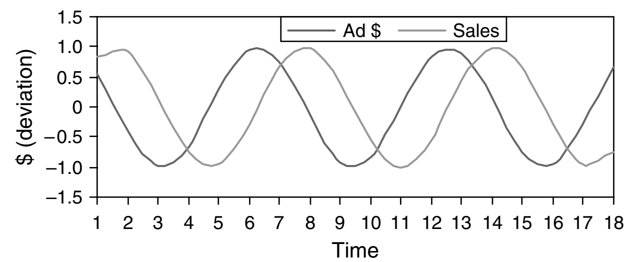
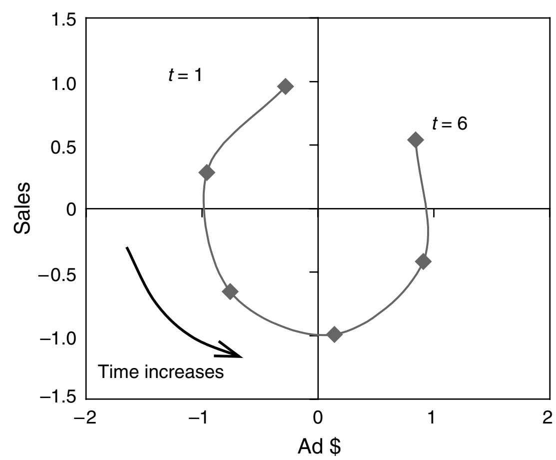



**Figure 2** A State-Space Diagram for the Sales-Lagging-Advertising Example




Note that this analysis requires $k > 0$, which is the case for many business phenomena. If $k < 0$, as is typically the case for price and sales, the logic and curvature are both reversed, but the pattern of curvature is still uniquely linked to the lead-lag relationship. In practice this means that before applying the method we must have some theory to assure us that the two series are either positively or negatively related. In cases where neither the sign of the relationship between the series, nor which is leading, is known, the method is indeterminate.

This explains the rationale behind the graphical method using sinusoids, a rather special type of function. We now present the rationale for arbitrary functions. Any time series can be approximated by a Fourier series composed of sine and cosine functions, much as any function can be approximated well by a few Taylor series components. The Fourier expansion of an arbitrary continuous function $z(t)$, which is periodic with period $T$ (or if nonperiodic and of finite length $T$, is assumed to repeat with period $T$), is given by:

$$z(t) = a_0 + \sum_{m=1}^{\infty} [a_m \cos(2\pi m f_0 t) + b_m \sin(2\pi m f_0 t)] \quad (2)$$

Each term on the right hand side of Equation 2 is a single-frequency sinusoid. The coefficients $a_m$ and $b_m$ are determined from the Fourier transform of the function $z(t)$, or in the discrete case, the discrete Fourier transform of the series $z_t$. A small number of the Fourier components (represented by their Fourier coefficients, $a_m$ and $b_m$) can usually approximate an arbitrary time series quite well over a limited range of $t$. It is thus technically possible to compare any two arbitrary time series by comparing their corresponding Fourier components in a series of state-space diagrams, with one diagram for each frequency of interest. The associated curvatures would determine which series is leading and which is lagging at each frequency. This is essentially the frequency-domain equivalent of the time-domain VARX methodology. However, while the frequency-domain approach demonstrates the rationale behind the interpretation of the state-space diagrams, we are not advocating estimating the Fourier coefficients. Such estimation is comparable to estimating the lag coefficients and suffers similar limitations. Rather, we present this approach as the theoretical basis for using the graphical method for arbitrary pairs of series. This also allows using Fourier theory to specify limitations of the method, as detailed in Appendix A; in particular, an underlying continuous function must be sampled "frequently enough" for the lag pattern to show up clearly in the state-space diagrams. In the next section, we build on this idea to design empirical validation tests.

## 2.2. Validation

To examine the lead-lag relationship between two time series, we could take an econometric approach by estimating a VARX model and testing for the existence of Granger causality. Besides the inability to work with very short time series, Granger causality analyses are very sensitive to the choices of the number of lags and the statistics used to test the null hypothesis (Hamilton 1994, p. 305; Hsiao 1982; Thornton and Batten 1985). The main advantage of the state-space diagrams, on the other hand, is that if there is a lead-lag relationship, the curvature direction can be quickly detected by eye, even with very few data points in the series. A full assessment of all possible circumstances is beyond the scope of this paper. Rather, in this section, we concentrate on conditions that will cause difficulty for a Granger causality test using a VARX model: first, when the time series are short, but stationary, and second, when the series are short and nonstationary.

**2.2.1. Validation and Smoothness Limitation with Short Stationary Series.** Appendix A describes the Nyquist limitation of the graphical method with periodic functions. In general, an underlying continuous function must be sampled "fast enough." When the underlying continuous function is not known, as is typical with discrete economic series, this condition translates as the discrete series being "smooth enough." If we take the perspective of an analyst who is given a discrete time series but either knows no details of the underlying continuous function or has a series that is intrinsically discrete—for example, weekly movie revenue—then the smoothness of the discrete series will be relevant in the application of the graphic method. We now provide an empirical validation of the method and show that the length of the lead or lag that can be detected increases as the "smoothness" of the series increases and that the detection is more reliable than Granger causality for short time series.

We first created time series with different smoothness and lengths as follows. Uniform i.i.d. random number series were generated. These series were then smoothed by convolution with either a 5-point or a 9-point triangular filter to give two levels of smoothness. These are referred to as the MA5 and MA9 series, and the MA9 series is smoother. For each series, pairs were then generated by creating a second series that was identical to the first but with a lead or lag of 1, 2, 4, or 6. Finally, shorter series pairs of either 11 points or 6 points were extracted from these longer stationary series.

In summary, we vary the smoothness (2 levels), the lead or lag (8 levels), and the length (2 levels) of the test series. These series pairs were displayed in state-space plots. Eight realizations of each combination were plotted, for a total of 256 state-space plots. Three judges who were blind to the test objectives but trained in recognizing clockwise and counterclockwise curvature in states-space plots then independently judged each of the 256 plots. The task was to assign them into one of three categories: dominantly clockwise (CW), dominantly counterclockwise (CCW), or indeterminate.[8] From the aggregate judgments, an overall "hit rate" was calculated. A hit was counted if the judged direction matched the lead or lag of the data generation process. A judgment of indeterminate was automatically a miss.

Table 1 summarizes the hit rates, indicating that first, increasing the lag systematically and significantly decreases the accuracy of detection for all conditions ($F = 19.01$, $p < 0.01$). Second, such a decrease is slower for the smoother MA9 series ($F = 5.82$, $p < 0.05$). On average, a smoother series allows better identification, as predicted. Finally, there is only minimal and insignificant difference between the hit rates for the 6-point series and the 11-point series ($F = 0.19$, $p = 0.67$). This confirms the usefulness of the graphical method when the time series becomes shorter.[9] The overall performance of the graphical method is impressive—even with a lead or lag of 2, and series as short as 6 data points, the accuracy of unambiguous judgments of which series leads and which lags is still more than 75%.

**Table 1 Comparisons of Hit Rate in Lead-Lag Detection**

| | | Series length and smoothness | | | |
|---|---|---|---|---|---|
| Lead-lag detection method | No. of lags | 6-point MA5 (%) | 6-point MA9 (%) | 11-point MA5 (%) | 11-point MA9 (%) |
| Graphical | 1 | 93.75 | 93.75 | 89.58 | 91.67 |
| | 2 | 75.00 | 85.42 | 85.42 | 85.42 |
| | 4 | 29.17 | 70.00 | 22.92 | 70.83 |
| | 6 | 43.75 | 41.00 | 22.92 | 41.67 |
| Granger causality | 1 | 62.50 | 50.00 | 31.25 | 6.25 |
| | 2 | 0.00 | 25.00 | 56.25 | 50.00 |
| | 4 | 25.00 | 37.50 | 12.50 | 12.50 |
| | 6 | 18.75 | 31.25 | 18.75 | 6.25 |

While the main purpose of this exercise was to validate the graphical method and to identify limitations, it is also interesting to compare it with the econometric approach. We estimated bivariate vector autoregressive models and tested for the existence of Granger causality. As Hamilton (1994, p. 304) summarizes, the simplest and probably best econometric test for Granger causality takes the following format:

$$\begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \begin{bmatrix} \phi_{11}^{(1)} & \phi_{12}^{(1)} \\ \phi_{21}^{(1)} & \phi_{22}^{(1)} \end{bmatrix} \begin{bmatrix} x_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} \phi_{11}^{(2)} & \phi_{12}^{(2)} \\ \phi_{21}^{(2)} & \phi_{22}^{(2)} \end{bmatrix} \cdot \begin{bmatrix} x_{t-2} \\ y_{t-2} \end{bmatrix} + \cdots + \begin{bmatrix} \phi_{11}^{(p)} & \phi_{12}^{(p)} \\ \phi_{21}^{(p)} & \phi_{22}^{(p)} \end{bmatrix} \begin{bmatrix} x_{t-p} \\ y_{t-p} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}. \quad (3)$$

If the coefficient matrices $\Phi_j$ ($j = 1, \ldots, p$) are all lower triangular, i.e., $\phi_{12}^{(j)} = 0$, then $y$ does not Granger-cause $x$. The most common implementation of this vector autoregression system is to estimate the following equation using OLS:

$$x_t = \alpha_0 + \alpha_1 x_{t-1} + \alpha_2 x_{t-2} + \cdots + \alpha_p x_{t-p} + \beta_1 y_{t-1} + \beta_2 y_{t-2} + \cdots + \beta_p y_{t-p} + u_t. \quad (4)$$

Then either the $F$ statistic or the Wald statistic can be calculated to test the null hypothesis $H_0$: $\beta_1 = \beta_2 = \cdots = \beta_p = 0$. If the null hypothesis is rejected,

[8] The reason for allowing these three categories is for greater comparability with the VARX method, which can and often does give significant results for both lead and lag, and, hence, is indeterminate in choosing a dominant direction.

[9] The 6-point series were subsets of the 11-point series to make the comparison between series lengths less variable. The plots were presented in groups of 64 that were either all 6-point or all 11-point, so that there was no possibility of judges recognizing that the 6-point series were subsets of the 11-point series.

then "$y$ Granger-causes $x$." Otherwise, "$y$ does not Granger-cause $x$."

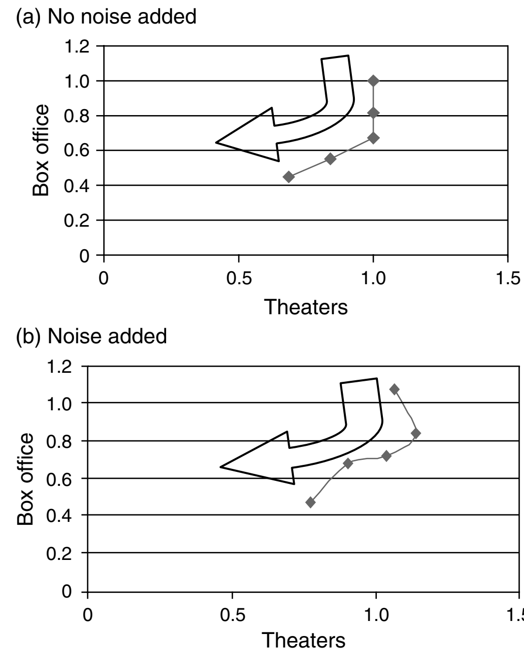
For each pair, a hit was recorded if the underlying direction was unambiguously determined (the maximum lead or lag to be tested was three for the 11-point series). The results are also summarized in Table 1. Since this is an asymptotic test on very short, even though stationary, series, we did not expect good results, and such was the case. Hit rates in almost all cases were below 50% and were less than that for the graphical method ($F = 34.22$, $p < 0.01$). The graphical method consistently outperforms the econometric method in recovering the lead-lag relationship.

**2.2.2. Validation with Short Nonstationary Series.** We next generate short nonstationary data that are inspired by our objective of understanding the demand and distribution relationship for movies (i.e., box office sales and the number of theaters showing the movie). We label the simulated series pairs as "box office" and "theaters." The model we use to generate weekly data assumes that box office revenues decline monotonically, and the number of theaters is the same as the previous week until box office drops below a threshold. Below the threshold, the number of theaters is a fixed proportion of the previous week's box office. Thus, theaters lag box office by one week. Five data points are generated, and the threshold is crossed in the third week. For illustration purpose, Figure 3(a) shows the state-space diagram of one of these short, nonstationary series. It clearly shows the clockwise curvature indicating that box office leads. We then add noise to the base series to generate 18 pairs of noisy data points. The noise level was chosen so that the clockwise direction could be identified most, but not all, of the time. An example of the noisy trace is shown in Figure 3(b). By looking at the state-space diagrams, three judges independently judged and agreed on the clockwise curvature for 15 of the 18 traces, while three were indeterminate.

This rate of identifying that the box office leads theaters was then compared to the Granger method. We estimated bivariate vector autoregressive models for both theaters leading and box office leading for each trace. Note that with only five data points (i.e., five weeks into a movie's life span), only one lag ($p = 1$) can be estimated. For the null hypothesis that box office does *not* lead screens, which should be rejected by an ideal test, the null was rejected in only 3 of the 18 cases.[10] As a result, for these simulated short nonstationary series, the graphical approach was able to recover the data generation process more consistently than the econometric approach.

[10] On the other hand, the null was rejected (i.e., false-positive) in two of the 18 cases for screens leading box office.

**Figure 3** **State-Space Diagram of the Simulated Threshold Model**




*Note.* The underlying simulation mechanism is that box office leads theaters.

## 3. Detecting the Demand-Distribution Pattern for Movies

In movie exhibition, the distribution intensity is typically adjusted weekly (Sanjeev et al. 1999). The demand for most movies, particularly wide-release movies, changes very smoothly and typically with a monotonic decline (Krider and Weinberg 1998). As a result, weekly sampling should be sufficiently smooth to allow the lead-lag pattern to appear.

The box office and theaters data for 407 movies playing between May 5, 2000 and December 7, 2001 were collected from a popular website of movie records (www.the-numbers.com). The full data set includes many narrow-release and short-run movies. Only movies with at least 5 weeks of data and a cumulative box office of 5 million dollars—231 movies in total—were retained for analysis. Key summary statistics of these movies are provided in Table 2.

To compare the graphical and econometric methods of determining whether theaters lead box office

**Table 2** **Summary Statistics of the Movie Sample**

| | Mean | Median | Standard deviation |
|---|---|---|---|
| Opening week number of theaters | 1,847 | 2,305 | 1,162 |
| Opening week box office revenue | 12,999,037 | 9,386,342 | 14,412,500 |
| Total number of weeks in theaters | 11.97 | 11.00 | 7.74 |
| Total box office revenue | 51,695,553 | 32,054,918 | 54,444,803 |

**Table 3 Demand-Distribution Relations for 231 Movies Classified by Granger Causality Tests**

| | Demand leads distribution | | |
|---|---|---|---|
| | Yes | No | Subtotal |
| Distribution leads demand | | | |
| Yes | **94** | **12** | 106 |
| No | **86** | **39** | 125 |
| Subtotal | 180 | 51 | 231 |

*Note.* Distribution is the number of theaters showing the movie, and demand is the box office revenue.

revenues or revenues lead theaters, we first estimate six VARX models for each movie. Three of the models assume that theaters lead revenue, and the other three assume box office leads theaters. In each case, lags of one, two, and three weeks are modeled. In cases where there is insufficient data, only one-week or two-week lags are estimated. For example, the VARX model for the three-week lag case is

$$y_t = \alpha_0 + \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \alpha_3 y_{t-3} + \beta_1 x_{t-1} + \beta_2 x_{t-2} + \beta_3 x_{t-3} + \varepsilon_t. \quad (5)$$

Then Wald tests on $\beta_1 = \beta_2 = \beta_3 = 0$ are conducted for Granger causality. If the model with the minimum AIC of the three models for theaters leading passes the Granger causality test, that movie was counted as "theaters leading." "Box office leading" was similarly defined. Any one movie could be classified as theaters-leading, box-office leading, neither, or both.

Table 3 shows the resulting classifications of the 231 movies based on the econometric test. It indicates that in 39 cases, no lead-lag relation can be detected. In 94 cases, neither direction can be rejected. In 86 cases, demand leads theaters unambiguously, and in the remaining 12 cases theaters lead demand unambiguously.[11]

Next, state-space diagrams of the movies are generated with the number of theaters on the horizontal axis. Three judges independently assigned each movie into one of five categories, depending on the curvature of the plots. Three broad categories were box office leads screens (clockwise), screens lead box office (counterclockwise), and indeterminate. The two curvature categories were further divided into strongly curved (with less than 20% of the points deviating from the curvature direction) and weakly curved (with more than 20% deviating). State-space diagrams for an example movie from each of the four cases in Table 3 are shown in Figure 4.

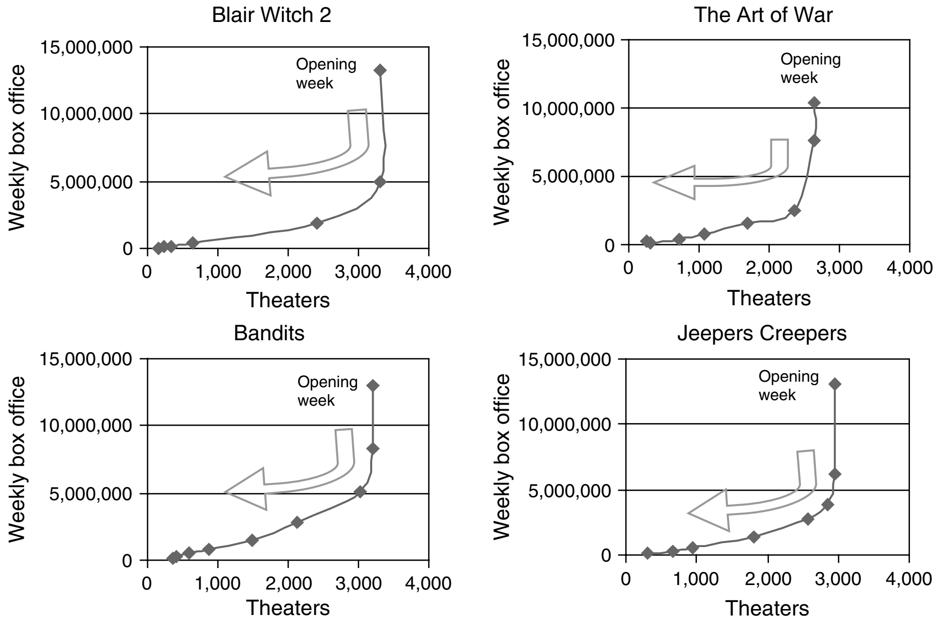
The three judges agreed on 82% of all the movies with regard to which of the five categories they belong to. If counting only the three broad categories, the agreement rose to 90%. Most of the disagreement occurs on whether or not the direction was indeterminate. The judges assigned the majority of the movies (187, 188, and 183 movies, respectively) to the strong-clockwise category, that is, box office leads screens. The state-space diagrams for the four movies in Figure 4 illustrate this pattern, which can readily be seen. As a result, we conclude that the demand-distribution dynamics for most movies are characterized by theater owners observing the demand in a week and adjusting screens in the following week.[12] Given the role of pattern recognition in making these assessments, we term this approach "Visually Inferred Causality."

As an example, consider the movie "The Art of War," which was determined to be strongly clockwise (demand leading) by all three judges (see Figure 4). Theaters remain constant in the first two weeks while demand is high but falling. In the third week, demand drops dramatically, even though only a few theaters have dropped the movie. In the fourth week, exhibitors respond to the previous week's drop in sales by reducing the number of theaters by more than one-third. The theater lag shows up clearly in clockwise curvature. Not only is this nonstationary behavior, but also the series has only enough data to estimate Granger causality with two lags. To make things worse, the Granger test classified this movie as theaters leading, not as demand leading. The state-space diagrams appear to offer results that have greater face validity than the econometric methods for this type of data.

[11] Another possible method is to treat the VARX parameters for each movie as random draws from a known distribution and to use random coefficient estimations to infer the *mean* lead-lag relationship in the sample. While it offers a different approach to analyzing the data, it has the same limitation as the common VARX method (e.g., short time series cannot be reliably estimated) and faces additional constraints. That is, the number of lags used in any one run must be the same for all the movies—it equals the *maximum* lag that allows Granger causality to be tested for all the movies. Nevertheless, we conducted random coefficient estimation using lag numbers one, two, and three, respectively for two specifications—box office revenue leads theaters and theaters lead revenue. Therefore, six random coefficient models are estimated. The results are not very informative: for both specifications and for all of the lag numbers, the Wald tests reject the hypothesis that the lagged exogenous variables are jointly zero.

[12] In a study of movies in domestic and international markets, Elberse and Eliashberg (2003) find that while the number of screens in a week influences that week's box office revenue, there also exists a significant effect of the *expected* box office revenue for a week on the actual number of screens. In other words, the studio/theater, managers appear to observe the trend in box office revenue and respond to it by setting the number of screens. Note that the realized demand cannot precede distribution in the first week, when distribution has to be set based on demand expectations.

**Figure 4 Movies That Show Clockwise Curvature but Classified Differently by Granger Causality Tests**




*Note.* The four movies are classified by the Granger causality test as different in lead-lag relationship between demand and distribution. The position of the four movies in the figure corresponds to the positions in the four cells of Table 3. For example, Blair Witch 2 is classified as both "demand leads distribution" and "distribution leads demand" in Table 3. Clockwise curvature of the state-space diagram is apparent for each movie.

# 4. Discussion and Conclusions

In this paper we develop a graphical method based on state-space diagrams to help identify and interpret the lead-lag relationship between movie distribution and box office sales. The method is easy to implement, can be conducted for short time series, and offers results that are less ambiguous and of greater face validity. Various simulations are conducted to examine the validity and limitations of the method.

## 4.1. Managerial Implications

Applying this graphical method, we consistently find that demand leads distribution for most movies. In other words, movie distributors and exhibitors appear to be monitoring the weekly box office sales and then responding by adjusting distribution intensity. There are at least two potential explanations for why this pattern of lead-lag dominates in the industry. First, as many movies open wide on thousands of theaters across the United States, there is low risk of "lost" sales due to insufficient distribution in the early weeks of a movie's run. The movie industry, especially theaters, is more concerned with "overdistribution" than "underdistribution." When there are many competing films that can be shown, theater owners tend to use the most revealing factor of a movie's potential—its box office sales—to decide whether to retain a particular movie in the subsequent week. (See Eliashberg et al. 2001, for example.) Second, a number of common practices in the movie industry enable (and encourage) the demand-leading-distribution pattern. For instance, the typical movie distribution contract includes a holdover term, which specifies the minimal level of box office revenue that a movie needs to achieve to be kept in the theater for the next week (Vogel 2001). The theaters, by observing the box office revenue, may drop the movie if the revenue falls below this threshold.[13] Moreover, since movie exhibitors typically have a meeting every Monday morning to make "shelf space" decisions for the upcoming week, this makes the weekend box office results and the use of the screens for the coming week critical subjects of discussion.

As a result, from the managerial point of view, it may be more effective for movie studios to focus on building demand for movies during the post-release period rather than trying to push the theaters. In other words, it is important for movie studios to consider adopting "pull"-oriented consumer promotional strategies. By building strong demand from potential audiences the studios can increase the opportunity for a movie to receive continuous distribution support from the theaters, or at least to not be withdrawn

[13] Recognizing that declining revenue acts a stimulus for theaters to drop a movie, the studios typically offer contracts in which the share of revenue the theaters must remit back to studios declines over time.

too early.[14] Advertising is one of the most important consumer promotional tools. The current industry practice is to heavily promote the movie before release and then allow the advertising budget to drop rapidly (Vogel 2001). The demand leading distribution result suggests changes to this practice—studios should consider spending more money on advertising after the opening week than they currently do (or in general, increasing post-release marketing effort aimed at pulling viewers into theaters).[15] This alternative strategy appears to be particularly relevant to the extent that subsequent sales of videos, video games, and other products are driven by the number of people who see the movie in a theater.

Moreover, these results imply that when negotiating contract terms with movie theaters, studios should be careful about trying to get distribution support through costly concessions that may not be necessary. If audiences can be motivated to come to the theaters in larger numbers and for a longer time (as a result of appropriate advertising and sales promotions), theaters will respond positively by keeping the movie. Ultimately, this calls for more careful planning during the production stage of new movies. For example, some movies include high-budget special effects to motivate moviegoers to see the movie in theaters more than once (e.g., King 2000). More generally, movie producers might place great emphasis on movie characteristics that are likely to generate strong, positive word of mouth.

Finally, the pattern of demand leading distribution suggests one negative aspect of the wide opening strategy. Despite its presumed ability to reduce the risk of a total box office flop by generating viewership before "true quality" is revealed and to crowd out competition by putting a movie on thousands of screens, opening wide may not allow sufficient time for a movie to build up awareness and potential demand. It can exhaust demand quickly, causing box office sales to decline rapidly and thus inducing movie theaters to cut distribution early.

### 4.2. Limitations and Future Research

Nearly every managerial decision requires some knowledge of causal relationships. As a particular example, if advertising causes sales (in contrast to advertising budget being set as a result of sales), then the firm should advertise as long as the net marginal returns are positive. Employing various theories and methodologies, researchers have tried to address the causality issue for different markets (e.g., Eliashberg and Shugan 1997 for movies and Chevalier and Mayzlin 2003 for book sales). It is important to note that temporal relationships alone do not establish "true" causality. Nevertheless, especially in the absence of controlled experiments (Lodish et al. 1995), temporal data do provide useful evidence for the underlying relationships. Furthermore, such evidence becomes stronger if either theoretical or institutional reasons exist to support it. While the graphical method of visually inferred causality proposed in this paper offers a new type of analytical tool to detect lead-lag relationships in short time series data, its usefulness and limitations for more general market conditions remain to be explored.

This paper focuses on the temporal relationship between the number of theaters and the box office revenue, two critical factors in the movie industry. There are, of course, other factors that may influence the movie distribution process. For instance, competition between studios (and, to a lesser degree, between movie theaters) during peak seasons may moderate the impact of box office revenue on subsequent distribution. Moreover, these data are from the North American market. Relationships between demand and distribution may vary in other locations where the intensity of distribution is lower or where channel arrangements differ. Richer data sets are clearly valuable to make a fuller assessment of these situations.

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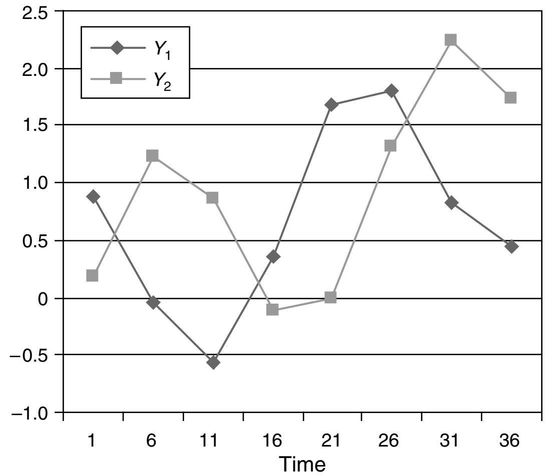
### Appendix A. Sample Rates

An important consideration for ease of graphical interpretation involves the relation between the series' sample interval and how rapidly the underlying process changes. Roughly speaking, the sample rate must be high enough to adequately represent the most rapid changes in the underlying data generation process. To make the relations specific, consider a time series resulting from regular sampling of a continuous time function with a well-defined frequency spectrum. Let the upper limit of the spectrum be $f_{\max}$, i.e., the highest frequency in the continuous time function. The sample interval $\Delta t$ determines the highest frequency that

[14] Since box office sales do not occur before the first week, movie theaters cannot observe demand to adjust distribution for that week (although they can use expectations, see Footnote 12). As a result, it is difficult to predict whether trade promotions or consumer promotions should dominate before opening. Our findings mainly support pull promotions for the weeks after opening.

[15] One approach to do so would be to provide cooperative advertising money to exhibitors, to motivate them to find marketing strategies that would increase demand for movies currently in theaters. Another would be to utilize the Internet to stimulate interest and word of mouth.

**Figure A.1** **Two Time Series with Sample Interval $\Delta t = 5$**




can be represented in the discrete sampled time series: $f_N = 1/(2\Delta t)$, the "Nyquist" frequency. The sample interval must be short enough so that $f_N \geq f_{max}$. Otherwise, the upper end of the original spectrum cannot be captured in the discrete series. Frequencies above the Nyquist in the continuous time function are shifted to frequencies below the Nyquist in the sampled discrete series (a process known as *aliasing*). In practice, most of the underlying spectrum should be below about half the Nyquist frequency for easy interpretation.

As an example of the problem of insufficient sampling frequency, consider the two discrete time series in Figure A.1 (and the associated state-space plot in Figure A.2). These two series are generated from two identical single-frequency continuous time functions, with the Series 2 leading[16]:

$$\text{Series 1: } Y_1(t) = \sin(t) + t/25 \tag{A.1}$$

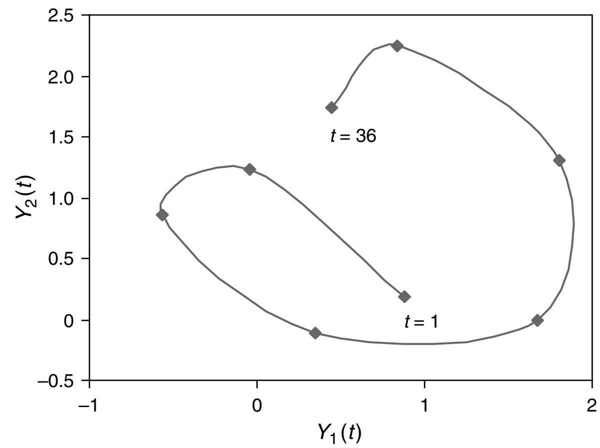
$$\text{Series 2: } Y_2(t) = \sin(t+2) + t/25 \tag{A.2}$$

Figure A.2 incorrectly suggests a counterclockwise pattern, indicating that Series 1 is leading. This arises because we have not sampled the underlying function often enough. The sample interval is 5, and the Nyquist frequency is 0.1. The time functions have a single frequency $1/2\pi$, approximately 0.16, which is greater than the Nyquist frequency.

By quintupling the sample rate and Nyquist frequency (to $f_N = 0.5$, which is greater than 0.16), we can adequately represent the continuous time function, and the underlying pattern becomes obvious (see Figure A.3). Similarly, the state-space diagram now shows the expected clockwise curvature, and the interpretation that Series 2 leads Series 1 is straightforward (Figure A.4). As indicated in the text, the theater scheduling decision is made on a weekly basis, so the state-space diagrams based on weekly information have the appropriate sampling frequency.

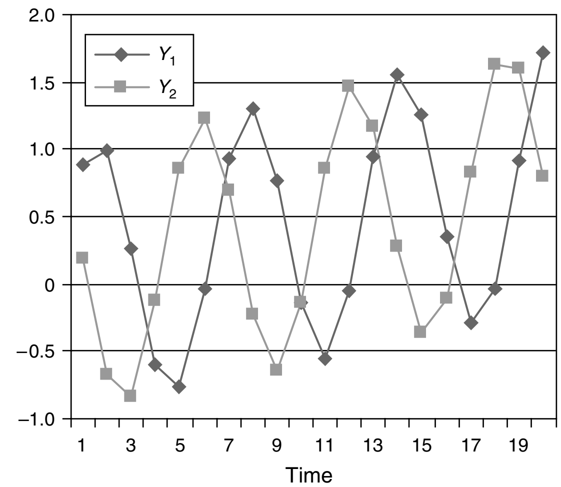
[16] The linear trend term $t/25$ in both series is included only to shift sequential cycles so that they do not plot on top of each other in the state-space diagram.

**Figure A.2** **The State-Space Diagram Associated with the Time Series of Figure A.1**



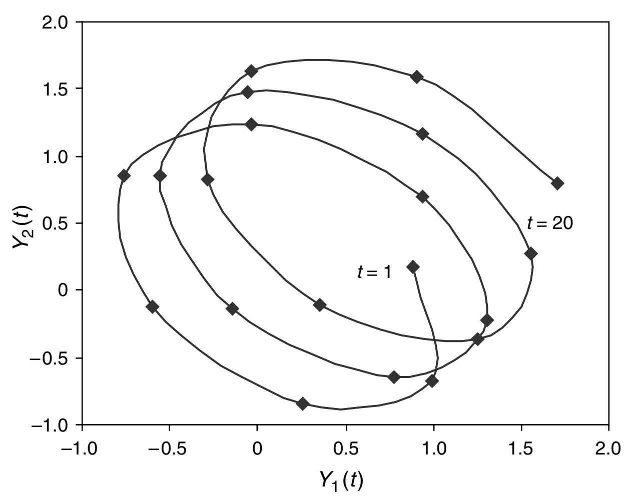

*Note.* Counterclockwise curvature is shown here.

**Figure A.3** **The Same Underlying Continuous Time Function as in Figure A.1, but with Sample Rate $\Delta t = 1$**




*Note.* The first 20 points are shown in $\Delta t = 1$.

**Figure A.4** **The State-Space Diagram Associated with the Time Series of Figure A.3**




*Note.* The correct clockwise curvature is shown here.

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