Phys101 Lecture 6
Circular Motion

Key points:

- **Centripetal acceleration**
- **Uniform Circular Motion - dynamics**

Ref: 5-1,2,3,5,6.

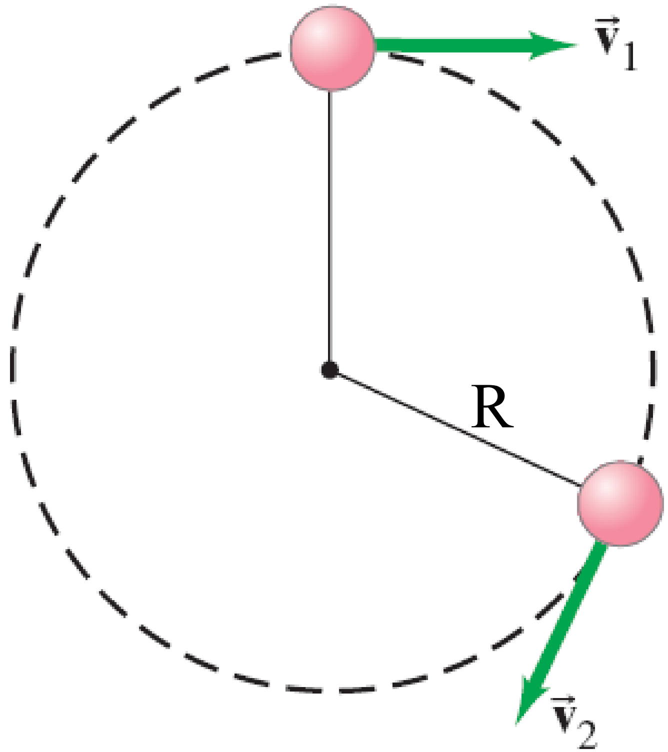
Uniform Circular Motion—Kinematics

Uniform circular motion: motion in a circle at constant speed

Instantaneous velocity is always tangent to the circle.

The magnitude of the velocity is constant:

$$v_1 = v_2 = v$$

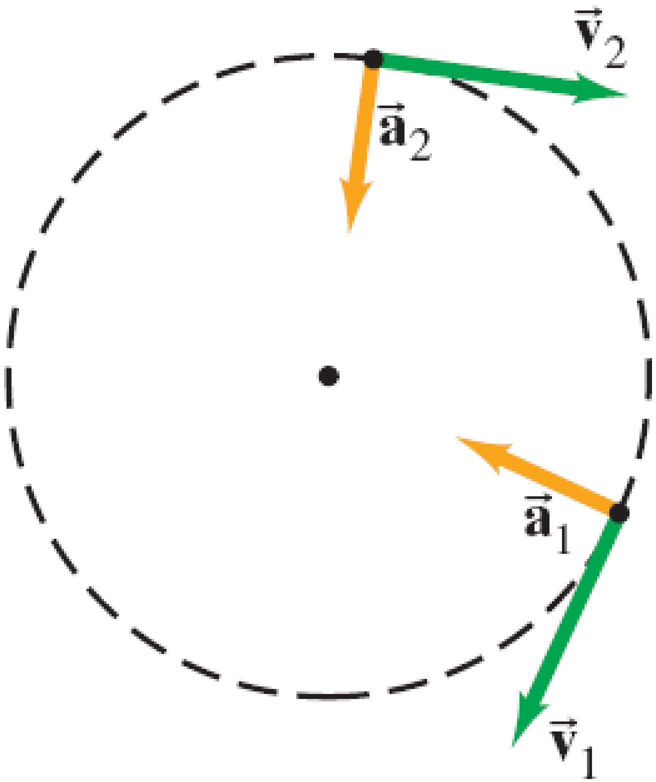


Centripetal acceleration

The acceleration, called the centripetal acceleration, points toward the center of the circle.

The magnitude of centripetal acceleration is:

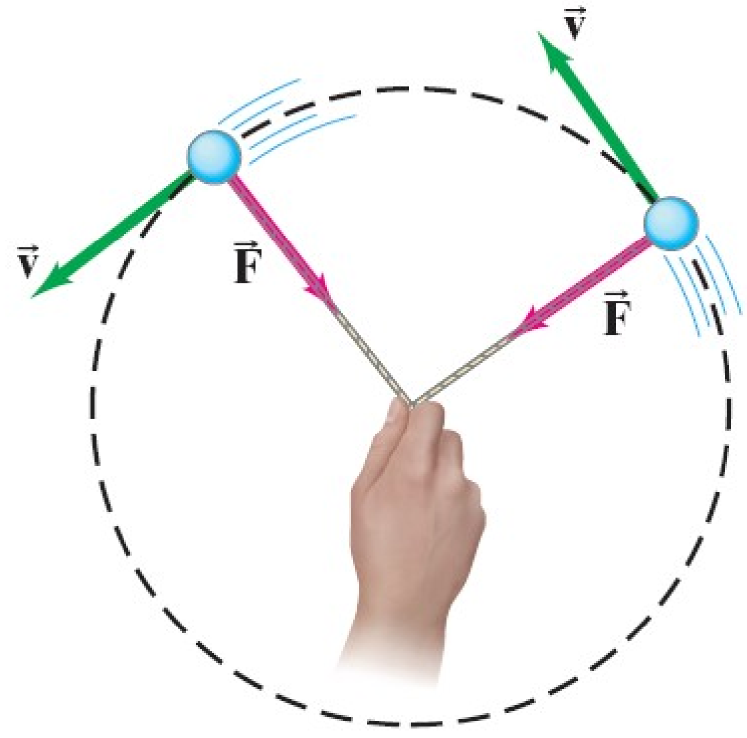
$$a_R = \frac{v^2}{R}$$



Dynamics of Uniform Circular Motion

For an object to be in uniform circular motion, Newton's 2nd law requires a net force acting on it. This net force is called centripetal force:

$$\Sigma F_R = ma_R = m\frac{v^2}{r}.$$



Physically, the centripetal force can be the tension in a string, the gravity on a satellite, the normal force of a ring, etc.

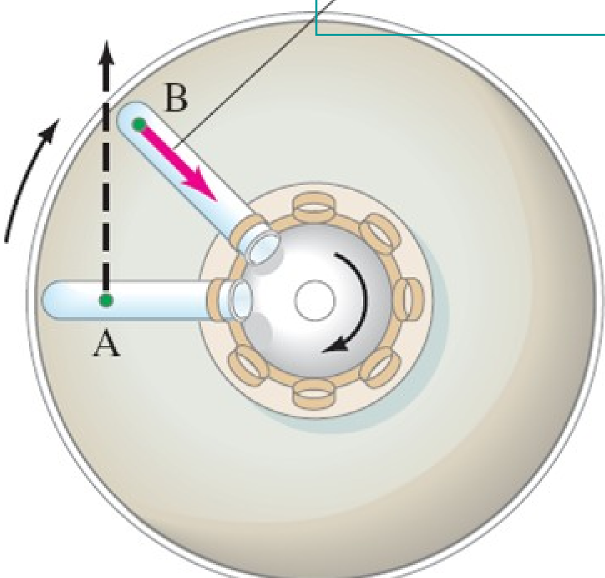
Note: Don't count the centripetal force as an additional force in the free-body-diagram! It refers to the required net force for circular motion.

Centrifuge

A centrifuge works by spinning very fast. An object in the tube requires a large **centripetal** force. When the liquid can't provide such a large force, the object will move (sink) to the end of the tube.

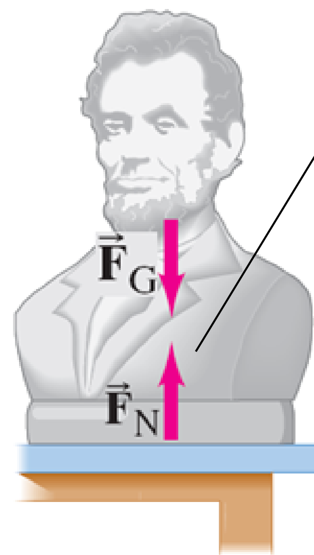
Force reuired for circular motion:

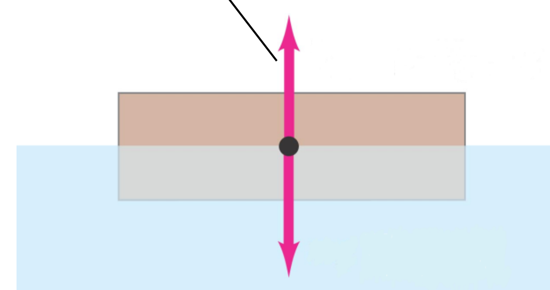
$$F_R = ma_R = \frac{mv^2}{R}$$



Force reuired for staying in position:

$$F_N = mg$$





Example: Ultracentrifuge.

The rotor of an ultracentrifuge rotates at 50,000 rpm (revolutions per minute). A particle at the top of a test tube is 6.00 cm from the rotation axis. Calculate its centripetal acceleration, in "*g*'s."

$$\text{Radius}: \quad R = 6.00cm = 0.0600m,$$

$$\text{Period}: \quad \mathrm{T} = \frac{1\,min}{50000rev} = \frac{60\,sec}{50000rev} = 1.2\times10^{-3}\ sec,$$

$$\text{Speed}: \quad v = \frac{2\pi R}{T} = \frac{2\pi(0.06)}{1.2\times10^{-3}} = 314m/s,$$

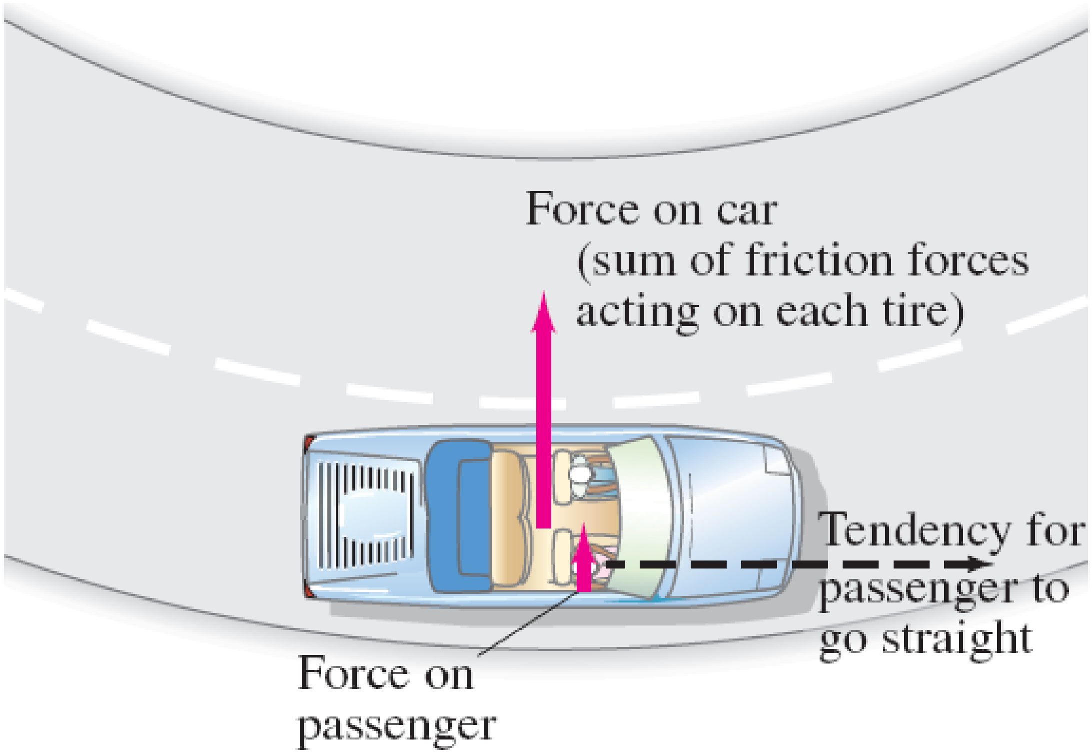
i-clicker question 6-1

An object is in a uniform circular motion. Which of the following statements must be true?

A. The net force acting on the object is zero.
B. The velocity of the object is constant.
C. The speed of the object is constant.
D. The acceleration of the object is constant.

Highway Curves: Banked and Unbanked

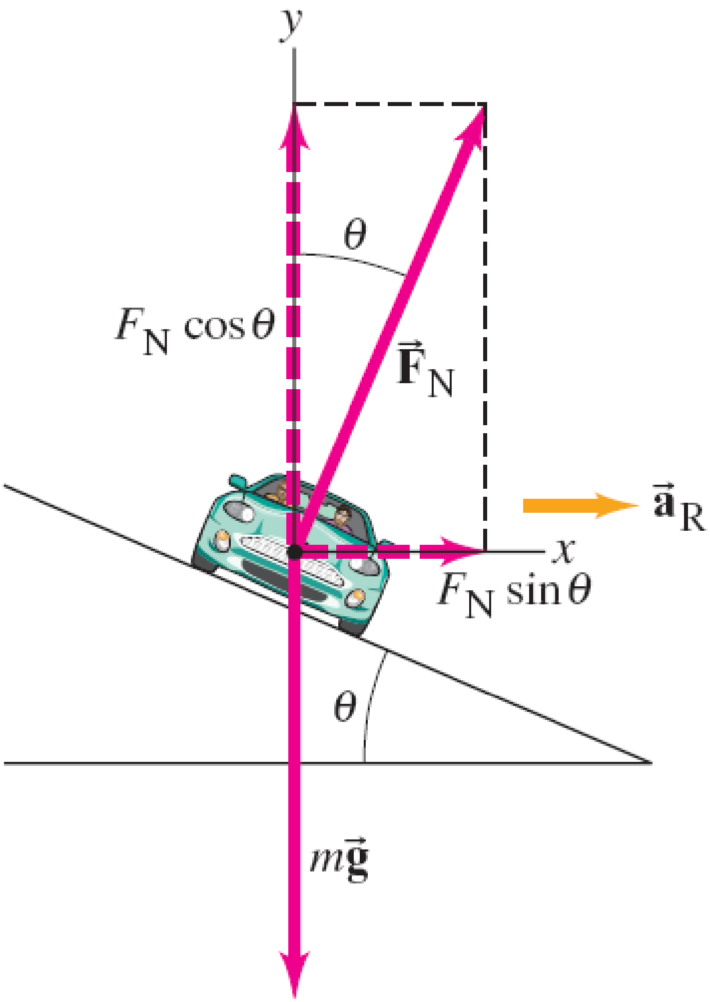
When a car goes around a **curve**, there must be a net force toward the center of the circle of which the curve is an arc. If the road is flat, that force is supplied by **friction**.



Highway Curves: Banked and Unbanked

If the frictional force is **insufficient**, the car will tend to move more nearly in a **straight line**, as the skid marks show.

Highway Curves: Banked and Unbanked



Banking the curve can help keep cars from skidding. When the curve is banked, the centripetal force can be supplied by the horizontal component of the **normal** force. In fact, for every banked curve, there is one speed at which the entire centripetal force is supplied by the horizontal component of the **normal** force, and no friction is required.

Example: Banking angle.

(a) For a car traveling with speed *v* around a curve of radius *r*, determine a formula for the angle at which a road should be banked so that no friction is required.
(b) What is this angle for an expressway off-ramp curve of radius 50 m at a design speed of 50 km/h?

(a) $F_N \sin\theta = m\frac{v^2}{R}$

$F_N \cos\theta - mg = 0$

$\tan\theta = \frac{v^2}{Rg}$

(b) $\mathrm{R} = 50\mathrm{m},\ v = 50 km/h = 13.89 m/s$

$\theta = \tan^{-1}\frac{v^2}{Rg} = \tan^{-1}\frac{13.89^2}{50g} = 22^\circ$