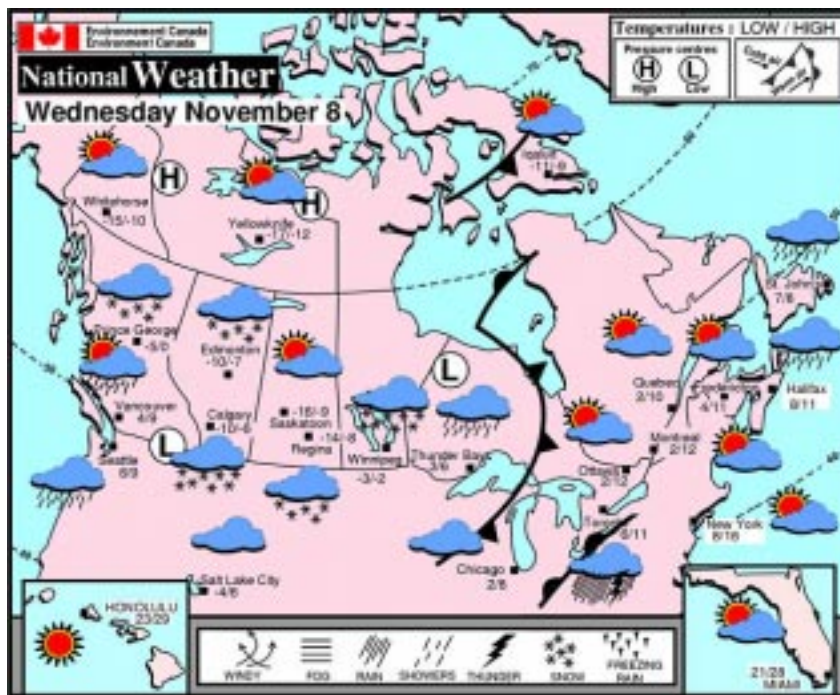


# Computational Models for Understanding Weather

## Mathematics for Atmospheric Science



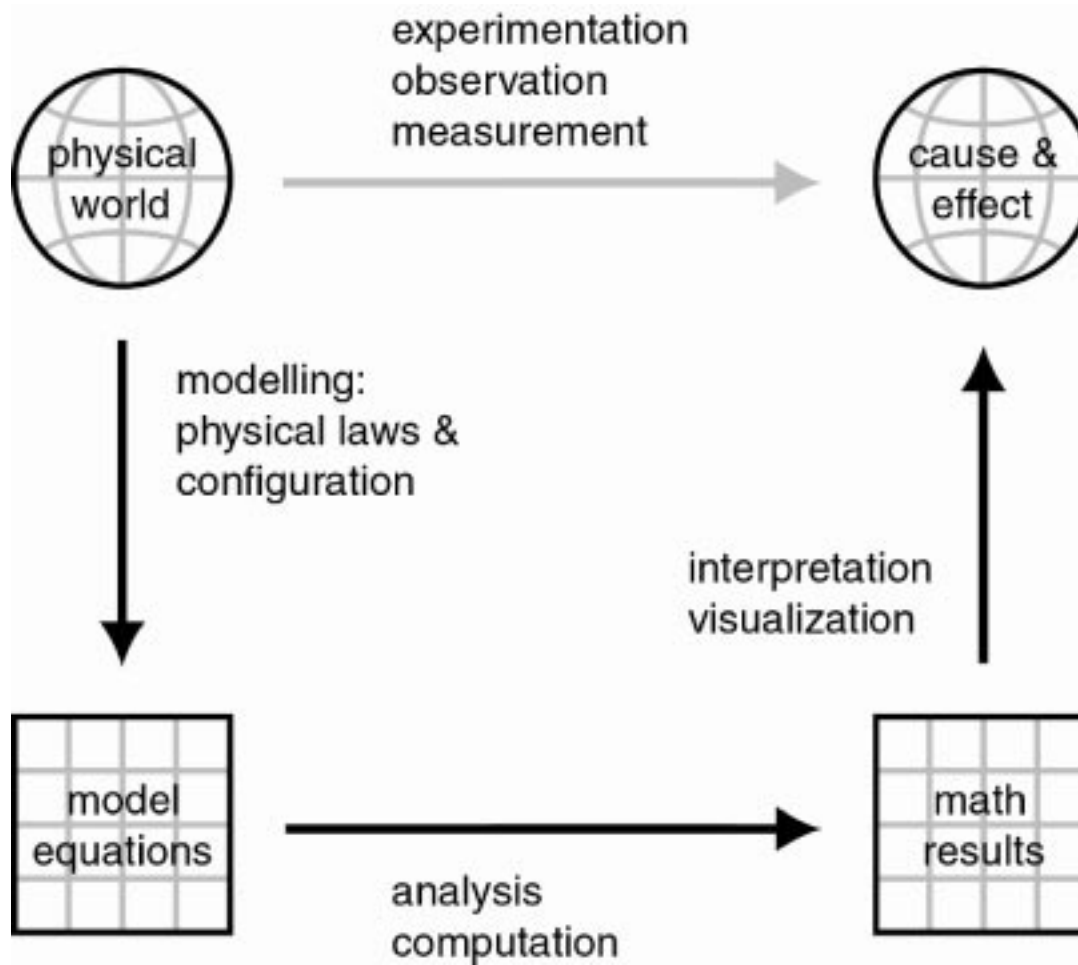
<http://weather.ec.gc.ca>

- ▷ Chris Snyder, Rich Rotunno (NCAR Boulder)
- ▷ Greg Hakim (Univ of Washington)

# Science via Mathematics

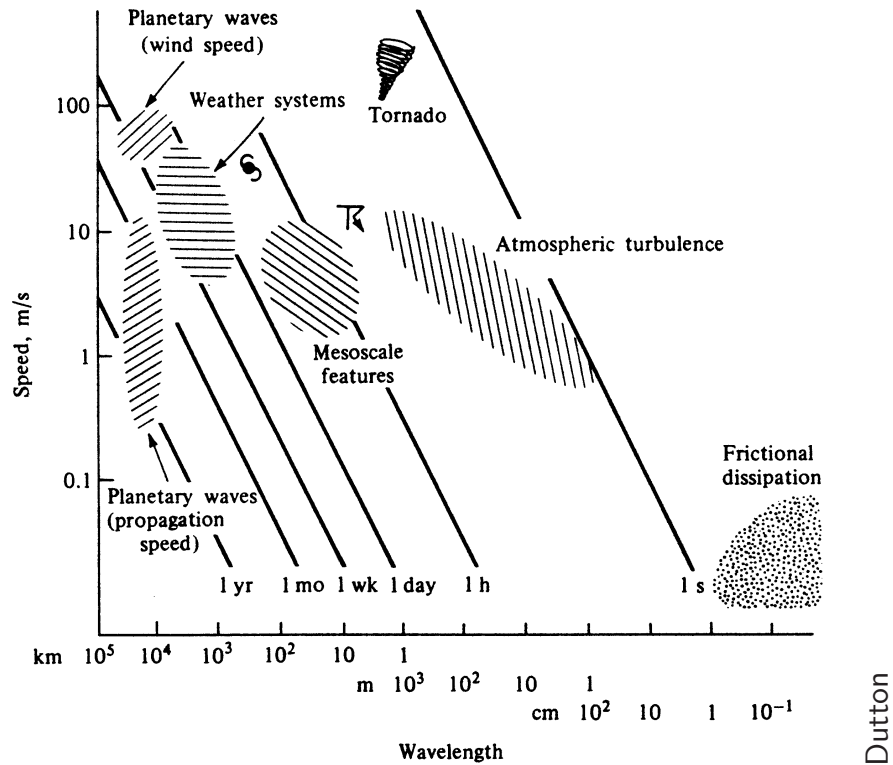
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## A 3-Step Process



# Atmospheric Science

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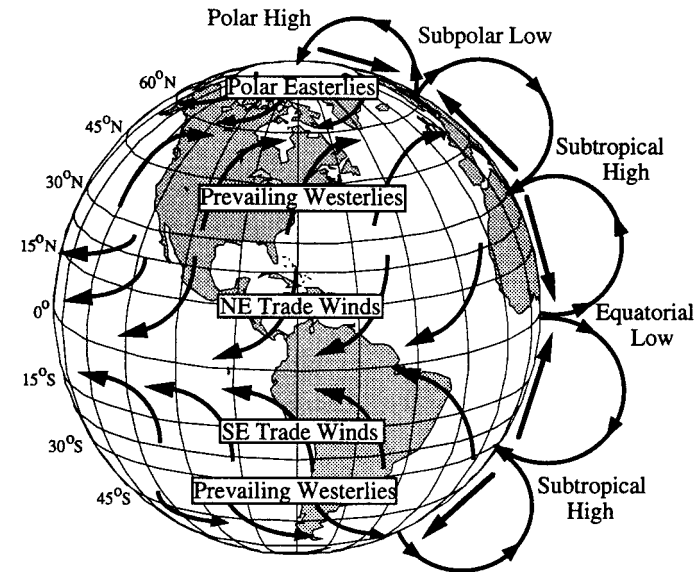
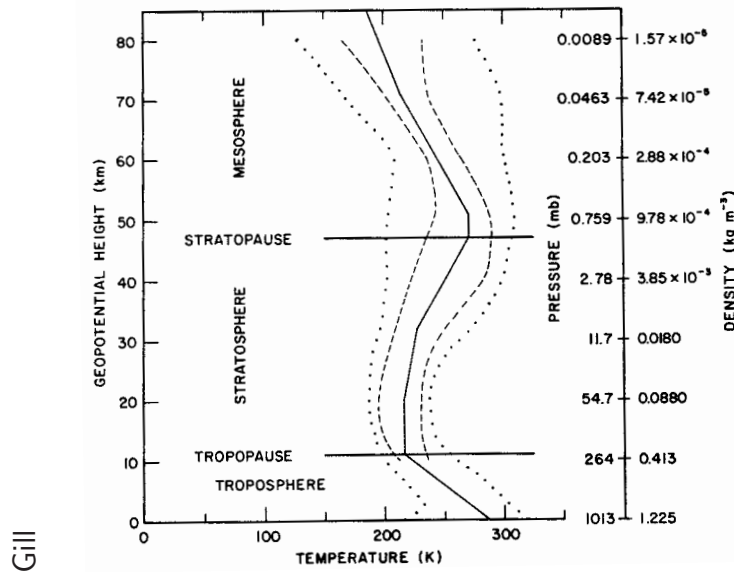


## A Matter of Scales

- ▷ climatology: long-term trends (seasons, years)
- ▷ meteorology: short-term forecast (hours, days)
  - synoptic (global), mesoscale (continental) & microscale (local)

# Atmospheric Structure

## A Problem in Three-Dimensions



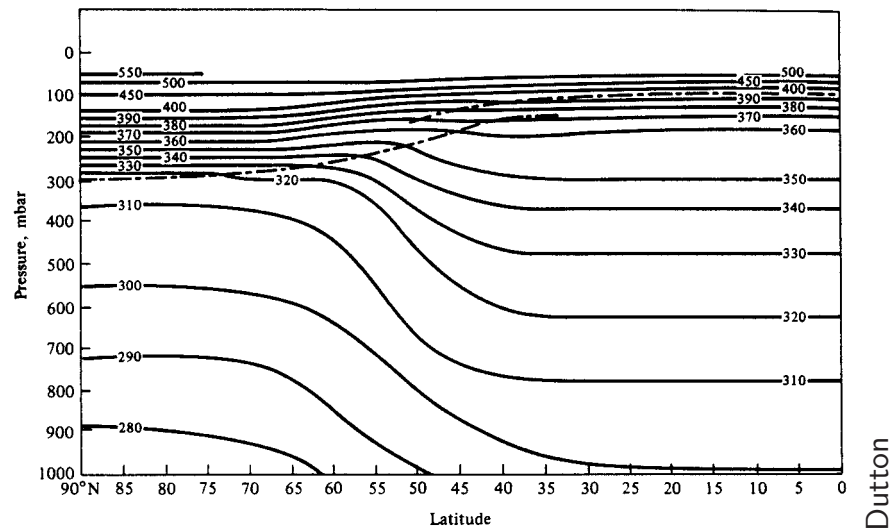
- ▷ vertical: troposphere, stratosphere, . . .
- ▷ global: tropics, midlatitudes, mountains, polar caps, . . .

# A Model for the Midlatitudes

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## Physical Laws & Configuration

- ▷ fluid mechanics (winds & pressure)
  - Coriolis **rotation** & density **stratification**
- ▷ thermodynamics (potential temperature & pressure)
  - NO humidity/moisture feedbacks
- ▷ troposphere: surface & **tropopause** (alt  $\approx$  10 km)

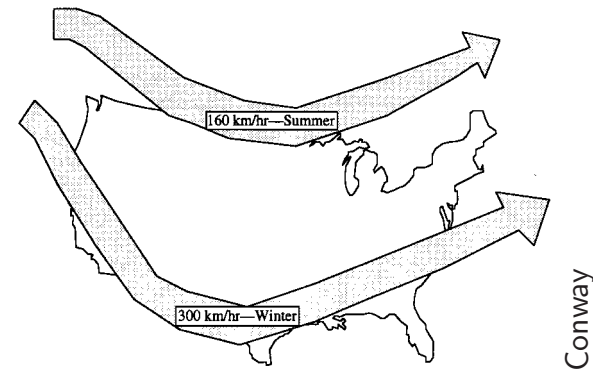
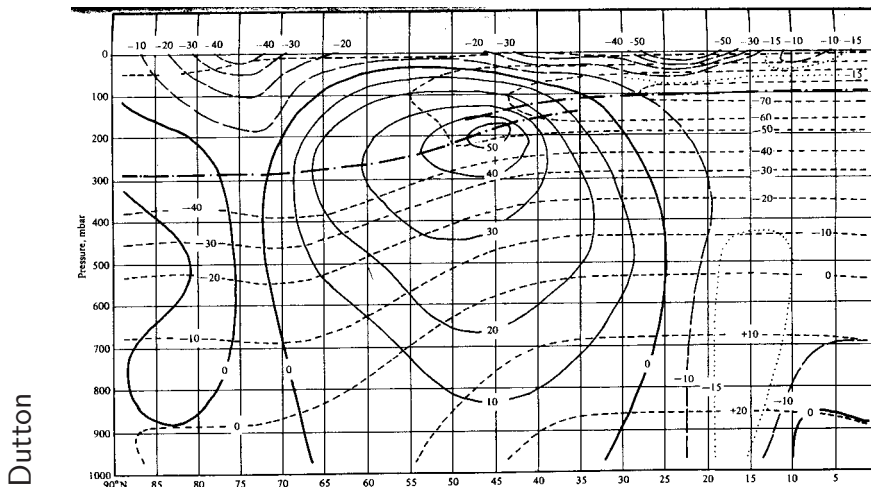


# Midlatitude Jetstream

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## Thermal Wind Balance

- ▷ pole-equator temperature gradient + Coriolis force → jetstream
- ▷ zonal (circumpolar) winds, maximal on tropopause
- ▷ seasonal N-S migration



- ▷ zonal jetstream in unstable → weather

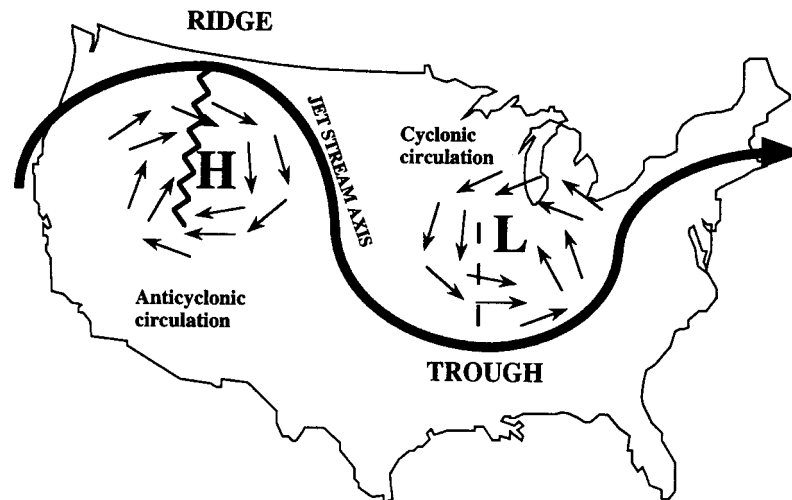
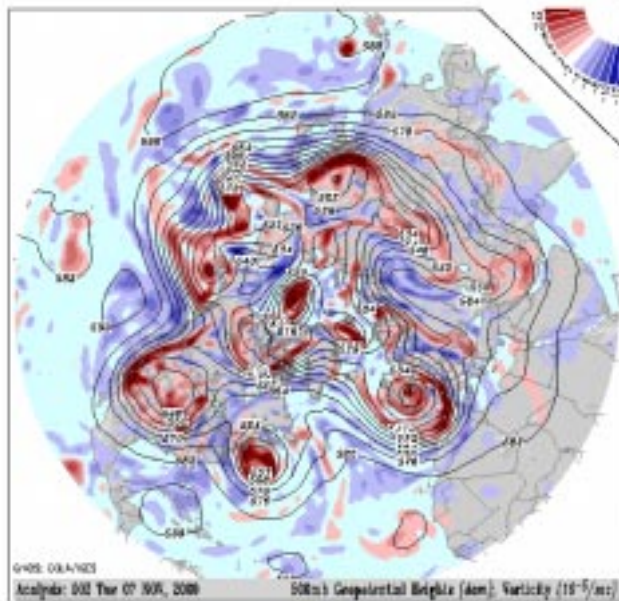
# Baroclinic Instability

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## Vortices in the Jetstream

- ▷ pressure variations + Coriolis force → vortical flows
- ▷ low pressures: **cyclones** (CCW in N hemisphere)
- ▷ high pressures: **anticyclones** (CW in N hemisphere)
- ▷ meandering of jetstream around vortices

<http://grads.iges.org/pix>



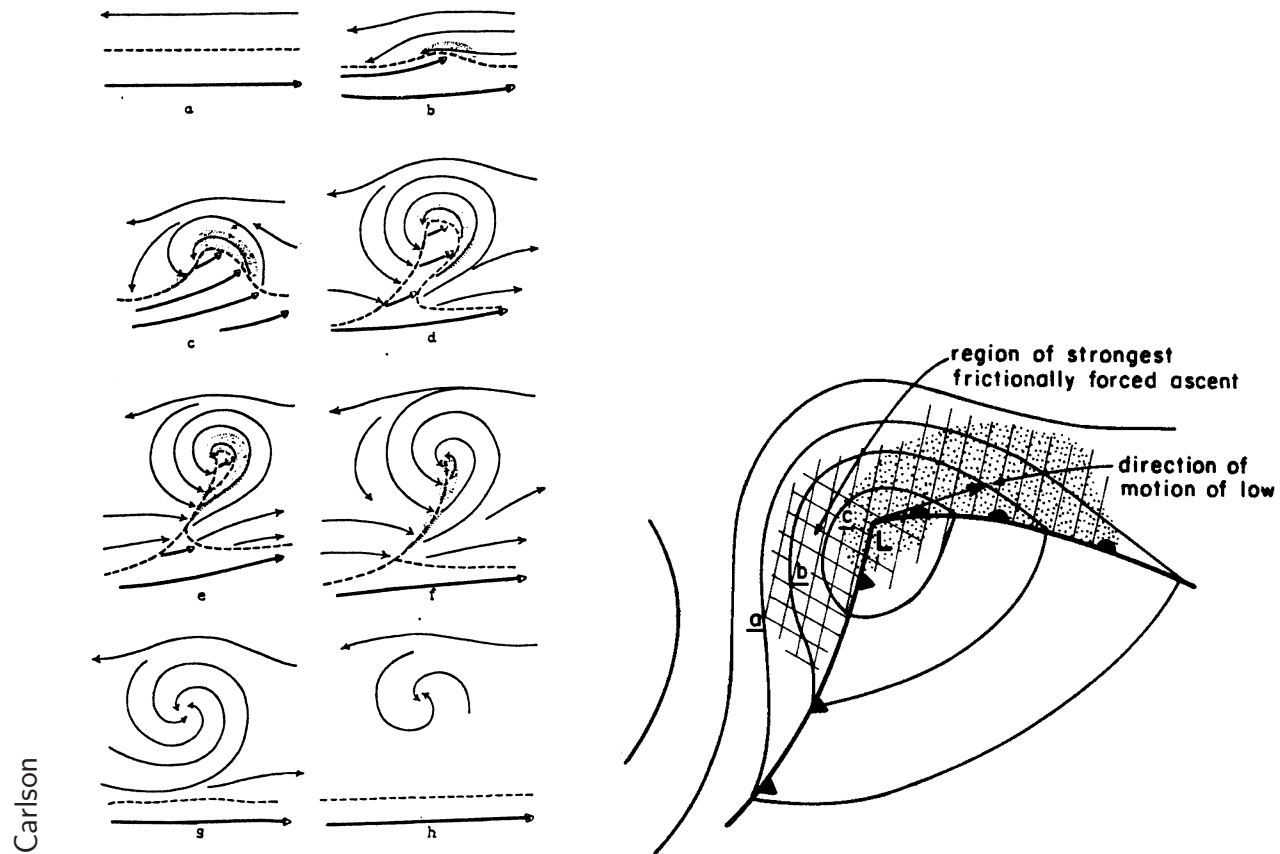
Conway

# Storm Development

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## Norwegian Cyclone Model

- ▷ Bjerknes & Solberg (1926): low pressure, cyclonic storm cells





# Prototypical Model Equations

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## Geophysical Fluid Dynamics

- ▷ Rossby number ( $\mathcal{R} \approx 0.1 - 1.0$ ), hydrostatic limit ( $\delta \rightarrow 0$ )

no divergence

$$u_x + v_y + \mathcal{R} w_z = 0$$

"time derivatives"

$$\mathcal{R} \{ u_t + u u_x + v u_y + \mathcal{R} w u_z \}$$

$$\mathcal{R} \{ v_t + u v_x + v v_y + \mathcal{R} w v_z \}$$

$$-\ v = -\phi_x$$

$$+ \ u = -\phi_y$$

$$\delta^2 \mathcal{R} \{ w_t + u w_x + v w_y + \mathcal{R} w w_z \}$$

$$-\ \theta = -\phi_z$$

$$\{ \theta_t + u \theta_x + v \theta_y + \mathcal{R} w \theta_z \} + w = 0$$

temperature advection      Coriolis force      pressure gradient      hydrostatic balance

# Symmetry-Breaking

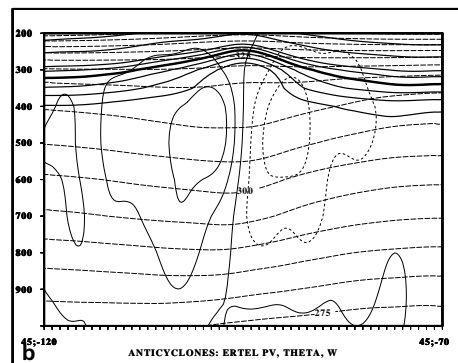
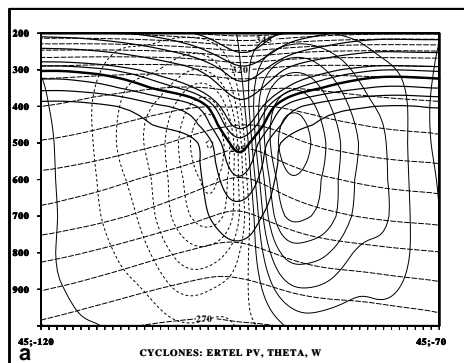
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## Cyclone/Anticyclone Asymmetry

- ▷ why do **low** pressure cells intensify preferentially over **high**s?
- ▷ textbook theory, *quasigeostrophy*, is symmetric ( $\mathcal{R}=0$ )
- ▷ new mathematics:  $QG^{+1}$ , small  $\mathcal{R}$  perturbation theory

## Tropopause Observations

- ▷ upper-level troughs as precursors to instability
- ▷ troughs from upper-level **cyclonic** vorticity, Sanders (1988)
- ▷ hypothesis: *is the tropopause an organizing level for vorticity?*



Hakim 2000

# Turbulent Organization of Vortices

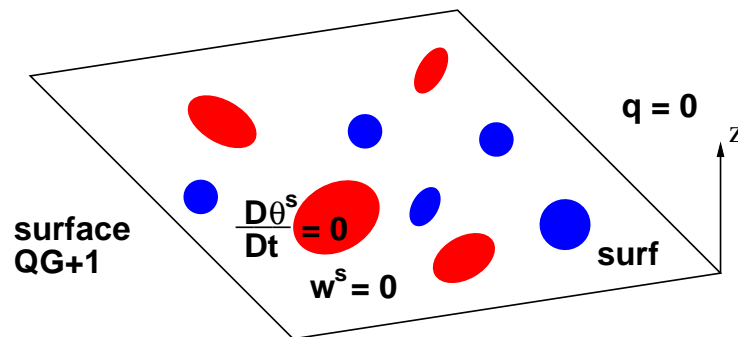
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“Evidently, the organization and growth of the system out of the small-scale chaos of the vorticity field is the most important process.”

Sanders (1988), on trough development

## Surface QG<sup>+1</sup> Dynamics

- ▷ new mathematics: sQG<sup>+1</sup> temperature dynamics on a rigid surface
- ▷ cyclonic bias from unbiased random vorticity
  - intense, localized cyclones vs weak, broad anticyclones
- ▷ prior models → weak anticyclonic bias at small Rossby numbers!

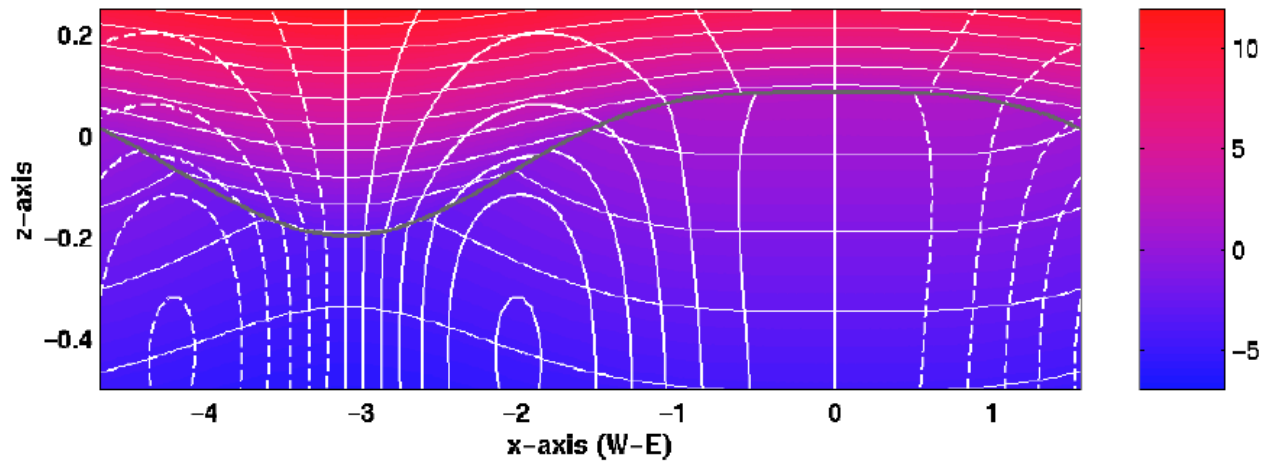
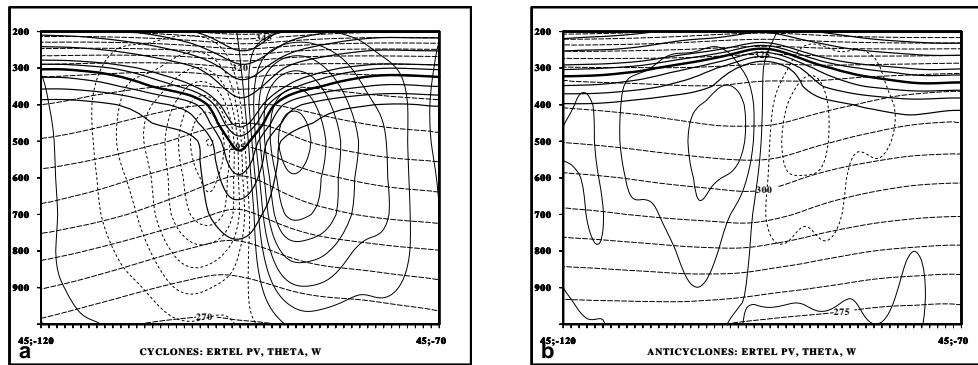


# Tropopause Wave

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## Observations vs Periodic Wave Solution

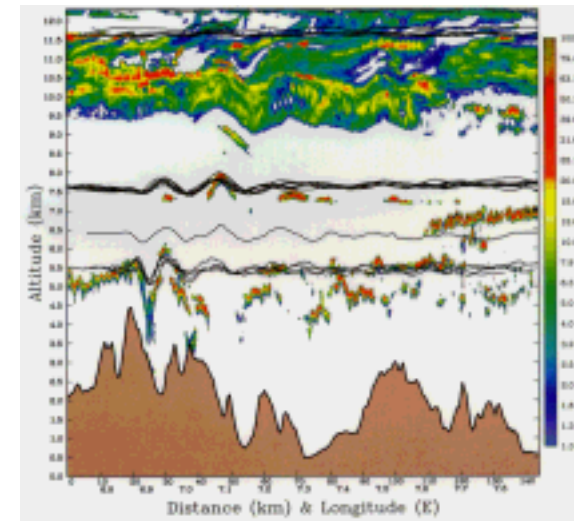
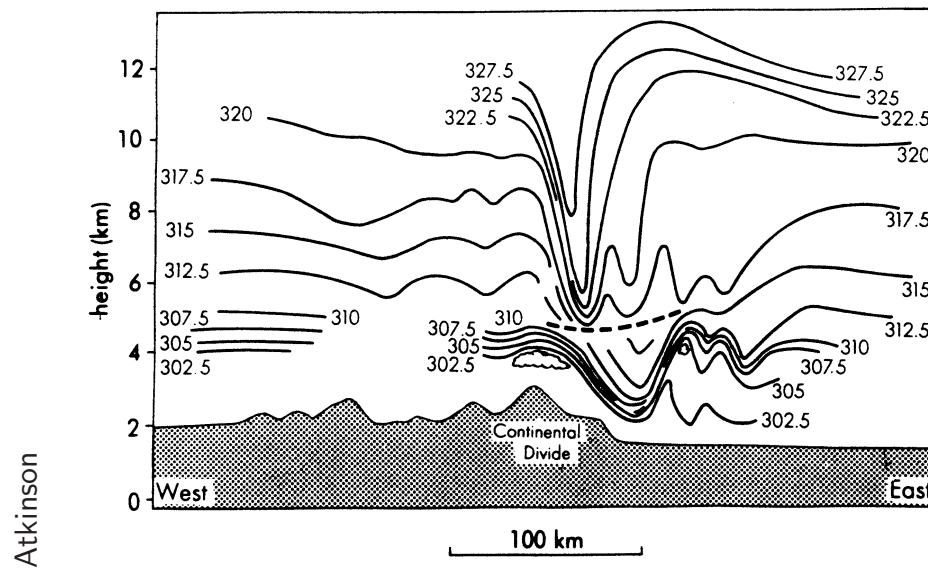
- ▷ new mathematics:  $sQG^{+1}$  tropopause model
- ▷ tropopause interface, potential temperature & vertical motion



# Airflow over Mountains

## Buoyancy Waves

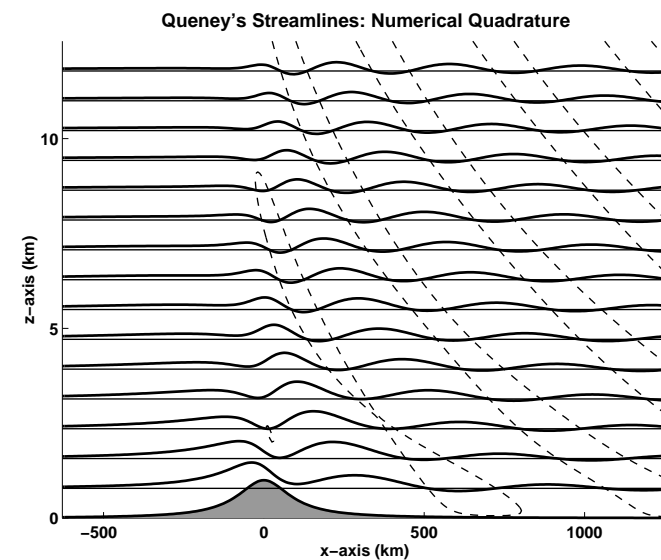
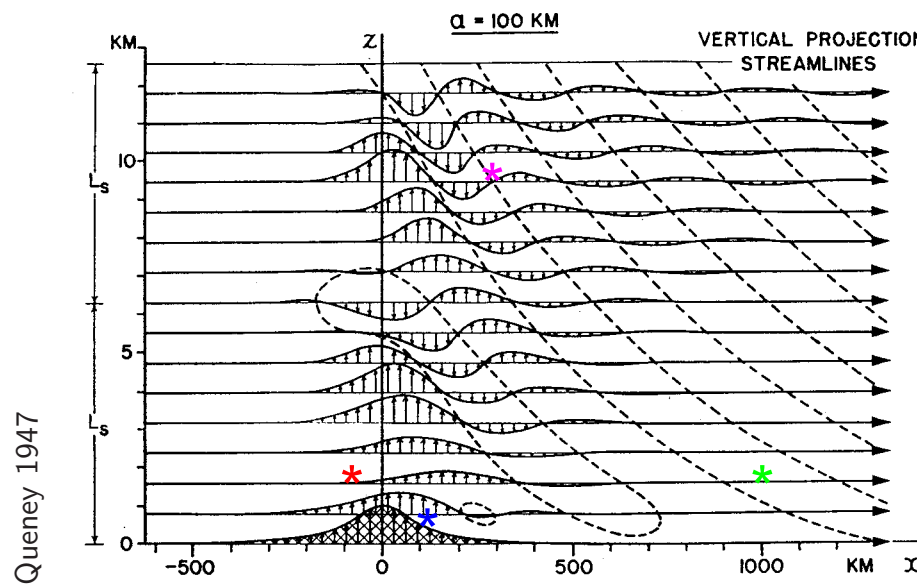
- ▷ wave response to dense air ascending over a mountain
- ▷ clear air turbulence over alpine regions (Colorado, the Alps)



# An Old Problem Revisited

## Flow over a Mesoscale Ridge

- ▷ original work: Queney 1947, *rotating* case ( $\mathcal{R}=1$ )
- ▷ new mathematics: efficient & accurate Fourier quadrature
- ▷ adaptable for 3D mountain flow computations



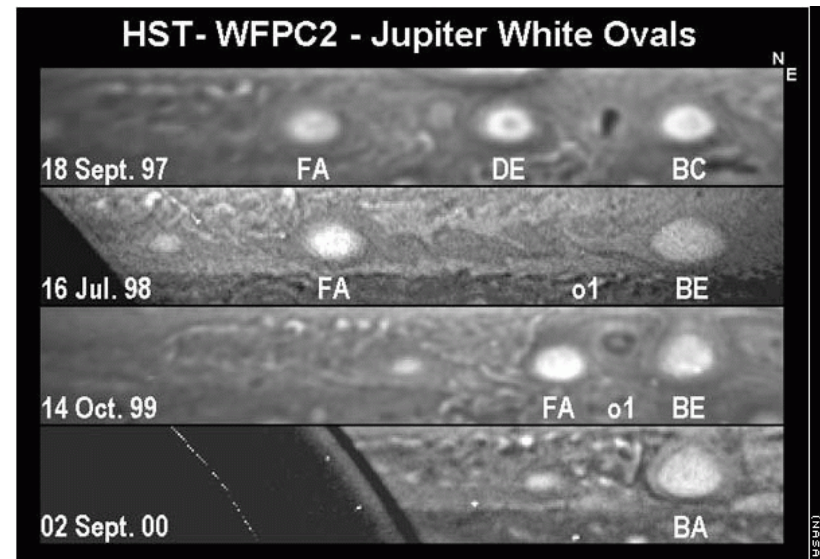
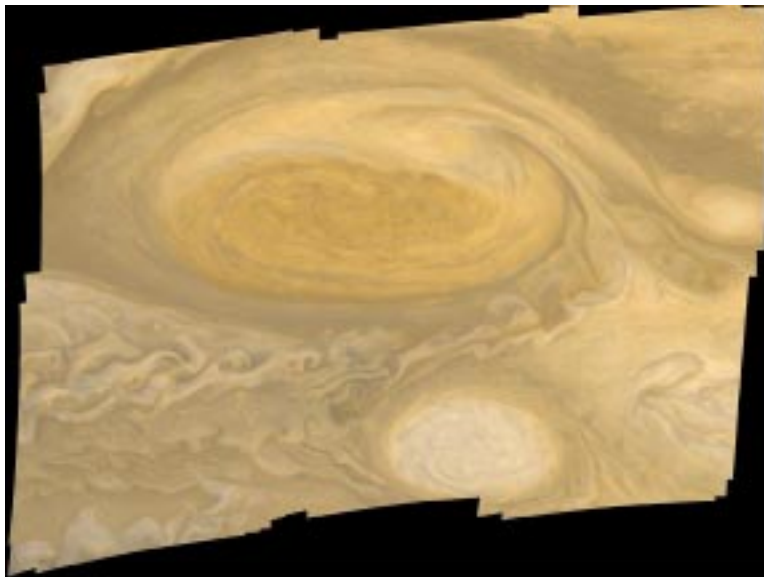
- ▷ new direction: nonlinear wave generation

# New Frontiers

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## Jovian Weather

- ▷ vorticity dynamics: filamentation & vortex merger



<http://pds.jpl.nasa.gov/planets/jpeg/jup/grsturb.jpg>

<http://www.cnn.com/2000/TECH/space/10/25/jupiter.pics>