Investigation #3 • APMA 935 • Wave Propagation

- please respect page limits.
- working group should present oral update 18 February.
- submit your final write-up Wednesday 25 February.

A) Dynamics of a Charged Particle (4 pages + plots) The electromagnetic force on a charged particle in the presence of electromagnetic fields is given by

$$\vec{F} = q\left(\vec{E} + \frac{\vec{v}}{c} \times \vec{B}\right)$$

where q is the charge of the particle and $\vec{v}$ its velocity. Consider the special case where the $\vec{E}$ and $\vec{B}$ fields are due to a simple electromagnetic wave. You may neglect the effect of the charge on the EM fields.

To investigate this problem, use exact, approximate or numerical techniques, or any combination thereof. As a place to start, you might consider neglecting the influence of the $\vec{B}$-field.

B) Refraction (4 pages + plots) Implement an ODE solver in matlab to numerically investigate the refraction of time-harmonic waves on a *variable-tension* string

$$\lambda(x)\,\eta_{tt} - \{\tau(x)\,\eta_x\}_x = 0\ .$$

Investigate cases where the mass and tension approach constants at $x \to \pm\infty$. Use combinations of hyperbolic tangents, $\tanh \alpha x$, to design your wavespeed profiles. The scattering problem has the asymptotic conditions

$$\eta = \begin{cases} \dfrac{1}{T}\, e^{i(k^- x - \omega t)} + \dfrac{R}{T}\, e^{i(k^- x + \omega t)} & \text{for } x \to -\infty \\ e^{i(k^+ x - \omega t)} & \text{for } x \to +\infty \end{cases} ,$$

so you may impose radiation IVs at a large, positive value of x, and solve the spatial ODE *backwards* to $x \to -\infty$. The reflection and transmission coefficients can then be obtained by solving a linear system at the incoming end.