Investigation #2 • APMA 935 • More from the Inverse Square Law

- please respect page limits.
- submit your write-up Wednesday 04 February.
- you are encouraged to use the webct discussion forum.
- refer to *Guidelines for Reports*.

A) Stability Timescales (5 pages + numerical summary page) First, nondimensionalize the restricted 3-body problem as presented in the lectures. You may do this part as a class group, since it will be advantageous for everyone to be working with the same equations for the stability computation (the group may submit one high-quality write-up). Please also agree on values of the physical constants required for the sun-earth dynamics.

Verify by computation, whether or not the in-plane instability timescales for the L1-3 Lagrange points correspond to the values as taken from the NASA webpage on the sun-earth system (L1 & L2 $\approx$ 23 days; L3 $\approx$ 150 years!?)[1]. Numerically, this will involve a zero finding procedure and an eigenvalue/vector calculation. (Note: it is possible that the fastest instability is associated with out-of-plane motion.)

B) Gravity of a Massive Shell (3 pages) Solve for the gravitational potential $\Phi(\vec{x})$ generated by a uniform surface distribution over a spherical shell having radius R with a total mass of $\mathcal{M}$. Give the corresponding force field. (Hint: how do you expect potential to behave at the centre of the spherical shell and as $|\vec{x}\,| \to \infty$?) Explain how the potential behaves when $|\vec{x}\,| = R$. Based on your results, make an educated guess as to a PDE problem statement which uniquely defines a gravitational potential for the general problem of an arbitrary surface distribution $\sigma(\vec{s})$ over an arbitrary shell $\mathcal{S}$.

[1] see both http://www.physics.montana.edu/faculty/cornish/lagrange.pdf and http://map.gsfc.nasa.gov/m_mm/ob_techorbit1.html