Investigation #1 • APMA 935 • Dynamics of the Inverse Square Law

- please respect page limits.
- submit your write-up Wednesday 21 January.
- you are encouraged to use the webct discussion forum.
- refer to *Guidelines for Reports*.

A) Newton's Falling Body Problem (3 pages + 3-5 plots) Construct a basic template script for numerically solving an ODE system, and producing solution plots and phase planes. As a first example, use the 1-D (dimensionless) gravity problem, and superimpose the numerically-generated solution trajectories onto the contours of the analytically-obtained first integral (*w01pplane.m*). Monitor the value of the first integral as one indicator of the error in the numerical solution — how is it affected by the specified tolerances? You are encouraged to incorporate features from existing ODE scripts that have been used in your other classes and the working group postings.

Use the *events* feature of Matlab's ODE solvers to implement a stopping criterion that permits the calculation of $\tau(s; s_0, s_0')$. Verify the results by integrating

$$s' \sqrt{\frac{s}{1+Cs}} = \pm 1$$

using a substitution like $Cs = \sinh^2 u$ or $Cs = \tan^2 u$. Plot the time it takes to reach the origin as a function of the initial height s_0 given that the fall begins from rest.

B) Electrostatic Repulsion (5 pages + plot) The electrostatic interaction between charged particles also involves an inverse square force law

$$F = \frac{\kappa\, q_1 q_2}{r^2}$$

where $\kappa = 1/4\pi\epsilon_0 \approx 8.9876 \times 10^9$ $\mathrm{Nm^2C^{-2}}$ is the Coulomb constant. The major difference between electrostatics and gravitation is that the charges q_1 and q_2 are signed quantities and allow for repulsive forces.

Consider two repelling charged particles (A and B) whose interaction is restricted to a line. Present the construction of a non-dimensional model which produces the ODEs

$$\begin{aligned} a'' &= -\frac{1}{2(a-b)^2} \\ b'' &= +\frac{K}{2(a-b)^2} \end{aligned}$$

where $a(t) < b(t)$ and K is a positive dimensionless constant. Consider the scattering problem where particle A is approaches particle B from the left with the asymptotic velocities

$$a' \to 1 \quad ; \quad b' \to 0 \quad \text{as} \quad t \to -\infty\ .$$

Construct formulas corresponding to the solution for which the time of closest approach occurs at $t = 0$. (Hint: the change of variables $b - a = (K+1)\cosh^2 u$ may be useful.) For this solution, the asymptotes of the particle positions before and after the interaction ($t = \pm\infty$) have the general form

$$a(t) = \alpha_0 t + \alpha_1 \ln t + \alpha_2 + o(1) \quad ; \quad b(t) = \beta_0 t + \beta_1 \ln t + \beta_2 + o(1)\ .$$

Find the constants for the asymptotes. For what value of K does particle A have an asymptotic velocity of zero after the interaction?