Math 495 • Special Topics in Applied Mathematics • Spring 2007

The Mathematical Origins of Patterns

Spatial patterns abound in nature. Principles of energy and entropy might suggest that the natural tendency of the universe is a steady decay to boring uniformity, yet we often find that Nature abhors the featureless background. Instead, Nature yields to a spontaneous appearance of spatial patterns.

This course will investigate mathematical models that use differential equations (ODEs & PDEs) to describe the variations in time and space that mathematically represent patterns. Diffusion and wave propagation are two familiar linear mechanisms involved in the dynamics of pattern development. Nonlinearity, however, is truly the main character in the pattern-formation story. Instability to bifurcations and resonance are two nonlinear mechanisms that are often associated with the initial appearance of spatial structure and temporal complexity. These ideas will be explored by numerical computation, and pursued mathematically through the development of further methods for solving differential equations.

The ultimate goal of the course is to demonstrate how mathematical models can mimick the formation of simple patterns like those seen in the physical world. Analyzing these models as differential equations leads to an understanding of the quantitative principles behind the evolution of spatial patterns.

The left image shows the spiral flow pattern of convection cells in a thin film of soap. The centre image shows concentric ring patterns of E. coli bacteria colonies. The right image shows nearly-regular stripes in drifting sand — the pattern defects are clearly evident.

Computer experiments will be an important accompaniment to the lectures and assigned work. The rudiments of numerical computing and graphics will be introduced through the use and modification of downloaded Matlab scripts.

This mathematics course will run in parallel with the physics course offered by Professor John Bechhoefer. This collaboration will involve some joint lectures and overlapping assignments with the Phys 484 class.

Course prerequisites: Math 310 (ODEs) & Math 314 (elementary PDEs), or equivalent. Interest in mathematical modelling of physics, and some basic experience with numerical computing.

Further information & updates: www.math.sfu.ca/∼muraki